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Applicability of the Korteweg-de Vries Equation for Description of the Statistics of Freak Waves
최극해파통계분석을 위한 Korteweg-de Vries 식의 적용성 검토

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Abstract □ The requirements to the numerical model of wind-generated waves in shallow water are discussed in the framework of the Korteweg-de Vries equation. The weakness of nonlinearity and dispersion required for the Korteweg-de Vries equation applicability is considered for fully developed sea, non-stationary wind waves and swell, including some experimental data. We note for sufficient evaluation of the freak wave statistics it is necessary to consider more than about 10,000 waves in the wave record, and this leads to the limitation of the numerical domain and number of realizations. The numerical modelling of irregular water waves is made to demonstrate the possibility of effective evaluation of the statistical properties of freak waves with heights equal to 2−2.3 significant wave height.

Keywords : freak waves, random processes, Korteweg-de Vries equation, numerical simulation

要 旨 : 본 논문에서는 Korteweg - de Vries(이하 KdV)방정식의 골격내에서 천해의 풍파의 수치모형요구조건에 대한 토의를 수행하였다. KdV식을 실험자료를 포함하는 발달된 해상상태, 비정상적 풍파와 나불상황에 적용시의 비선형성과 분산성의 취약점을 논하였다. 최극해파통계의 충분한 평가를 위해서는 파고기록이 적어도 10,000개정도의 해파를 다루어야 하는데 이는 숫적으로 다루기 힘들다. 따라서 유의파의 2-2.3배에 상응하는 최극해파의 통계적 특성을 효과적으로 평가할 수 있는 가능성을 제시하는 불규칙해파의 수치적 모형을 제시하였다.

핵심용어 : 삼각파도, 확률과정, KdV방정식, 수치실험

1. Introduction

It is well known that wind-generated waves in the deep water are categorized as a Gaussian random process for which the probability distribution of wave profile obeys the normal probability law (Massel, 1996). It is assumed that all the spectral components of the irregular surface are independent (because nonlinear wave components have weak correlation). The wave profile of the wind-generated waves changes very quickly due to the strong dispersion effect of the water waves. And as a result its spectrum represents slow varied characteristics of wind-generated waves, which is described by the *kinetic equation* (Massel, 1996; Zakharov *et al.*, 1982; Lavrenov, 1998). It is obvious that it is impossible to restore the wave profile within the spectral approach because we have to know the phases of spectral components. And this problem is quite important for the prediction of the abnormal waves (freak or rogue waves) appearance due to great interest to it (Lawton, 2001).

The dispersion effects become weaker in shallow water, and the wave profile conserves its individuality for a long time (comparing to the wave period). The most widespread examples of the spectral correlation conservation are the quasi-stationary waves (as cnoidal waves and solitons) and

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wave breaking of the plunging type. The spectral description of shallow water waves is possible and corresponding kinetic equations have been already obtained (Zaslavsky, 1998). However, its applicability is not fully understood because of strong nonlinearity compared to the dispersion. An alternative approach for shallow water dynamics description is related to the different generalizations of shallow water equations (Mei, 1993) in which the initial perturbation is represented as a deterministic shape wave. The irregular waves are not very well investigated, and here the numerical simulations and laboratory models are effective (Bychkov *et al.*, 1971; Osborne, 1993; Osborne, 1995; Pelinovsky *et al.*, 2000).

Here we will consider the problem of numerical simulations of irregular waves in shallow water to obtain the data of freak waves formation probability. In section 2 the observed data of the freak waves in coastal zone are presented and the importance of the freak waves prediction is addressed. In section 3 the estimations of applicability of the Korteweg - de Vries equation for wind-generated waves (fully developed sea, non stationary waves and swell) are considered. An important problem regarding of the requirements to the numerical model for wind-generated waves is discussed in section 4 for the effective abnormal wave prediction.

2. The observed data of freak waves in coastal zone

In recent years much information about abnormal waves (freak waves) formation in different areas of the World Ocean is collected. Large number of observations shows that freak wave appears and disappears very suddenly; usually it is one - three waves with high crests and low troughs. Recently, the statistics of disasters tanker collisions with freak waves was published: in last 25 years (1968-1994) 22 super-tankers were lost and 525 people were dead (Lawton, 2001). The geography of these disasters is shown in Fig. 1.

