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선호강도를 고려한 그룹의사결정지원 앨고리듬*

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An Interactive Group Decision Support Procedure Considering Preference Strength*

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■ Abstract ■

This paper presents an interactive decision procedure to aggregate each group member's preferences when each group member articulates his or her preference information incompletely. An index, an indicative for the preference strength between alternatives, is derived to aid each decision maker to articulate preference information about alternatives. We develop a mathematical programming model that can establish dominance relations when the preference information about values of alternatives, attribute weights, and group member's importance weights are provided incompletely. Also, the preference relation between alternatives is to be considered in the model. Based on the preference strength measure and mathematical model, we develop an interactive group decision support procedure.

Keyword : Group Decision Support, Multi-Attribute Decision Making, Interactive Algorithm

1. Introduction

The decision making problem with multiple attributes consists of a multitude of subjects such as decision alternatives, criteria and preference information for consequences. Moving from single decision maker to multiple decision makers setting introduces a great deal of complexity into the analysis. Since each decision maker has different preference structure about

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alternatives, criteria and consequences, the aggregation methods are needed that can aggregate the different preference information of an individual decision maker.

The decision maker is willing or able to provide only incomplete information on parameters such as attribute weights and values of consequences. The reasons may be that (1) a decision should be made under time pressure and lack of knowledge or data, (2) many of attributes are intangible or non-monetary because they reflect social and environmental impacts, and (3) the decision maker has limited attention and information processing capabilities [1]. These may take the form of linear partial information such as rankings, interval descriptions, and so on.

Initially explored by Fishburn [2], there have been a number of studies [3-7] for the multi-attribute decision making with incomplete information. The multi-attribute decision making in single decision maker considers the incomplete information about two parameters, attribute weights and values of consequences. Although many researches which help decision making in multiple decision makers setting have been provided, there have been just a few studies considering incomplete information in group decision making problems. Anandaligam [8] presents that even without specifying exact preference weights for the attributes, dominance relationships can be established between alternatives, and Nash bargaining solution can be established among the affected states. Salo [9] develops an interactive approach for the aggregation of group member's preference judgments in the context of an evolving value representation. He puts an emphasis on interactive decision support in decision analytic techniques. Under the assumption that individual and group preferences for multi-attribute consequences fulfill the requirement imposed by additive representation and imprecise preference model, he suggests strong or weak dominance relations and in case of weak dominance, additional information on weights and values are elicited for further analysis. Kim and Ahn [10] suggest a procedure that takes account of individual decision maker's preference strength. The individual decision maker's preference strength obtained by solving a series of Linear Programming (LP) problem is used to form a group consensus. They consider the incomplete information about three parameters, attribute weight, value and group member's importance weight.

So far only a few studies have employed incomplete information in group setting. We consider the model in which three parameters (attribute weight, value and group member's importance weight) are articulated incompletely by each decision maker. Furthermore, this research suggests the preference strength measure that indicates the strength of the preference (dominance) between two alternatives. Similarly HOPIE by Weber [12] suggested the probabilistic measure in single decision maker setting by the assumption that a single value of the alternatives is a random variable within value interval between two alternatives. The measure may be helpful and dedicate information for each decision maker to provide preference information about alternatives.

Decision maker's articulating the preference relation between alternatives gives rise to a new mathematical programming model that is to be used for establishing dominance relation between alternatives. Since the model including a decision maker's preference information about alternatives is a non-linear programming model, an appropriate solution method is needed for establishing dominance. A solution method will be proposed that can guarantee the optimal solution of the original non-LP model.

Using the solution method, we will develop an interactive group decision making procedure considering the strength of the preference (dominance). Usually group decision making is not made successfully at a single trial, so it is necessary to feedback the aggregated results to each group member for a more informative group consensus. In the decision making procedure, we provide all decision makers with the measure (index) of dominance strength for more helpful decision support.

The rest of this paper is organized as follows. In section 2, we briefly describe basic definitions for multiple attribute group decision making problem with incomplete information and a basic solution technique for establishing dominance relation. In section 3, we derive an index for the dominance strength and provide two properties of the index. In section 4, we develop a mathematical programming model for our interactive procedure. In section 5, we summarize an interactive group decision procedure. An illustrative example is included in section 6, and the conclusion is included in section 7.

2. Definitions

2.1 Incomplete Information

Each group member can specify preference information about weights and values, etc. Among the types of information, we consider the incomplete information which can be classified as follows :

Form 1. $\{ w_i \geq w_j \}$

Form 2. $\{ w_i - w_j \geq \alpha_i \}$

Form 3. $\{ w_i \geq \alpha_i w_j \}$

Form 4. $\{ \alpha_i \leq w_i \leq \alpha_i + \varepsilon_i \}$

Form 5. $\{ w_i - w_j \geq w_k - w_l \}$ for $j \neq k \neq l$

where $\{ \alpha_i \}$ and $\{ \varepsilon_i \}$are non-negative constants.

A difficulty in taking the information of Forms 2-4 is to precisely justify their constants, since these forms contain numerical values such as α_i and ε_i [12]. Form 1 is widely used to construct ordinal ranking, because it is one of the most simple forms. Form 5 is a ranking of differences of adjacent parameters obtained by ranking between two parameters, which can be subsequently constructed based on Form 1.

2.2 Additive value function and basic solution technique

With the certainty, the group's value function over the consequence x can be represented by $v(x) = v_D(v_1(x), v_2(x), \cdots, v_K(x))$ where v_i, $i = 1, \cdots, K$ is respective individual's value function. This formulation has strong implicit assumption that the group's preferences for consequence x are entirely captured through the v_i's and individual i's preference structure is completely specified by v_i, for all i. Also, in order that the group value function is decomposed into the form of an additive value function, we need two assumptions : preferential independence and ordinal positive association. The former implies that for any two individuals i and j, $j \neq i$, if all other K-2 individuals are indifferent between a pair of consequences, then the preferences of the group for these consequences should be governed by the preferences of individuals *i* and *j*. The latter says that v_D is a positive monotonic function of each of its arguments, v_i's. Please refer to [13-15] for more details.

