## **Intelligent Digitally Redesigned Fuzzy Controller**

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#### Abstract

In this paper, we develop the intelligent digitally redesigned fuzzy controller for nonlinear systems. Takagi-Sugeno (TS) fuzzy model is used to model the nonlinear systems and a continuous-time fuzzy-model-based controller is designed based on the extended parallel-distributed-compensation(EPDC) method. The digital controllers are determined from existing analogue controllers. The proposed method provides an accurate and effective method for digital control of continuous-time nonlinear systems and enables us to efficiently implement a digital controller via the pre-determined continuous-time TS fuzzy-model-based controller. We have applied the proposed method to the duffing forced oscillation system to show the effectiveness and feasibility of the proposed method.

Key Words: Fuzzy control, pulse-width-modulation control, intelligent digital redesign, linear matrix inequality (LMI)

#### 1. Introduction

Fuzzy logic control is one of most useful approaches for the control of complex and ill-defined nonlinear systems. The main drawback of the fuzzy logic control is the empirical design procedures, which are based on trial-and-error process. Therefore, the recent trend in fuzzy logic control is to develop systematic method to design the fuzzy logic controller. The studies on the systematic design of fuzzy logic controller have largely been devoted two approaches: model-based control [1-8] and model-free control methods.

Since a continuous-time framework formulates the most practical systems and industrial control processes, it is natural to design a controller in the continuous-time domain. At the same time, we have been witnessed rapid development of flexible, low-cost microprocessors in the electronics fields. Therefore, it is desirable to implement the recent advanced controller in digital [9-12].

The PWM controller, which produces a series of discontinuous pulses with a fixed amplitude and variable width, has become popular in industry for on-off control of DC power converters and stepper motors (widely used in robotics), satellite station-keeping (with on-off rejection jets) etc. Since the conventional direct digital design approach takes into account only the sampling instants of the continuous-time system [13, 14], the resulting PWM controllers could produce degradation in the sample behavior of the closed-loop sampled-data system [15-17].

In this paper, we develop an intelligent digitally designed PWM fuzzy controller for continuous-time nonlinear systems. TS fuzzy model is adopted for this purpose since it is suitable for incorporating the conventional linear control theory. We first apply the digital redesign technique to each linear model.

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By the proposed method, the digital redesign method developed in linear system field will be then extended and applied to the control of nonlinear systems. The advanced fuzzy logic control theory can be finally implemented in the digital controller. Finally, we apply the proposed method to the duffing forced oscillation system to show the effectiveness and feasibility of the proposed method.

## 2. Fuzzy-Model-Based Controller

Consider a nonlinear dynamic system in the canonical form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}(t) \tag{1}$$

where the scalar  $\mathbf{x}^{(n)}$  is the output state variable of interest,  $\mathbf{x} = [x \ \dot{x} \ \dots \ x^{(n-1)}]^T$  is the state vector, and the vector  $\mathbf{u}$  is the system control input. In Eq. (1), the nonlinear function  $\mathbf{f}(\mathbf{x})$  is a known nonlinear continuous function of  $\mathbf{x}$ , and the control gain  $\mathbf{g}(\mathbf{x})$  is a known nonlinear continuous and locally invertible function of  $\mathbf{x}$ . The TS fuzzy model, which combines the fuzzy inference rule and the local linear state space model can approximate this nonlinear system [14, 18]. The *i*th rule of the TS fuzzy model in the continuous-time case is formulated in the following form:

Plant Rule i:

IF 
$$x(t)$$
 is  $F_1^i$  and ... and  $x^{(n-1)}(t)$  is  $F_n^i$   
THEN  $\dot{\mathbf{x}}(t) = \mathbf{A}_i \mathbf{x}(t) + \mathbf{B}_i u(t)$   $(i = 1, 2, ..., q)$  (2)

where,  $F_j^i(j=1,...,n)$  is the fuzzy set, Rule *i* denotes the *i*th fuzzy inference rule,  $\mathbf{x}(t) \in \mathbb{R}^n$  is the state vector,  $\mathbf{u}(t) \in \mathbb{R}^m$  is the control input vector,  $\mathbf{A}_i \in \mathbb{R}^{n \times n}$  and

 $\mathbf{B}_i \in R^{n \times m}$ , q is the number of fuzzy IF-THEN rules.