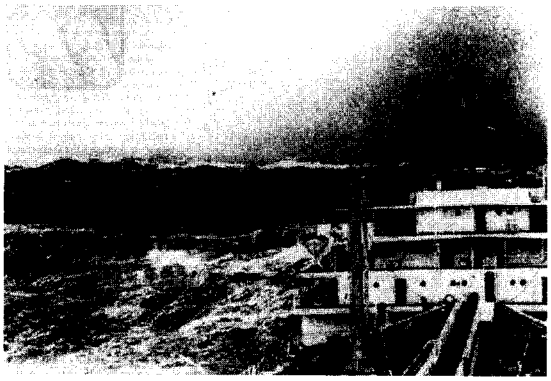

Fig. 2. Observed freak wave.

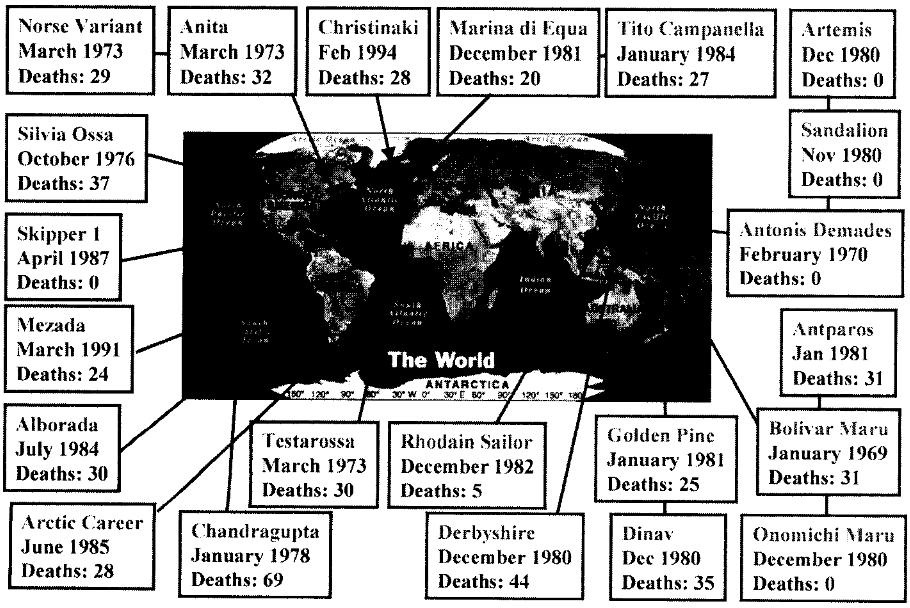

Fig. 1. The statistics of tanker collisions with freak waves during 1968-1994.

Table 1. Data of abnormal waves in coastal zone (Sand *et al.*, 1990)

Place	h(m)	H_s(m)	H_{max}(m)	H_{max}/H_s	Year
Gork, Eire	20	5	12.8	2.6	1969
Hanstholm	20	2	6	3	1985
Gorm Field	40	6.8	17.8	2.6	1981
Gorm Field	40	7.8	16.5	2.1	1981
Gorm Field	40	5	12	2.4	1984
Gorm Field	40	5	11.3	2.3	1984
Gorm Field	40	5	11	2.2	1984
Gorm Field	40	4.8	13.1	2.7	1984

Fig. 2 presents the picture of the real abnormal wave, which demonstrates its strong nonlinear character.

The same abnormal waves occur in shallow water. Some observations of big waves were made in Danish shelf of the North Sea, which are presented in Table 1 (Sand *et al.*, 1990). Here h is the water depth, H_s is the significant wave height, and H_{max} is maximum observed wave height. As it could be seen, the abnormal waves exceed the significant wave height more than twice (this criterion is used for the freak wave definition in ocean engineering now).

The prediction of such waves is considered to be key point now in oceanography due to its importance in practice. Some theoretical models of the freak wave phenomenon are investigated within the Korteweg - de Vries equation (Pelinovsky *et al.*, 2000; Pelinovsky *et al.*, 2001). Its applicability to description of the freak wave formation in random wind-generated waves should be specially analysed which will be given below.