Under the above assumptions and another assumption that attributes of each decision maker are preferentially independent [13], the group's aggregated of alternative a_j under certainty is given as follows :

$$\phi(a_j) = \sum_{k=1}^{k} p^k \sum_{i \in I} w_i^k v_i^k(a_j) \qquad (2.1)$$

where

$I = \{i\}_{i=1,N}$: a set of index of N criteria

$A = \{a_j\}_{j=1,M}$: a set of M possible alternatives

K : the number of group members, $k = 1, \cdots, K$

w_j^k : a weight by group member k on attribute i

$v_j^k(a_j)$: a value of alternative j by group member k on the attribute i.

p^k : importance weight of group member k.

Group member's importance weight p^k means that all participants do not have equal expertise about problem domain [16].

The main purpose of decision making under multiple attributes is to calculate the aggregated value about each alternative, $\phi(a_j)$, and to specify dominance relationships between alternatives by comparing the magnitude of each value among alternatives. However, when group members give incomplete information about w_i^k, $v_i^k(a_j)$ and p^k, the group's consensus can be built by pairwise dominance relationship between alternatives. Now, we define two types of pairwise dominance, strict dominance and weak dominance.

Definition 1 : Strict dominance

If $\phi_{min}(a_j, a_l) \geq 0$ or $\phi_{max}(a_l, a_j) \leq 0$ in (2.2) and (2.3), then *strict dominance*, $a_j \underset{S}{>} a_l$, occurs, and an alternative a_j can be said to strictly dominate an alternative a_l.

$$\phi_{min}(a_j, a_l) =$$

$$\min \sum_{k=1}^{K} p^k \sum_{i \in I} w_i^k [\, v_i^k(a_j) - v_i^k(a_l)] \qquad (2.2)$$

$$s.t \quad P,\ W,\ V$$

$$\phi_{max}(a_l, a_j) =$$

$$\max \sum_{k=1}^{K} p^k \sum_{i \in I} w_i^k [\, v_i^k(a_l) - v_i^k(a_j)] \qquad (2.3)$$

$$s.t \quad P,\ W,\ V$$

where

W : constraints set of attribute weights

V : constraints set of vlaues

P : constraints set of group member's importance weights.

The simple meaning of strict dominance, $a_j \underset{S}{>} a_l$, is the value of exceeds that of a_l for all the feasible region of (2.2) or (2.3). Although this strict dominance is widely used in multi-attribute decision making problem with incomplete information, the strict dominance condition between alternatives rarely happens in most decisions with incomplete information. Hence, the following weak dominance was introduced and used for final decision making.

Definition 2 : Weak dominance

If $\phi_{min}(a_j, a_l) \geq \phi_{min}(a_l, a_j)$ or $\phi_{max}(a_j, a_l) \geq \phi_{max}(a_l, a_j)$ in (2.2) and (2.3), then *weak dominance*, $a_j \underset{W}{>} a_l$ occurs, and an alternative

a_j can be said to *weakly dominate* an alternative a_l.

Since $\phi_{\min}(a_j, a_l)$ is equal to $-\phi_{\max}(a_l, a_j)$, $\phi_{\min}(a_j, a_l) \geq \phi_{\min}(a_l, a_j)$ is converted equivalently as $\phi_{\max}(a_j, a_l) \geq \phi_{\max}(a_l, a_j)$. Simply, the worst (best) possible value for a_j is greater or equal than the worst (best) possible value for a_l, for the feasible region denoted by P, W and V. This is, selecting a_j yields less 'regret' than selecting a_l (see [17] for the proof). Additionally the weak dominance is always identified for two competing alternatives [12].

For establishing the strict/weak dominance, we have to solve the nonlinear model (2.2) and (2.3). Based on Kim and Ahn's research [10], the nonlinear model (2.2) and (2.3) can be solved as follows :

$$\phi_{\min}(a_j, a_l) = \min \sum_{k=1}^{k} p^k \underline{v}^k(a_j, a_l) \quad (2.4)$$
$$s.t \quad P$$

$$\phi_{\max}(a_l, a_j) = \max \sum_{k=1}^{k} p^k \overline{v}^k(a_l, a_j) \quad (2.5)$$
$$s.t \quad P$$

where

$\underline{v}^k(a_j, a_l) = \min \sum_{i \in I} w_i^k \underline{z}_i^k(a_j, a_l) \quad s.t\ W^k$

$\underline{z}_i^z(a_j, a_l) = \min[v_i^k(a_j) - v_i^k(a_l)] \quad s.t\ V_i^k$

$\overline{v}^k(a_l, a_j) = \max \sum_{i \in I} w_i^k z_i^k(a_l, a_j) \quad s.t\ W^k$

$\overline{z}_i^k(a_l, a_j) = \max[v_i^k(a_l) - v_i^k(a_j)] \quad s.t\ V_i^k$

$W^k = \Phi_w^k \cup \{\sum w_i^k = 1,\ w_i^k \geq 0\}$: the set of constraints or all possible values of group member k on the attribute weights, where Φ_w^k is the set obtained from DM k's information regarding the relative importance of attributes.

V_i^k : the set of constraints of group member k on the values, for the consequences when attribute i is given, $\{v_i^k(a_j), v_i^k(a_l)\} \in V_i^k$.

These linearization (2.4) and (2.5) not only require that value differences can be optimized separately for each criterion, but that minimization/maximization using the weights in the second step (i.e in the calculation of $\underline{v}^k(a_j, a_l)$, $\overline{v}^k(a_l, a_j)$) can be performed independently for each group member.

3. Development of the preference strength index

As mentioned in section 2, the strict dominance condition between alternatives rarely happens in most decisions with incomplete information, and the weak dominance is always identified for two competing alternatives. Hence, the weak dominance rule may be valuable to support a final decision, when the strict dominance of two competing alternatives is not established and the decision maker is not willing or able to give more information. However, providing the degree or strength when alternative a_j weakly dominate a_l is further expected for more helpful and delicate decision support.