Using the center of gravity defuzzification, product inference, and singleton fuzzifier, the final output of the overall fuzzy system is given by

$$\dot{\mathbf{x}}(t) = \sum_{i=1}^{q} \mu_i(\mathbf{x}(t))(\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t))$$
(3a)

where.

$$w_{i}(\mathbf{x}(t)) = \prod_{j=1}^{n} F_{j}^{i}(\mathbf{x}^{(j-1)}(t)),$$

$$\mu_{i}(\mathbf{x}(t)) = \frac{w_{i}(\mathbf{x}(t))}{\sum_{i=1}^{q} w_{i}(\mathbf{x}(t))}$$
(3b)

In order to design a global controller for the TS fuzzy model Eq. (2) of the original nonlinear system Eq. (1), the extended parallel distributed compensation (EPDC) technique is adopted in this paper [19].

Using the same premise as Eq. (2), the EPDC fuzzy controller in continuous-time model has the following rules structure:

Controller Rule i:

IF 
$$x(t)$$
 is  $F_1^i$  and ... and  $x^{(n-1)}(t)$  is  $F_n^i$ ,  
THEN  $u(t) = -\mathbf{K}_i \mathbf{x}(t) + \mathbf{E}_i(t) \mathbf{r}(t)$   $(i = 1, 2, ..., q)$  (4)

where  $\mathbf{K}_i = [k_1^i, \dots, k_n^i]$  and  $\mathbf{E}_i = [e_1^i \dots e_n^i]$  are the feedback and feedforward gain vector in *i*th subspace, respectively.  $\mathbf{r}(t)$  is the reference input. The fuzzy controller is then represented by

$$\mathbf{u}(t) = \frac{\sum_{i=1}^{q} w_i(\mathbf{x}(t))(-\mathbf{K}_i \mathbf{x}(t) + \mathbf{E}_i \mathbf{r}(t))}{\sum_{i=1}^{q} w_i(\mathbf{x}(t))}$$
(5)

$$= -\sum_{i=1}^q \mu_i(\mathbf{x}(t)) \mathbf{K}_i \mathbf{x}(t) + \sum_{i=1}^q \mu_i(\mathbf{x}(t)) \mathbf{E}_i \mathbf{r}(t)$$

The overall closed-loop fuzzy system obtained by combining Eq. (2) and Eq. (5) becomes

$$\dot{\mathbf{x}} = \sum_{i=1}^{q} \sum_{j=1}^{q} \mu_i(\mathbf{x}) \mu_j(\mathbf{x}) ((\mathbf{A}_i - \mathbf{B}_i \mathbf{K}_j) \mathbf{x}(t) + \mathbf{B}_i \mathbf{E}_j \mathbf{r}(t))$$
(6)

# 3. Intelligent Digitally Redesigned Fuzzy Controller

In general, when applying the dual-rate sampling method to

a dynamic system, the fast sampling rate  $T_f$  is used for the system parameter identification without losing the information of the dynamic system, and the slow sampling rate  $T_s$  is used for the computation of advanced controllers in real time.

For the implementation of a digital control law, it needs to find a digital control law from the obtained optimal analogue control law. This can be carried out using the digital redesign technique. This technique matches the continuous-time closed-loop state  $\mathbf{x}_c(t)$  with the discrete-time closed-loop state  $\mathbf{x}_c(t)$  at  $t = kT_s$ , where  $T_s$  is the slow sampling time. We can expect that this method consider the system responses only at the slow-rate sampling time,  $t = k_sT_s$ . If the slow sampling time  $T_s$  is not sufficiently small, then the control law cannot capture the system's behavior during the slow sampling time  $T_s$ . Thus, it needs to consider the system's behavior during the slow-rate sampling and reflect it to the control law. We adopt a new digital redesign method that considers the inter-sampling behaviors [10].

Subscript *i* representing *i*th subspace is omitted to avoid the complexity. Consider a controllable and observable analogue plant represented by

$$\dot{\mathbf{x}}_{c}(t) = \mathbf{A}\mathbf{x}_{c}(t) + \mathbf{B}u_{c}(t), \quad \mathbf{x}_{c}(0) = \mathbf{x}_{c0} \tag{7}$$

$$\mathbf{y}_{c}(t) = \mathbf{C}\mathbf{x}_{c}(t) + \mathbf{D}u_{c}(t) \tag{8}$$

where  $\mathbf{x}_{c}(t) \in \mathbb{R}^{n \times 1}$ ,  $\mathbf{u}_{c}(t) \in \mathbb{R}^{m \times 1}$ .