3. Applicability of the Korteweg - de Vries equation

The Korteweg - de Vries equation was developed in 1895

$$\frac{\partial \eta}{\partial t} + c\left(1 + \frac{3\eta}{2h}\right)\frac{\partial \eta}{\partial x} + \frac{ch^2}{6}\frac{\partial^3 \eta}{\partial x^3} = 0 \qquad (1)$$

and has become the basic equation in the nonlinear theory in recent years due to its integrability within the inverse-scattering method (Drazin *et al.*, 1993). Here η is the sea surface elevation, h is a water depth assuming being constant, g is gravitational acceleration, $c = (gh)^{1/2}$ is a maximal velocity of long wave propagation, x is horizontal coordinate and t is time. Eq. (1) was derived for simple geometry of unbounded (in horizontal plane) basin of constant depth and two-dimensional progressive wave with plain fronts. The main approximations used for derivation Eq. (1) are the weakness of nonlinearity, characterized by parameter ε.

$$\varepsilon = a/h \qquad (2)$$

and weakness of dispersion

$$\mu = kh \qquad (3)$$

where a and k are the local wave amplitude and wave number. The relation between them is defined by the Ursell parameter

$$Ur = \frac{\varepsilon}{\mu^2} \qquad (4)$$

values of which could be different: small Ursell parameter (almost linear dispersive waves) and large Ursell parameter, which corresponds to the nonlinear long waves. If the Ursell parameter is equal to 1, it characterizes the balance between nonlinearity and dispersion, which leads to the steady-state wave formation (cnoidal waves and solitons). The analytical expression for solitons shape is

$$\eta(x,t) = H\,\mathrm{sech}^2\left[\sqrt{\frac{3H}{4h}}\frac{x-Vt}{h}\right], \quad V = c\left(1 + \frac{H}{2h}\right) \qquad (5)$$

which is defined by only free parameter - its height, H (in fact, there is also another parameter corresponding to the phase or initial soliton location).

All expressions above are well known in the theory of nonlinear waves. The solitary wave was first observed by Russell about 150 years ago, and soliton solution Eq. (5) well describes it. Different experiments were carried out in laboratory tanks, which verified the applicability of the Korteweg - de Vries equation for describing regular water shallow waves. The processes of initial disturbance transformation into solitons, recurrence effects of the periodical perturbation, wave group propagation and wave breaking were investigated. But the direct using these results of hydromodelling applying to the real processes in the sea is impossible due to scale effect (the similarity parameter in tank depth and length, wave amplitude and wave length should the same) and, as usual, the waves simulated in the laboratory tank, are too nonlinear and too dispersive.

In more details the applicability of the Korteweg - de Vries equation was considered in the tsunamis problem (Pelinovsky, 1982). It turned out that for most of the tsunamis of the seismic nature the Ursell parameter is quite large, and that is why the dispersion effects are too small and don't influence the wave transformation and propagation. For tsunamis of the volcanic nature, on the contrary, the dispersion effects are more important (the nonlinearity influences only on the first step of the wave formation).

For our purposes it is important to discuss the applicability of the Korteweg - de Vries equation to describe random wind-generated waves. The main problem is that the wind wave field is more complicated; it is characterized by wind velocity, fetch length, depth and geometry of the basin. We will discuss some limiting cases.

3.1 Fully developed sea

As it exists only in open sea in the deep water and far from the shoreline, the wave filed is characterized by only parameter - wind velocity, W. Wave field parameters are (these evaluations are based on the Pierson-Moskowitz spectrum).

$$H_s \approx 0.2\frac{W^2}{g}, \quad T_s \approx 5\frac{W}{g} \tag{6}$$

where H_s and T_s is the height and period of the significant wave, and numerical coefficients are obtained from experimental data (Massel, 1996; Bowden, 1983). The conversion of wave characteristics from deep to shallow water is not trivial problem. We can consider very roughly that the wave period will not be changed greatly (nominally within the asymptotic approach the wave period is not changed if the wave is sinusoidal), and wave amplitude could increase due to energy flux conservation. We will assume that the wave amplitude is constant and wave length decreases adiabatically, following the relation

$$\lambda_s = \sqrt{gh}\,T_s \tag{7}$$

Then the parameters of nonlinearity and dispersion depend on the relation of wind velocity to the long waves velocity

$$\varepsilon \approx 0.2\left(\frac{W}{c}\right)^2, \quad \mu \approx \frac{c}{W} \tag{8}$$