We start with a value interval $[\phi_{\min}(a_j, a_l), \phi_{\max}(a_j, a_l)]$. That represents a range of all possible differences between overall evaluation scores of alternatives a_j and a_l for a feasible region denoted by P, W and V. The range of value interval can be represented as the denominator of (3.1.b). Also, if we divide the value interval into two range, $[0, \phi_{\max}(a_j, a_l)]$ and $[\phi_{\min}(a_j, a_l), 0]$, then the absolute value of the first range means the degree that a_j is more preferred than a_l. Similarly, the absolute value of the second range means the degree that a_l is more preferred than a_j. Now, we can say the ratio, $\phi_{\max}(a_j, a_l) / [\phi_{\max}(a_j, a_l)]$, means that the overall evaluation of a_j is greater or equal than that of a_l.

$$Index(a_j, a_l) = \begin{cases} 1, \text{ if } \phi_{\min}(a_j, a_l) \geq 0 & (3.1.a) \\ \dfrac{\phi_{\max}(a_j, a_l)}{\phi_{\max}(a_j, a_l) - \phi(a_j, a_l)}, \\ \quad \text{if } \phi_{\max}(a_j, a_l) > 0 \text{ and } \phi_{\min}(a_j, a_l) < 0 & (3.1.b) \\ 0, \text{ if } \phi_{\max}(a_j, a_l) \leq 0 & (3.1.c) \end{cases}$$

Based on the above mentioned concept, $Index(a_j, a_l)$ in (3.1) can be used as an interesting measure which represents the preference (dominance) strength indicative between two alternatives. In the case of (3.1.*a*), by the definition of strict dominance a_j strictly dominates a_l. Similarly, in the case of (3.1.*c*), a_l strictly dominates a_j. Dislike the above two cases, (3.1.*a*) and (3.1.*c*), formula (3.1.*b*) has the following two properties.

Property 1 : The value of index (3.1.b) is greater than 0 and less than 1,
$0 < Index(a_j, a_l) < 1$.

Proof : Since $-1 < \phi_{\min}(a_j, a_l) < 0$ and in (3.1.*b*), we have $0 < \phi_{\max}(a_j, a_l) - \phi_{\min}(a_j, a_l) < 2$ and $0 < \dfrac{\phi_{\max}(a_j, a_l)}{\phi_{\max}(a_j, a_l) - \phi_{\min}(a_j, a_l)} < 1$. Hence, the value of $Index(a_j, a_l)$ can be from 0 to 1, $0 < Index(a_j, a_l) < 1$.

Property 2 : In case (3.1.*b*), If $Index(a_j, a_l) > 0.5$, then a_j weakly dominates a_l.

Proof : $Index(a_j, a_l) > 0.5$ means

$$\frac{\phi_{\max}(a_j, a_l)}{\phi_{\max}(a_j, a_l) - \phi_{\min}(a_j, a_l)} > 0.5 \quad (3.2)$$

Firstly doubling the two sides of the expression, (3.2), and transposing the denominator of the expression and arranging each term, the following yields :

$$\phi_{\max}(a_j, a_l) > -\phi_{\min}(a_j, a_l). \quad (3.3)$$

Since $\phi_{\min}(a_j, a_l) = -\phi_{\max}(a_l, a_j)$, (3.3) is transformed into as following :

$$\phi_{\max}(a_j, a_l) > \phi_{\max}(a_l, a_j). \quad (3.4)$$

Now, by the definition of weak dominance, a_j weakly dominates a_l.

Before demonstrating the usage of the index, we note that the indexes for all ordered pairs of alternatives have to be calculated and compared for decision analyst's correctly using of the index. Since the value interval is not symmetric for alternatives pair, the values of two indexes, $Index(a_j, a_l)$ and $Index(a_l, a_j)$, are not same. This is different from deciding strict or weak dominance relationships between alternatives.

We now demonstrate with an example how the measure could be used to aid decision. For example, consider three alternatives, a_1, a_2, a_3 and the value intervals between alternatives as follows :

$$[\phi_{\min}(a_1, a_2), \phi_{\max}(a_1, a_2)] = [-0.211, 0.764]$$
$$[\phi_{\min}(a_1, a_3), \phi_{\max}(a_1, a_3)] = [-0.465, 0.501]$$
$$[\phi_{\min}(a_2, a_3), \phi_{\max}(a_2, a_3)] = [-0.684, 0.276]$$

From the value intervals, let us further compute the strength measure, then the following indexes are obtained :

$$Index(a_1, a_2) = 0.784$$
$$Index(a_3, a_2) = 0.712$$
$$Index(a_1, a_3) = 0.519$$
$$Index(a_3, a_1) = 0.418$$
$$Index(a_2, a_3) = 0.288$$
$$Index(a_2, a_1) = 0.216$$

With the information, each member can find the strengths of the weak dominance. These strengths between the alternatives are different. A confidence alternative a_1 dominate a_2 is relatively high as about 0.784, but confidence a_3 dominate a_2 is low as about 0.712. Each group member can decide which alternative is more attractive to himself from the indexes of dominance strength. For example, if one member of the group is provided with the above results, three indexes, and then he can conclude that is more a_1 preferred than a_2, $a_1 \succ a_2$. In this case, we can use the preference relation, $a_1 \succ a_2$, which is provided from the individual decision maker. The preference relation, $a_1 \succ a_2$, can be said that $\phi^k(a_1) \geq \phi^k(a_2)$, where $\phi^k(a_j) = \sum_{i \in I} w_i^k v_i^k(a_j)$.

From such a point of view, the preference strength measure may be valuable in individual decision making. Additionally, the measure makes it possible that we can obtain additional information from each decision maker in group decision making. Each group member may have a different confidence about the strength of dominance. Hence, when the measure is provided for each group member, various results may happen as follows :

Case 1 : Some or all members provide that alternative a_x is more preferred than alternative a_y from the measure (index). We can obtain the preference relation between alternative a_x and a_y, $\phi^k(a_x) \geq \phi^k(a_y)$, $x, y \in \Theta^k$ where Θ^k is the set of alternative pairs that is assessed from the kth group member.