The optimal state-feedback control law, which minimizes the performance index

$$J = \int_0^\infty e^{2at} \{ [\mathbf{x}_c(t) - \mathbf{r}(t)]^T \mathbf{Q} [\mathbf{x}_c(t) - \mathbf{r}(t)] + \mathbf{u}_c(t) \mathbf{R} \mathbf{u}_c(t) \} dt$$
(9)

with  $Q \ge 0, R > 0$ ,

$$u_c(t) = -\mathbf{K}_c \mathbf{x}_c(t) + \mathbf{E}_c \mathbf{r}(t)$$
(10)

where  $\mathbf{r}(\mathbf{t})$  is a reference input vector and

$$\mathbf{K}_c = \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} \tag{11}$$

$$\mathbf{E}_{c} = -\mathbf{R}^{-1}\mathbf{B}^{T}(\mathbf{A} - \mathbf{B}\mathbf{K}_{c})^{-T}\mathbf{Q}$$
 (12)

where P is the solution to

$$\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} - \mathbf{P} \mathbf{B} \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P} + \mathbf{Q} = \mathbf{0}$$
 (13)

The analogue control law  $\mathbf{u}_c(t)$  in Eq. (10) can be approximated as

$$\mathbf{u}_{c}(t) \cong W_{k_{f}} \Phi_{k_{f}}(t) = \mathbf{u}_{dk_{f}}(k_{f}T_{f})$$
for  $\mathbf{r}(t) = \mathbf{r}(k_{f}T_{f})$  with  $k_{f}T_{f} \leq t < k_{f}T_{f} + T_{f}$ , where,

$$W_{k_f} = \frac{1}{T_f} \int_{k_f T_f}^{K_f T_f + T_f} \mathbf{u}_c(t) dt$$

$$\approx \mathbf{u}_c(t) \Big|_{t = k_f T_f}$$
(15)

where  $\Phi_{k_f}(t)$  is orthonormal series and  $T_f (= T_s/N)$  is the fast sampling time, N is the number of fast sampling times during a slow sampling time  $T_s$ .

Consider an analogue system in Eq. (7) and its closed loop system represented by

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}_{C,L} \mathbf{x}_c(t) + \mathbf{B} \mathbf{E}_c \mathbf{r}(k_f T_f)$$
(16)

and its and discrete-time system represented by

$$\mathbf{x}_{c}(k_{f}T_{f} + T_{f}) = \mathbf{G}_{c C.L}\mathbf{x}_{c}(k_{f}T_{f}) + \mathbf{H}_{c C.L}\mathbf{E}_{c}\mathbf{r}(k_{f}T_{f})$$
(17)

where  $\mathbf{A}_{C,L} = \mathbf{A} - \mathbf{B}\mathbf{K}_{C}$ 

$$\mathbf{G}_{c\,C.L} = e^{\mathbf{A}_{C.L}T_f}$$
,  $\mathbf{H}_{c\,C.L} = [\mathbf{G}_{cC.L} - \mathbf{I}_n]\mathbf{A}_{C.L}^{-1}\mathbf{B}$ .

Then consider the digital system with a piecewise constant input  $\mathbf{u}_{dk_f}(k_f T_f)$  as

$$\mathbf{x}_d(K_f T_f + T_f) = \mathbf{G}_f \mathbf{x}_d(k_f T_f) + \mathbf{H}_f \mathbf{u}_{df}(k_f T_f)$$
 (18)

where 
$$\mathbf{G}_f = e^{\mathbf{A}T_f}$$
,  $\mathbf{H}_f = [\mathbf{G}_f - \mathbf{I}_n]\mathbf{A}^{-1}\mathbf{B}$ .

Let the desired digitally redesigned control law be

$$\mathbf{u}_{df}(k_f T_f) = -\mathbf{K}_{df} \mathbf{x}_d(k_f T_f) + \mathbf{E}_{df} \mathbf{r}(k_f T_f)$$
 (19)

Its closed loop digital system becomes

$$\mathbf{x}_{d}(k_{f}T_{f} + T_{f}) = \mathbf{G}_{f \ C.L}\mathbf{x}_{d}(k_{f}T_{f}) + \mathbf{H}_{f \ C.L}\mathbf{r}(k_{f}T_{f})$$
(20)

where 
$$\mathbf{G}_{f C.L} = \mathbf{G}_{f} - \mathbf{H}_{f} \mathbf{K}_{df}$$
,  
 $\mathbf{H}_{f C.L} = \mathbf{H}_{f} \mathbf{E}_{df}$ .