Simultaneous weakness of nonlinearity and dispersion is contradictory, and if the wave is weakly nonlinear, it is strong dispersive. Theoretically, due to small coefficient 0.2 in Eq. (8) it is possible to choose the wind velocity, for instance, equal to 1.5 s, $W \sim 1.5\, s$, then $\varepsilon \sim \mu^2 \sim 0.5$ and both of this parameters could be considered to be weak. It is clear, that all of this estimations follow from spectral models of fully developed sea and the small difference in present empirical spectrums will affect on the coefficients in Eq. (6) and Eq. (8). So, it follows from the primary estimations that the Korteweg - de Vries equation has very limited applicability for the fully developed sea description.

3.2 Non-stationary wind-waves

The wave parameters are defined by the wind velocity and the distance from the shore (fetch length). Fetch length is characterized by dimensionless length $F = gX/W^2$. The empirical formulas for perturbation characteristics within the significant wave terms are the following (Bowden, 1983):

$$H_s = 0.3\frac{W^2}{g}[1-(1+0.004F^{1/2})^{-2}] \tag{9}$$

$$T_s = 8.6\frac{W}{g}[1-(1+0.008F^{1/3})^{-5}] \tag{10}$$

For the large fetch length formulas (9) and (10) agree with (6) with small difference in coefficients. For the small fetch length we have.

$$H_s \approx 0.02\frac{W^2}{g}F^{1/2}, \quad T_s \approx 3\frac{W}{g}F^{1/3} \tag{11}$$

In this case nonlinearity and dispersion parameters are

$$\varepsilon \approx 0.02\frac{W^2}{c^2}F^{1/2}, \quad \mu^2 \approx 4\frac{c^2}{W^2}F^{-2/3} \tag{12}$$

In general, the weakness of both of the parameters is arbitrary for wind velocities and fetch lengths. Nevertheless due to different range of the fetch in Eq. (12) there is parameter space, where ε and μ are less then unity (the space between two lines in Fig. 3). So, the domain of applicability of the Korteweg - de Vries equation for non-stationary wind-generated waves is larger than that for fully developed sea.

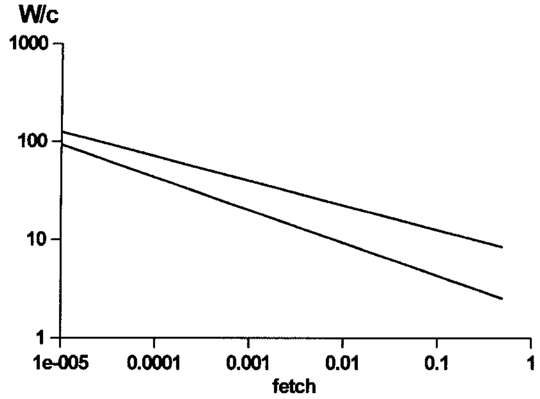

Fig. 3. Domain applicability of the Korteweg - de Vries equation for non-stationary wind-generated waves (between two lines).

3.3 Swell

The swell height and its period depend on the wind velocity in the storm area and on the distance from it. Due to the dispersion the irregular disturbance transforms into dispersive packet, where waves with greater length are speeder. The characteristic wave period could be found through the stationary phase approximation (Bowden, 1983).

$$T = \frac{x}{\pi g t} \tag{13}$$

The wave height decreases as $x^{-1/2}$ because of dispersion. It follows, that wave becomes longer and more linear with distance. Note that taking into account real turbulent viscosity the spectrum becomes more narrow, due to exponential decay of the spectral components with the decrement proportionate to ω^2. Due to this the swell characterized by long waves of weak steepness and long crests in the area far from the storm zone. So, the irregular waves corresponding to the swell have to satisfy to the Korteweg - de Vries equation.