Case 2 : No preference relation is provided by all members from the measure (index).

4. Model for establishing dominance relation

In this section, we propose a mathematical programming model that can be used to establish dominance relations in Case 1. In Case 2, each group member is requested to provide further information about attribute weights and values.

Firstly, we propose a mathematical model that can be used in Case 1. In Case 1, the preference relations from a group member, $\phi^k(a_x) \geq \phi^k(a_y)$ $x, y \in \Theta^k$, mean

$$\sum_{i \in I} w_i^k v_i^k(a_x) - \sum_{i \in I} w_i^k v_i^k(a_y) \geq 0 \qquad (4.1)$$
$$x, y \in \Theta^k$$

When (4.1) is added to the constraints set of the kth group member, mathematical formulation of the kth group member for establishing dominance between alternatives a_x and a_y is given as the following :

$$\underline{v}^k(a_x, a_y) =$$

$$\min \sum_{i\in I} w_i^k v_i^k(a_j) - \sum_{i\in I} w_i^k v_i^k(a_l) \qquad (4.2)$$

$$s.t \ \sum_{i\in I} w_i^k v_i^k(a_x) - \sum_{i\in I} w_i^k v_i^k(a_y) \geq 0,$$

$$x, y \in \Theta^k$$

$$W^k = \{w^k \in R^N \mid Aw^k \leq b\}$$

$$V_i^k = \{w_i^k \in R^M \mid D_i v_i^k \leq c_i\}, \quad i \in I$$

where $w^k = (w_1^k, w_2^k, \cdots w_1^k, \cdots, w_N^k)^T$, $v_i^k = (v_i^k(a_1), v_i^k(a_2), \cdots, v_i^k(a_g), \cdots, v_i^k(a_M))^T$, A is a $s \times N$ matrix and b is a $s \times 1$ matrix (s is the number of constraints in the set W^k), D_i is a $q_i \times M$ matrix and c_i is a $q_i \times 1$ matrix (q_i is the number of constraints in the set V_i^k).

Observe that the model (4.2) is a non-LP within which exist the product forms of the weights and utility values which are partially known. Consequently, the model (4.2) is not easily solved, hence an appropriate solution method is needed.

Theorem 1 : If the incomplete information about the utility values for each attribute are functionally independent, which is formally denoted by a notation, $V_i \perp V_j$, $i, j \in I$, $i \neq j$, then the non-LP model (4.2), can be converted into the LP as in (4.3) without changing the optimal solution of (4.2).

Proof : Put $X_i^k(a_g) = w_i^k v_i^k(a_g)$, and the value of the best consequence on the ith attribute equals one, $v_i^k(a^{*i}) = 1$, then the attribute weights of group member k will be, $w_i^k = X_1^k(a^{*1})$, $X_2^k(a^{*2})$, $w^k = (X_i^k(a^{*2}), \cdots X_i^k(a^{*i}), \cdots X_N^k(a^{*N}))^T$. And then, by the assumption of functional independence, it follows that the denominators of constraints in each V_i^k will be equal,

$$v_i^k(a_g) = \frac{X_i^k(a_g)}{w_i^k} = \frac{X_i^k(a_g)}{X_i^k(a^{*i})},$$

$$v_i^k = \left(\frac{X_i^k(a_1)}{X_i^k(a^{*i})}, \frac{X_i^k(a_2)}{X_i^k(a^{*i})}, \cdots, \frac{X_i^k(a_g)}{X_i^k(a^{*i})}, \cdots, \frac{X_i^k(a_M)}{X_i^k(a^{*i})} \right),$$

The multiplication of the entire constraints in each V_i^k by $X_i^k(a^{*i})$ will lead to linear constraint. In conclusion, the non-linear model (4.2) can be converted into the following LP model.

$$\underline{v}^k(a_x, a_y) =$$

$$\min \sum_{i\in I} X_i^k(a_j) \sum_{i\in I} X_i^k(a_l) \qquad (4.3)$$

$$s.t \ \sum_{i\in I} X_i^k(a_x) - \sum_{i\in I} X_i^k(a_y) \geq 0 \ , x, y \in \Theta^k$$

$$W^k = \{\pi^k \in R^N \mid A\pi^k \leq b\}$$

$$V^k = \{\mu_i^k \in R^M \mid D_i \mu_i^k \leq X_i^k(a^{*i}) \cdot c_i\} \ , i \in I$$

where $\pi^k = (X_1^k(a^{*1}), X_2^k(a^{*2}), \cdots X_i^k(a^{*i}), \cdots X_N^k(a^{*N}))^T$

$\mu_i^k = (X_i^k(a_1), X_i^k(a_2), \cdots, X_i^k(a_g), \cdots, X_i^k(a_M))^T$

We can aggregate the individual decision results, $\underline{v}^k(a_x, a_y)$ and $\bar{v}^k(a_x, a_y)$, by (2.4) and (2.5). Then, we can establish value intervals and dominance relations.

Note 1 : Because adding a new constraint reduces or does not change the feasible space in mathematical programming model, adding (4.1) to the previous constraints set does not decrease (increase) the previously determined $\phi_{\min}(a_j, a_l)$ ($\phi_{\max}(a_j, a_l)$). That is, once the strict dominance was established between alternatives, ($\phi_{\min}(a_j, a_l)$ > 0, in the previous steps, then the previously determined strict dominance relations are maintained in the following steps.

Note 2 : It is also possible that some members of the group provide the information that an alternative a_x is indifferent to an alternative a_y from the preference strength index. In this case, the model (4.3) can be formulated with constraints including equality (=).

In Case 2, each group member should provide additional information about weights, w_i^k,or values, $v_i^k(a_j)$. However, it is not always possible for an individual decision maker to provide additional information. Accordingly, we divide Case 2 into two subcases given as follows :

Subcase 1 : There is no information from all members of group.

Subcase 2 : There are additional information about values and weights from some members of group.