Matching the resulting system's state with Eq. (18), we have

$$\mathbf{K}_{df} = \mathbf{K}_{c} (\mathbf{A}_{C.L} \mathbf{T}_{f})^{-1} (\mathbf{G}_{cN} - \mathbf{I})$$

$$\mathbf{E}_{df} = [\mathbf{I} + (\mathbf{K}_{c} - \mathbf{K}_{df}) \mathbf{A}_{C.L}^{-1} \mathbf{B}] \mathbf{E}_{c}$$
(21)

To improve the performance during the slow-rate sampling time, it is desired to find the digitally redesigned control law that matches both the digital closed-loop state with the analogue closed-loop state, and has the slow sampling-rate time.

It is desired to find the digital control law  $\overline{\mathbf{u}}_{ds}$  with the dual-rate sampling that is inputted at the slow-rate sampling time. The slow sampled digital system with the digitally

redesigned fast sampling control law  $\overline{\mathbf{u}}_{ds}$  can be described as

$$\mathbf{x}_{d}(k_{s}T_{s} + T_{s}) = \mathbf{G}_{s}\mathbf{x}_{d}(k_{s}T_{s}) + \overline{\mathbf{H}}_{s}\overline{\mathbf{u}}_{ds}(k_{s}T_{s})$$

$$= \mathbf{G}_{s}\mathbf{x}_{d}(k_{s}T_{s}) + \sum_{i=1}^{N} \overline{\mathbf{H}}_{si}u_{dsi}(k_{s}T_{s} + (i-1)T_{f})$$
(22)

where

$$\mathbf{G}_{s} = (\mathbf{G}_{f})^{N} \tag{23}$$

$$\overline{\mathbf{H}}_{s} = [\overline{\mathbf{H}}_{s1} \quad \overline{\mathbf{H}}_{s2} \quad \cdots \quad \overline{\mathbf{H}}_{sN-1} \quad \overline{\mathbf{H}}_{sN}] 
= [\mathbf{G}_{f}^{N-1}\mathbf{H}_{f} \quad \mathbf{G}_{f}^{N-2}\mathbf{H}_{f} \quad \cdots \quad \mathbf{G}_{f}\mathbf{H}_{f} \quad \mathbf{H}_{f}]$$
(24)

$$\overline{\mathbf{u}}_{ds}(k_s T_s) = -\overline{\mathbf{K}}_{ds} \mathbf{x}_d(k_s T_s) + \overline{\mathbf{E}}_{ds} \mathbf{r}(k_s T_s)$$
 (25)

$$\overline{\mathbf{K}}_{ds} = \begin{bmatrix} \overline{\mathbf{K}}_{ds1}^T & \overline{\mathbf{K}}_{ds2}^T & \cdots & \overline{\mathbf{K}}_{dsN}^T \end{bmatrix}^T \\
= \begin{bmatrix} \mathbf{K}_{df}^T & (\mathbf{K}_{df} \mathbf{G}_{fC.L})^T & \cdots & (\mathbf{K}_{df} \mathbf{G}_{fC.L}^{N-1})^T \end{bmatrix}^T \tag{26}$$

$$\overline{\mathbf{E}}_{d}^{(N)} = \begin{bmatrix} \overline{\mathbf{E}}_{ds1}^{T} & \overline{\mathbf{E}}_{ds2}^{T} & \cdots & \overline{\mathbf{E}}_{dsN}^{T} \end{bmatrix}^{T} \\
= \begin{bmatrix} \mathbf{E}_{df}^{T} & (\mathbf{E}_{df} - \mathbf{K}_{df} \mathbf{H}_{dfC.L})^{T} & \cdots \\ \cdots & \left( \mathbf{E}_{df} - \sum_{j=1}^{N-1} \mathbf{K}_{df} \mathbf{G}_{fC.L}^{N-1-j} \mathbf{H}_{fC.L} \right)^{T} \end{bmatrix}^{T}$$
(27)

Let a PWM controlled sampled-data system be represented as

$$\dot{\mathbf{x}}_{w}(t) = \mathbf{A}\mathbf{x}_{w}(t) + \sum_{j=1}^{m} \mathbf{B}^{(j)} u_{w}^{(j)}(\lambda)$$
 (28a)

where the PWM mode is

$$\mathbf{u}_{dwi}^{(j)}(t) = \begin{cases} \mathbf{u}_{M}^{(j)}, & k_{s}T_{s} + (i-1)T_{f} \leq t < k_{s}T_{s} + (i-1)T_{f} + \tau_{i}^{(j)} \\ \mathbf{u}_{M}^{(j)}, & k_{s}T_{s} + (i-1)T_{f} + \tau_{i}^{(j)} \leq t < k_{s}T_{s} + (i-1)T_{f} + \tau_{i}^{(j)} + \delta_{i}^{(j)} \\ 0, & k_{s}T_{s} + (i-1)T_{f} + \tau_{i}^{(j)} + \delta_{i}^{(j)} \leq t < k_{s}T_{s} + iT_{f} \end{cases}$$