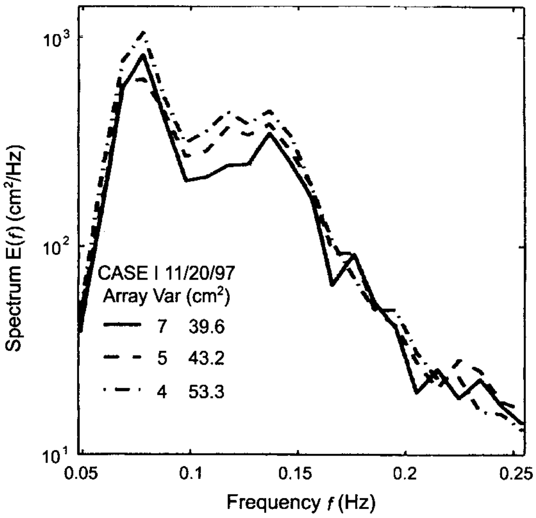

Fig. 4. Frequency spectra of swell in shallow water($f=\omega/2\pi$, H_s=0.3 m, f_p=0.07 Hz, W<5 m/s).

Low-energy swell with a peak frequency of 0.07 Hz ($f=\omega/2\pi$, $E(f)=2\pi S(\omega)$) (Fig. 4), a significant wave height of 0.3 m was observed in light wind conditions (speeds<5 m s^{-1}) (Herbers *et al.*, 2002). The wave measurements were made with sensors (4, 5, 7) located along a cross-shore transect 100, 150 and 400 m from the shoreline in nominal depths of 2.5, 3.5×5.0 m. For instance, for the water depth equal to 8 m calculated parameters of nonlinearity and dispersion are $\varepsilon = 0.038$ and μ=0.391. The Ursell number is $Ur = 0.274$. These estimates confirm the applicability of the Korteweg - de Vries equation for the swell description in shallow water.

Wind-generated wave measurements in shallow water. The spectra of the wind-generated waves in shallow water, obtained in the North Sea near the Netherlands (*Haringv-*

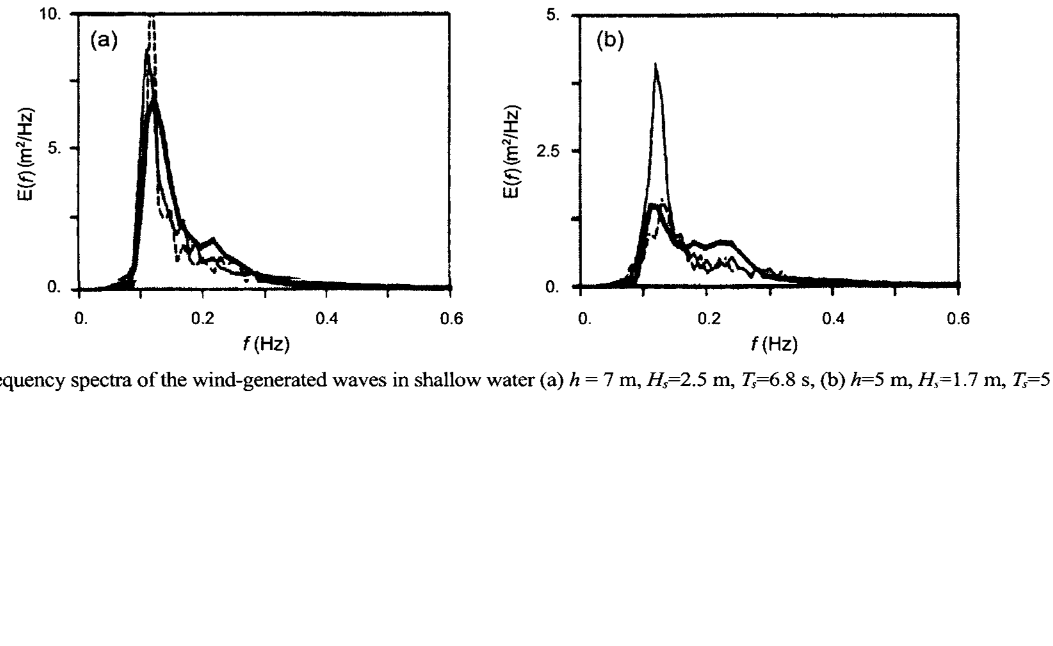

Fig. 5. Frequency spectra of the wind-generated waves in shallow water (a) h = 7 m, H_s=2.5 m, T_s=6.8 s, (b) h=5 m, H_s=1.7 m, T_s=5 s.