In Subcase 1, we use weak dominance for final decision making. In Subcase 2, we compute the value intervals for each pair of alternatives, including the additional information about weight, w_i^k, or values, $v_i^k(a_j)$. Then, we establish the dominance relations and compute the preference strength index.

5. An interactive procedure for group decision making

Based on the preference strength measure and the mathematical programming model in section 3 and 4, we summarize the procedure for group decision making. The procedure is composed of the following steps.

Step 0 : Assess the information about values and weights from the decision makers.

Step 1 : Compute the value interval, $[\phi_{\min}(a_j, a_l), \phi_{\max}(a_j, a_l)]$, for each pair of alternatives.

Step 2 : Construct the strict dominance set, Ω, using the strict dominance relation.

Situation 1 : When complete ranking of all alternatives is required : If the strict dominance relations are established for all pair of alternatives, then stop the procedure. Otherwise, go to step 3.

Situation 2 : When only one best alternative is required : If an alternative a_x is such that $(a_x, a_i) \in \Omega$ and $(a_i, a_x) \in \Omega \forall a_i \neq a_x$, then a_x is the most preferred alternative and stop the procedure. Otherwise, go to step 3.

Step 3 : Compute the preference strength indexes and show the indexes to all decision makers.

Step 4 : Assess the information from the decision makers.
If there is additional information about values or weights or alternatives, then go to Step 1. Otherwise, make a final decision using weak dominance.

The LP models (2.4) and (2.5) can be used to compute the value intervals in Step 1 of the first iteration of the procedure. Once preference information about alternatives is assessed from decision makers, only the LP model (4.3) can be used for computing the value intervals. If the preference information about some pairs of alternatives are assessed in Step 0, then we may use model (4.3).

In Step 3, the preference strength indexes can be computed using (3.1). Since the preference strength indexes must be one for the pair of alternatives which are included in Ω, we may have to compute the indexes for the pairs of alternatives which are not included in Ω.

As we pointed out in Note 1, the strict dominance relations determined previously are maintained in the following steps or iterations. However, adding new constraints like (4.1) may incur an inconsistent feasible region. For example, when the dominance relations of an individual decision maker are obtained as $\{a_1 > a_2,\ a_1 > a_4\}$ and the group's value interval is obtained as [-0.52, 0.15], if the decision maker articulates $a_2 > a_1$ from the preference strength index in Step 4, then the decision maker's feasible region of (4.3) becomes inconsistent. Hence, we must check the inconsistent preference information about the alternatives. We have to construct the dominance set of each decision maker, Ω^k, and then check the inconsistency between Ω^k and preference information about alternative pairs articulated from the preference strength indexes.

6. An illustrative example

This section illustrates features of the interactive decision procedure in the context of an international supplier selection problem[18, 19]. Suppose three departments (decision makers) of a company are considering 5 suppliers (alternatives) denoted by a_1, a_2, a_2, a_4, a_5. Suppliers are evaluated by 3 criteria : 1) product quality, 2) financial term, and 3) information system capability.

The incomplete information about the values articulated from the three departments are shown in the following <Table 1> and <Table 2>. Furthermore, the incomplete information about the attribute weights articulated from the departments are summarized in <Table 3> and group member's importance weights are given by following inequalities,

〈Table 1〉 Incomplete information about values : Form 1

DMs	Attribute	Incomplete information about values (Form 1.)
DM 1	product quality	$1 = v_1^1(a_1) \ge v_1^1(a_2) \ge v_1^1(a_3) \ge v_1^1(a_4) \ge v_1^1(a_5) = 0$
	financial term	$1 = v_2^1(a_2) \ge v_2^1(a_1) = v_2^1(a_4) \ge v_2^1(a_3) \ge v_2^1(a_5) = 0$
	IS capability	$1 = v_3^1(a_3) \ge v_3^1(a_1) \ge v_3^1(a_2) = v_3^1(a_5) \ge v_3^1(a_4) = 0$
DM 2	product quality	$1 = v_1^2(a_1) \ge v_1^2(a_2) \ge v_1^2(a_3) \ge v_1^2(a_4) \ge v_1^2(a_5) = 0$
	financial term	$1 = v_2^2(a_3) \ge v_2^2(a_1) = v_2^2(a_5) \ge v_2^2(a_2) \ge v_2^2(a_4) = 0$
	IS capability	$1 = v_3^2(a_2) \ge v_3^2(a_1) = v_3^2(a_4) \ge v_3^2(a_3) \ge v_3^2(a_5) = 0$
DM 3	product quality	$1 = v_1^3(a_1) \ge v_1^3(a_2) \ge v_1^3(a_3) \ge v_1^3(a_4) \ge v_1^3(a_5) = 0$
	financial term	$1 = v_2^3(a_3) \ge v_2^3(a_5) = v_2^3(a_4) \ge v_2^3(a_2) \ge v_2^3(a_1) = 0$
	IS capability	$1 = v_3^3(a_4) \ge v_3^3(a_5) \ge v_3^3(a_1) \ge v_3^3(a_3) \ge v_3^3(a_2) = 0$