$$(28b)$$

for  $j=1,\,2,\,...,\,$  m. Here,  $\mathbf{u}_{M}^{(j)}$ ,  $\tau_{i}^{(j)}$ , and  $\boldsymbol{\delta}_{i}^{(j)}$  are the jth input's fixed pulse amplitude, firing delay and firing duration in the PWM mode at the (k-1)th sampling, respectively. The graphical illustration of PWM inputs is shown in Fig. 1. The values of  $\tau_{i}^{(j)}$  and  $\boldsymbol{\delta}_{i}^{(j)}$  can be determined as follows:

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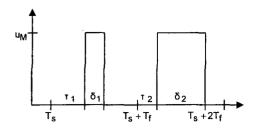


Fig. 1 PWM control inputs

The corresponding discrete-time model of the sampled-data system in Eq. (22) can be written as

$$\mathbf{x}_{d}(k_{s}T_{s} + T_{s}) = \mathbf{G}_{s}\mathbf{x}_{d}(k_{s}T_{s}) + \sum_{i=1}^{N} \sum_{j=1}^{m} \overline{\mathbf{H}}_{si}^{(j)} \overline{\mathbf{u}}_{dsi}^{(j)}(k_{s}T_{s})$$
(29)
$$\mathbf{x}_{d}(k_{s}T_{s} + T_{s})$$

$$= \mathbf{G}_{s}\mathbf{x}_{d}(k_{s}T_{s}) + \sum_{i=1}^{N} \sum_{j=1}^{m} \sum_{k_{s}T_{s} + (i-1)T_{f}}^{k_{s}T_{s} + iT_{f}} e^{\mathbf{A}(k_{s}T_{s} + T_{s} - \lambda)} \mathbf{B}^{(j)} \mathbf{u}_{dwi}^{(j)} d\lambda$$

$$= \mathbf{G}_{s}\mathbf{x}_{d}(k_{s}T_{s})$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{m} \sum_{k_{s}T_{s} + (i-1)T_{f} + \tau_{k_{s}}^{(j)} + \delta_{k_{s}i}^{(j)} e^{\mathbf{A}(k_{s}T_{s} + T_{s} - \lambda)} \mathbf{B}^{(j)} \mathbf{u}_{dwi}^{(j)} d\lambda$$

$$= \mathbf{G}_{s}\mathbf{x}_{d}(k_{s}T_{s}) + \sum_{i=1}^{N} \sum_{j=1}^{m} \overline{\mathbf{H}}_{swi}^{(j)} \mathbf{u}_{d}^{(j)}(k_{s}T_{s})$$

$$= \mathbf{G}_{s}\mathbf{x}_{d}(k_{s}T_{s}) + \sum_{i=1}^{N} \sum_{j=1}^{m} \overline{\mathbf{H}}_{swi}^{(j)} \mathbf{u}_{d}^{(j)}(k_{s}T_{s})$$

$$(30)$$

where,

$$\overline{\mathbf{H}}_{SWi}^{(j)} = (\mathbf{G}_f)^{N-i} [e^{\mathbf{A}(T_f - \tau_{k_Si}^{(j)} - \delta_{k_Si}^{(j)})}] [e^{\mathbf{A}(\delta_{k_Si}^{(j)} - \mathbf{I}_n)}] \mathbf{A}^{-1} \mathbf{B}^{(j)}.$$

To find the firing delay  $\tau_i^{(j)}$  and firing duration  $\delta_i^{(j)}$  with the fixed pulse amplitude  $\mathbf{u}_M^{(j)}$ , we match the state in Eq. (28a) with the state in Eq. (29), we have

$$\overline{\mathbf{H}}_{Si}^{(j)}\overline{\mathbf{u}}_{dSi} = \overline{\mathbf{H}}_{Swi}^{(j)}u_{M} . \tag{31}$$

Hence, we can obtain

$$\delta_i^{(j)} = T_f \frac{\overline{\mathbf{u}}_{dsi}^{(j)}(k_S T_S)}{u_M}$$

$$\tau_i^{(j)} = \frac{1}{2} (T_f - \delta_i^{(j)})$$
(32)

#### 4. Simulations

Let's consider the duffing forced oscillation system as follows:

$$\dot{x}_1 = x_2 
\dot{x}_2 = -0.1x_2 + x_1 - x_1^3 + 7.2\cos(t) + u(t)$$
(33)

Eq. (33) can be modeled by some operating points as follows:

Plant Rule:

Rule 1: IF  $x_1$  is about 0, THEN  $\dot{x} = A_1x + B_1u$ .