liet), are presented in Fig. 5 (depth is 5-7 m, wind velocity is $W = 14$ m/s) where in-situ measurements are marked with dot line. Solid line is the result of numerical simulations using SWAN model (Ris *et al.*, 1999). The calculated parameters of nonlinearity, dispersion and Ursell number are equal: (a) $\varepsilon = 0.36$, $\mu = 0.64$, $Ur = 0.70$, (b) $\varepsilon = 0.34$, $\mu=0.54$, $Ur=0.90$. So, our estimates based on the experimental data show the applicability for the wind-generated waves in shallow water.

4. Statistical approach of the freak wave formation

As it was noted above, the randomly changing wind-generated waves are considered as a stochastic process, and different statistical methods are applied to describe it (Bychkov *et al.*, 1971; Lavrenov, 1998; Massel, 1996).

In the first approach a wind-generated wave process could be regarded as a narrow band random process with a Gaussian statistics. Then the probability density function of the wave amplitudes is described by a Rayleigh distribution.

$$p(A) = \frac{A}{\sigma^2}\exp\left(-\frac{A^2}{2\sigma^2}\right) \tag{14}$$

where σ^2 is a wave field variance. It could be found through the Gaussian distribution of the surface elevation $\eta(x, t)$.

$$p(\eta) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\eta^2}{2\sigma^2}\right) \tag{15}$$

or through the spectrum, $S(\omega)$

$$\sigma^2 = \int_0^\infty S(\omega)d\omega \tag{16}$$

It is clear that all this formulas are valid for the stationary random process, but doesn't hold true in reality. Especially for freak waves due to the rarity of this event, it is hard to say if this process is stochastic or determinate. Nevertheless, first we will discuss freak wave formation and prediction within the Rayleigh probability distribution.

As it is known, the probability of the wave appearance with given amplitude could be found from Eq. (15) through the integral distribution function

$$P(A) = \exp\left(-\frac{A^2}{2\sigma^2}\right) \tag{17}$$

Within the approximation of a narrow-band random process the wave height is defined through the wave amplitude as $H=2A$. Then the distribution function of wave heights will be defined as

$$P(H) = \exp\left(-\frac{H^2}{8\sigma^2}\right) \tag{18}$$

The relation between mean wave height and variance will be the following.

$$H_m = \sqrt{2\pi}\sigma \tag{19}$$

The significant wave height, H_s could be defined as one third of the biggest waves observed in the ocean. Using the Rayleigh distribution, the significant wave height is (Bychkov *et al.*, 1971; Massel, 1996)

$$H_s \cong 4\sigma \tag{20}$$

The mean wave height relates to the significant height as

$$H_s = \frac{4}{\sqrt{2\pi}}H_m \approx 1.6H_m \tag{21}$$

Distribution function allows to estimate the probability of the abnormal waves formation. Note, that according to formula (18), such probability is not zero, what means that such waves have to appear in reality. A freak wave is defined through its height H_f.

$$H_f > 2H_s \tag{22}$$

We can simple evaluate the probability of its formation within the Rayleigh distribution Eq. (18) which could be written through the significant wave height.

$$P(H) = \exp\left(-\frac{2H^2}{H_s^2}\right) \tag{23}$$

This dependence is presented in Fig. 6. According to it, the probability of extreme waves formation is not more then $P(2H_s) = 0.000336$ or one wave from 3000 waves. Taking into account that the period of wind-generated waves is about of 10 s, we obtain that each 8–9 hours the probability of such wave formation is rather big. As most of the storms continue about 24 hours and more, abnormal waves have to appear rather often in ocean. So, it follows from the Rayleigh statistics that extreme waves with height equal to 2–2.3 of significant wave height have to be observed in the ocean quite often.