〈Table 2〉 Incomplete information about values : Form 5

DMs	Attribute	Incomplete information about values (Form 5.)
	product quality	$v_1^1(a_1)-v_1^1(a_2) \geq v_1^1(a_2)-v_1^1(a_3) \geq v_1^1(a_3)-v_1^1(a_4) \geq v_1^1(a_4)-v_1^1(a_5)$
DM 1	financial term	$v_2^1(a_2)-v_2^1(a_1) \leq v_2^1(a_1)-v_2^1(a_3)$ $v_2^1(a_1)-v_2^1(a_3) \geq v_2^1(a_3)-v_2^1(a_5)$
	IS capability	$v_3^1(a_3)-v_3^1(a_1) \leq v_3^1(a_1)-v_3^1(a_2)$ $v_3^1(a_1)-v_3^1(a_2) \leq v_3^1(a_2)-v_3^1(a_4)$
	product quality	$v_1^2(a_1)-v_1^2(a_2) \geq v_1^2(a_2)-v_1^2(a_3) \geq v_1^2(a_3)-v_1^2(a_4) \geq v_1^2(a_4)-v_1^2(a_5)$
DM 2	financial term	$v_2^2(a_2)-v_2^2(a_4) \geq v_2^2(a_1)-v_2^2(a_2) \geq v_2^2(a_3)-v_2^2(a_1)$
	IS capability	$v_3^2(a_2)-v_3^2(a_1) \geq v_3^2(a_1)-v_3^2(a_3)$ $v_3^2(a_1)-v_3^2(a_3) \geq v_3^2(a_3)-v_3^2(a_5)$
	product quality	$v_1^3(a_1)-v_1^3(a_2) \geq v_1^3(a_2)-v_1^3(a_3) \geq v_1^3(a_3)-v_1^3(a_4) \geq v_1^3(a_4)-v_1^3(a_5)$
DM 3	financial term	$v_2^3(a_3)-v_2^3(a_5) \geq v_2^3(a_5)-v_2^3(a_2) \geq v_2^3(a_2)-v_2^3(a_1)$
	IS capability	$v_3^3(a_4)-v_3^3(a_5) \geq v_3^3(a_5)-v_3^3(a_1)$ $v_3^3(a_3)-v_3^3(a_2) \geq v_3^3(a_1)-v_3^3(a_3) \geq v_3^3(a_5)-v_3^3(a_1)$

〈Table 3〉 Incomplete information about attribute weight

Department 1	Department 2	Department 3
$w_1^1 \geq w_2^1 \geq w_3^1$ $w_2^1 - w_3^1 \geq 0.15$ $0.2 \leq w_2^1 \leq 0.5$ $w_1^1 - w_2^1 \geq w_2^1 - w_3^1$	$w_1^2 = w_2^2 \geq w_3^2$ $0.1 \leq w_3^2 \leq 0.3$ $w_1^2 - w_3^2 \geq 0.1$	$w_2^3 \geq w_3^3 \geq w_1^3$ $w_2^3 - w_3^3 \geq w_3^3 - w_1^3$ $w_3^3 - w_1^3 \geq 0.2$

$$P = \{p^2 \geq p^1 = p^3,\ p^2 - p^1 \geq 0.1,\ 0.15 \leq p^1 \leq 0.4,\ p^1 + p^2 + p^3 = 1,\ 0 \leq p^1,\ p^2,\ p^3 \leq 1\}$$

The incomplete information in <Table 1>, <Table 2>, and <Table 3> will be used in the first iteration of the procedure. Then, further information will be appended in the next iterations. However, further information will not be appended to the set of the group member's importance weights, P.

In the first iteration, for example, we demonstrate the formulation for obtaining $\underline{v}^1(a_3, a_4)$ of department 1. From the <Table 3>, the following set of attribute weights, W^1, is established.

$$W^1 = \begin{cases} X_1^1(a_1) \geq X_2^1(a_2) \geq X_3^1(a_3) \\ X_2^1(a_2) - X_3^1(a_3) \geq 0.15 \\ 0.2 \leq X_2^1(a_2) \leq 0.5 \\ X_1^1(a_1) - X_2^1(a_2) \geq X_2^1(a_2) \\ \quad - X_3^1(a_3) \\ X_1^1(a_1) + X_2^1(a_2) + X_3^1(a_3) = 1 \\ 0 \leq X_1^1(a_1),\ X_2^1(a_2),\ X_3^1(a_3) \leq 1 \end{cases}$$

Also, from the <Table 1> and <Table 2>, the following sets of values, V_1^1, V_2^1, and V_3^1, are established.

$$V_1^1 = \{1 = v_1^1(a_1) \geq v_1^1(a_2) \geq v_1^1(a_3) \geq v_1^1(a_4) \geq v_1^1(a_5) = 0,\ v_1^1(a_1) - v_1^1(a_2) - v_1^1(a_3) \geq v_1^1(a_3) - v_1^1(a_4) \geq v_1^1(a_4) - v_1^1(a_5)\}$$

$$V_2^1 = \{1 = v_2^1(a_2) \geq v_2^1(a_1) = v_2^1(a_4) \geq v_2^1(a_3) \geq v_2^1(a_5) = 0,\ v_2^1(a_2) - v_2^1(a_1) \leq v_2^1(a_1) - v_2^1(a_3),\ v_2^1(a_1) - v_2^1(a_3) \geq v_2^1(a_3) - v_2^1(a_5)\}$$

$$V_3^1 = \{1 = v_3^1(a_3) \geq v_3^1(a_1) \geq v_3^1(a_2) = v_3^1(a_5) \geq v_3^1(a_4) = 0,\ v_3^1(a_3) - v_3^1(a_1) \leq v_3^1(a_1) - v_3^1(a_2),\ v_3^1(a_1) - v_3^1(a_2) \geq v_3^1(a_2) - v_3^1(a_4)\}$$

An example of the formulation for alternatives a_3 and a_4 is

$$\underline{v}^1(a_3, a_4) = \min w_1^1 \underline{z}_1^1(a_3, a_4) + w_2^1 \underline{z}_2^1(a_3, a_4) + w_3^1 \underline{z}_3^1(a_3, a_4) \quad s.t \quad W^1 \qquad (5.1.a)$$

where $\underline{z}_1^1(a_3, a_4) = \min v_1^1(a_3) - v_1^1(a_4)$

$$s.t \quad V_1^1 \qquad (5.1.b)$$

$$\underline{z}_2^1(a_3, a_4) = \min v_2^1(a_3) - v_2^1(a_4) \quad s.t \quad V_2^1 \qquad (5.1.c)$$

$$\underline{z}_3^1(a_3, a_4) = \min v_3^1(a_3) - v_3^1(a_4) \quad s.t \quad V_3^1 \qquad (5.1.d)$$

The values of $z_1^1(a_3, a_4) = 0$, $\underline{z}_3^1(a_3, a_4) = 1$, and $\underline{z}_3^1(a_3, a_4) = 1$ are obtained by solving 3 LPs. By the substitution of the value $\underline{z}_i^1(a_3, a_4)$ into the top objective function of (5.1.a) and solving a LP, $\underline{v}^1(a_3, a_4) = -0.3333$ is obtained. Similarly, $\underline{v}^2(a_3, a_4) = 0.2778$ and $\underline{v}^3(a_3, a_4) = -0.1556$ can be derived.