Rule 2: IF  $x_1$  is about  $\pm 1.5$ , THEN  $\dot{x} = A_2x + B_2u$ .

Rule 3: IF  $x_1$  is about  $\pm 3$ , THEN  $\dot{x} = A_3x + B_3u$ .

Rule 4: IF  $x_1$  is about  $\pm 4.5$ , THEN  $\dot{x} = A_4x + B_4u$ .

$$\mathbf{A}_{1} = \begin{bmatrix} 0 & 1 \\ 1 & -0.1 \end{bmatrix}, \quad \mathbf{B}_{1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ \mathbf{A}_{2} = \begin{bmatrix} 0 & 1 \\ -5.75 & -0.1 \end{bmatrix}, \quad \mathbf{B}_{2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ \mathbf{A}_{3} = \begin{bmatrix} 0 & 1 \\ -26 & -0.1 \end{bmatrix}, \quad \mathbf{B}_{3} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ \mathbf{A}_{4} = \begin{bmatrix} 0 & 1 \\ -59.75 & -0.1 \end{bmatrix}, \quad \mathbf{B}_{4} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

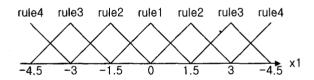


Fig. 2. Membership functions

We choose the slow-rate sampling period  $T_s$  as 0.04 and the fast-rate sampling period  $T_f$  as 0.01, thus  $N = T_s / T_f$  is 4. The initial conditions are  $\mathbf{x} = \begin{bmatrix} 2 & 2 \end{bmatrix}^T$ . In order to check the stability of the global fuzzy control system, based on LMI [11], we found the common positive definite matrix  $\mathbf{P}_c$  to be

$$\mathbf{P}_c = \begin{bmatrix} 1.5017 & 0.0048 \\ 0.0048 & 0.0147 \end{bmatrix}$$

The other conditions are also satisfied. Therefore, the overall continuous-time fuzzy system is stable in the sense of Lyapunov.

Three pictures of front, Fig. 3, Fig. 4 and Fig. 5, are results of the computer simulation. The solid and the dotted line types are resulted by digital controller and analogue controller, respectively. As seen in these figures, the proposed PWM controller is successful for digital control of nonlinear systems.

Moreover, it is important to emphasize that the stability is guaranteed not only for the TS fuzzy model but also for the original nonlinear systems.

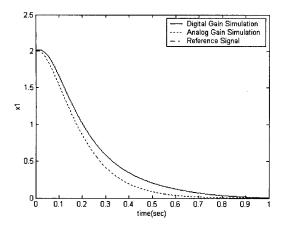


Fig. 3.  $\mathbf{x}_1(t)$  with PWM controller.

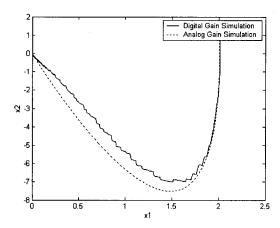


Fig. 4. Trajectories of  $\mathbf{x}_1(t)$  and  $\mathbf{x}_2(t)$ .

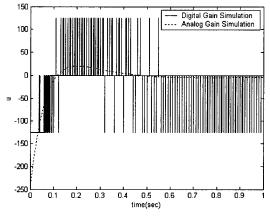


Fig. 5. Control input of the PWM controller.

### 5. Conclusion

In this paper, we proposed the intelligent digitally redesigned fuzzy controller design method for a nonlinear

system. We represent the nonlinear system as a TS fuzzy-model-based-system, and the EPDC technique is then utilized to design a fuzzy-model-based controller. In the step of analogue control law design, the optimal regional pole assignment technique is adopted and extended with some new stability conditions to construct multiple local linear systems and the LMI based stability analysis in to satisfy the stability of an overall system. The PWM digital redesign method is carried out to obtain the digital control law for control of each local analogue system, where a new state matching has been developed. We finally construct a global digitally redesigned fuzzy-model-based controller. The duffing forced oscillation system showed that the proposed intelligent digitally redesign method is very effective in controlling a nonlinear system with a satisfactory performance.

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