For obtaining the value $\phi_{\min}(a_3, a_4)$, the following formulation is used.

$$\phi_{\min}(a_3, a_4) = \min p^1 \underline{v}^1(a_3, a_4) + p^2 \underline{v}^2(a_3, a_4) + p^2 \underline{v}^3(a_3, a_4) \quad s.t \quad P. \qquad (5.2)$$

By the substitution of the value $\underline{v}^k(a_3, a_4)$ into the objective function of (5.2) and solving a LP, $\phi_{\min}(a_3, a_4) = -0.03556$ is obtained. $\phi_{\max}(a_3, a_4) = -0.57499$ is obtained by using "max" in the place of "min" in (5.1) and (5.2). Applying the similar process which is used to obtain

〈Table 4〉 Value intervals obtained from first iteration

Pair of alternative	Value interval	Pair of alternative	Value interval
(a_1, a_2)	[-0.1694,0.6625]	(a_2, a_4)	[-0.1511,0.6852]
(a_1, a_3)	[-0.1281,0.4533]	(a_2, a_5)	[-0.1900,0.5192]
(a_1, a_4)	[0.1201,0.7700]	(a_3, a_4)	[-0.0356,0.5749]
(a_1, a_5)	[0.1667,0.6133]	(a_3, a_5)	[0.0017,0.6281]
(a_2, a_3)	[-0.4067,0.3717]	(a_4, a_5)	[-0.3175,0.3056]

$\phi_{\min}(a_3, a_4)$ and $\phi_{\max}(a_3, a_4)$, the value intervals, $[\phi_{\min}(a_j, a_l), \phi_{\max}(a_j, a_l)]$, in <Table 4> are obtained.

From the results in <Table 4>, we can establish the strict dominance relations, $\Omega = \{(a_1, a_4), (a_1, a_5), (a_3, a_4)\}$. Also, the preference strength index can be calculated for the pair of alternatives that is not established as the strict dominance relation. For example, following the three highest values of the indexes are obtained by the use of (3.1).

$$index(a_3, a_4) = 0.9417$$

$$index(a_2, a_4) = 0.8193$$

$$index(a_1, a_2) = 0.7964$$

Now, we proceed with the procedure in the case that further information, $a_3 > a_4$, is provided by the department 1 and 3. For the department 1, the following formulation is need to obtain the value $\underline{v}(a_2, a_4)$.

$$\underline{v}(a_2, a_4) = \min \; w_1^1 v_1^1(a_2) + w_2^1 v_2^1(a_2) + w_3^1 v_3^1(a_2) - w_1^1 v_1^1(a_4) - w_2^1 v_2^1(a_4) - w_3^1 v_3^1(a_4) \quad (5.3.a)$$

$$s.t \quad W^1, W_1^1, V_2^1, V_3^1 \quad (5.3.b)$$

$$w_1^1 v_1^1(a_3) + w_2^1 v_2^1(a_3) + w_3^1 v_3^1(a_3) - w_1^1 v_1^1(a_4) - w_2^1 v_2^1(a_4) - w_3^1 v_3^1(a_4) \geq 0 \quad (5.3.c)$$

Since the formulation (5.3) is a non-LP form, we convert the (5.3) into a LP form by applying the Theorem 1.

Since $v_1^1(a_1)$, $v_2^1(a_2)$, and $v_3^1(a_3)$ are the best consequences on each set V_1^1, V_2^1, and V_3^1, the equalities are derived as follows :

$w_1^1 = X_1^1(a^{*i}) = X_1^1(a_1)$, $w_2^1 = X_2^1(a_2)$, and $w_3^1 = X_3^1(a_3)$. By applying the Theorem 1, the sets in (5.3.b) is transformed as follows :

$$XW^1 = \begin{cases} X_1^1(a_1) \geq X_2^1(a_2) \geq X_3^1(a_3) \\ X_2^1(a_2) - X_3^1(a_3) \geq 0.15 \\ 0.2 \leq X_2^1(a_2) \leq 0.5 \\ X_1^1(a_1) - X_2^1(a_2) \geq X_2^1(a_2) \\ \quad - X_3^1(a_3) \\ X_1^1(a_1) + X_2^1(a_2) + X_3^1(a_3) = 1 \\ 0 \leq X_1^1(a_1), X_2^1(a_2), X_3^1(a_3) \leq 1 \end{cases}$$

$$XU_1^1 = \{X_1^1(a_1) \geq X_1^1(a_2) \geq X_1^1(a_3) \geq X_1^1(a_4) \geq X_1^1(a_5) = 0, \; X_1^1(a_1) - X_1^1(a_2) \geq X_1^1(a_2) - X_1^1(a_3) \geq X_1^1(a_3) - X_1^1(a_4) \geq X_1^1(a_4) - X_1^1(a_5)\}$$

$$XU_2^1 = \{X_2^1(a_2) \geq X_2^1(a_1) = X_2^1(a_4) \geq X_2^1(a_3) \geq X_2^1(a_5) = 0, \; X_2^1(a_2) - X_2^1(a_1) \leq X_2^1(a_1) - X_2^1(a_3), \; X_2^1(a_1) - X_2^1(a_3) \geq X_2^1(a_3) - X_2^1(a_5)\}$$

$$XU_3^1 = \{X_3^1(a_3) \geq X_3^1(a_1) \geq X_3^1(a_2) = X_3^1(a_5) \geq X_3^1(a_4) = 0, \; X_3^1(a_3) - X_3^1(a_1) \leq X_3^1(a_1) - X_3^1(a_2), \; X_3^1(a_1) - X_3^1(a_2) \leq X_3^1(a_2) - X_3^1(a_4)\}.$$

Also, the constraint (5.3.c) is transformed as the following inequality :

$$X_1^1(a_3) + X_2^1(a_3) + X_3^1(a_3) - X_1^1(a_4) - X_2^1(a_4) - X_3^1(a_4) \geq 0.$$

Accordingly, the formulation (5.3) is converted as the following LP :

$$\underline{v}^1(a_2, a_4) = \min X_1^1(a_2) + X_2^1(a_2) + X_3^1(a_2) - X_1^1(a_4) - X_2^1(a_4) - X_3^1(a_4) \quad (5.4.a)$$

$$s.t \quad XW^1, XU_1^1, XU_2^1, XU_3^1 \quad (5.4.b)$$

$$X_1^1(a_3) + X_2^1(a_3) + X_3^1(a_3) - X_1^1(a_4) - X_2^1(a_4) - X_3^1(a_4) \geq 0 \quad (5.4.c)$$

$\underline{v}^1(a_2, a_4) = 0.05$ is obtained by solving (5.4). The value of $\underline{v}^1(a_2, a_4)$ is 0 in previous iteration of the procedure in which the constraints (5.4.c) was not considered. With the same constraints set, (5.4.b) and (5.4.c), the values, $\underline{v}^1(a_j, a_l)$ and, $\overline{v}^1(a_j, a_l)$ for the other pairs of alternatives can be obtained by solving LPs like (5.4).

Similarly, by adding the following constraint,

$$X_1^3(a_3) + X_2^3(a_3) + X_3^3(a_3) - X_1^3(a_4) - X_2^3(a_4) - X_3^3(a_4) \geq 0,$$

to the constraints set, the values of department 3, $\underline{v}^3(a_j, a_l)$ and $\overline{v}^3(a_j, a_l)\ \forall j, l, j \neq l$, can be obtained. The values of department 2, $\underline{v}^2(a_j, a_l)$ and $\overline{v}^2(a_j, a_l)\ \forall j, l, j \neq l$, is not changed from that of the first iteration.

With the values, $\underline{v}^k(a_j, a_l)\ \forall k, j, l \neq l$, new value intervals for each pair of alternatives, $[\phi_{\min}(a_j, a_l), \phi_{\max}(a_j, a_l)]$, are obtained by applying the LP like (5.2). The computational results of the value intervals are shown in <Table 5>.

For the alternative pair (a_3, a_4), the strict dominance is newly established. Also, the three highest values of the indexes are obtained as follows :

$$index\ (a_2, a_4) = 0.8343$$

$$index\ (a_1, a_2) = 0.7917$$

$$index\ (a_1, a_3) = 0.7677$$

Alternative pair (a_2, a_4) has the highest value of the preference strength indexes. However, we can not obtain further information from the probabilistic measure of alternative pair, (a_2, a_4). The group's aggregated value interval, $[\phi_{\min}(a_2, a_4), \phi_{\max}(a_2, a_4)] = [-0.1361, 0.6852]$, does not represent strict dominance relation, but each department's value intervals represent the strict dominance relation. The following value

<Table 5> Value intervals obtained from second iteration

Pair of alternative	Value interval	Pair of alternative	Value interval
(a_1, a_2)	[-0.1694, **0.6437***]	(a_2, a_4)	[**-0.1361***, 0.6852]
(a_1, a_3)	[-0.1281, **0.4233***]	(a_2, a_5)	[**-0.1825***, 0.5192]
(a_1, a_4)	[**0.1282***, 0.7700]	(a_3, a_4)	[**0.0111***, 0.5749]
(a_1, a_5)	[0.1667, 0.6133]	(a_3, a_5)	[0.0017, 0.6281]
(a_2, a_3)	[-0.4067, **0.3475***]	(a_4, a_5)	[-0.3175, 0.3056]

Note) * represent the number that is changed from first iteration.

intervals represent each department's value intervals.

Department 1's value interval :
$[\underline{v}^1(a_2, a_4),\ \overline{v}^1(a_2, a_4)] = [0.01,\ 0.672]$

Department 2's value interval :
$[\underline{v}^2(a_2, a_4),\ \overline{v}^2(a_2, a_4)] = [0.1222,\ 0.8777]$

Department 3's value interval :
$[\underline{v}^3(a_2, a_4),\ \overline{v}^3(a_2, a_4)] = [-0.6667,\ -0.2]$

Since each department's value interval represents the strict dominance relation between a_2 and a_4, further assessment of information about alternative pair (a_2, a_4) may incur inconsistent or redundant constraints. For example, if the further information, $a_2 > a_4$, is assessed from all departments, the new constraint that represent $a_2 > a_4$ is redundant in the mathematical programming model of department 1 and 2 and inconsistent in the mathematical programming model of decision maker 3.

If there is no further information from all decision makers, then we may choose a_1 as the most preferred alternative by applying the weak dominance. Finally, wezz have the set of dominance relation, $\Omega = \{(a_1, a_4), (a_1, a_5), (a_3, a_5), (a_3, a_4)\}$, and we have the value intervals, which are shown in <Table 5>, for the alternative pairs that are not included in Ω.

7. Conclusion

Since a selection is not generally made in a single step, an interactive decision procedure is suitable for group decision decision making problem with incomplete information. However, there are only a few studies about the interactive decision procedure resolving multi-attribute group decision making problems with incomplete information.

This paper presented an interactive group decision support procedure to aggregate each group member's incompletely identified information. In order to aid each decision maker to articulate preference information about alternatives, we proposed the preference strength index that is an indicative for the strength of preference or dominance relation between alternatives. Since the incomplete information about the alternatives was a non-linear inequality in the constraints set, we transformed the non-linear model into the linear model without changing the optimal solution.

Although commercial software associated with LP such as What's Best and LINDO can check inconsistency of each the decision maker's incomplete information about values and weights, this study assumes a consistent information set. Consequently, this still leaves a systematic approach to be developed which will check the inconsistencies and redundancies in each decision maker's judgments. Also, the implementation of a group decision support system based on the procedure described in this paper may be valuable to support the group decision with multiple attributes.

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