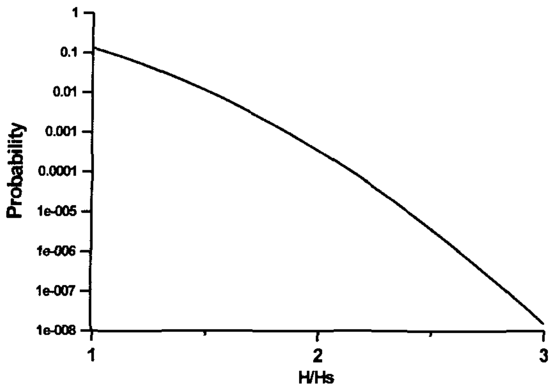

Fig. 6. Probability of the freak wave formation.

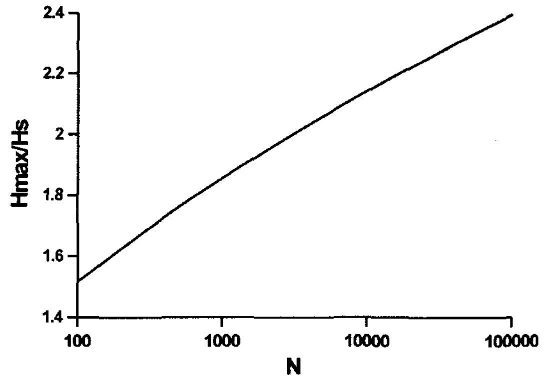

Fig. 7. Relation between maximal wave height and number of waves in a group.

Eq. (23) could be simply interpreted. If we choose a wave with a maximal height H_{max} in a group of N waves, its probability will be $P = 1/N$. Substituting it to Eq. (23), the last one could be rewritten as (Bychkov *et al.*, 1971; Massel, 1996).

$$H_{max} \cong \sqrt{\frac{\ln N}{2}} H_s \tag{24}$$

This dependence is presented in Fig. 7. From this relation it follows that increasing the record length (number of waves) weakly influences the maximal amplitude growing. The analysis of the short time record will not give true prediction of abnormal wave formation. Thus, for more reliable prediction of freak waves it is necessary to consider large number of waves (more then 10000). It could be made through the growing of the numerical domain or increasing number of realizations with the following averaging.

It is clear that abnormal waves, as any distribution functions tails, usually do not satisfy the statistical hypothesis, which the waves properties are based on. That's why the empirical data are the primary base for freak wave formation analysis. It is not clear enough what the abnormal wave is: it is the random realization of typical stochastic process or the typical realization of the very rare random process (Haver *et al.*, 2000).

Other difficulties join with the shoal, where freak waves are observed. In particular, near the offshore platforms in the North Sea the water depth is about 20-70 m, and storm waves can "feel" the bottom. The shallow waves, as it was noted, are weakly dispersive, that is why there is a phase correlation of different spectral components, broken the Gaussian approximation of the random process. In this cases different generalizations of the Rayleigh distribution are used (Morf *et al.*, 2000; Tomita *et al.*, 2000).

So, some requirements to the numerical models of freak waves formation follow from the statistical hypothesis. The simplest way is to consider rather large domain for computing contained 3000 local waves and more. In this case each realization will contain the abnormal wave and its statistics will be increased. But it is difficult to carry out such experiments due to computational constraints. To decrease the number of waves in numerical domain, for instance in 10 times, we have to have no less than 10 realizations. More over, in reality we have to carry out several sets of numerical experiments to increase number of random variables. Series of simulations will be set to define the optimal conditions for modelling of random process with real wind wave spectrum.

5. Wind-generated wave modelling

We use the Longuet-Higgins model (Massel, 1996) to describe wind-generated waves, according to which the sea surface elevation $\eta(x, t)$ is constructed as a following process with random amplitudes, a_i, and phases, φ_i.

$$\eta(x,t) = \sum_i a_i \cos(k_i x - \omega_i t + \varphi_i) \tag{25}$$

where $\eta(x, t)$ is a Gaussian process with a spectral density $S(\omega)$. The discrete amplitude of series Eq. (25) is a random variable related to the spectrum $S(\omega)$ by

$$a_i = \sqrt{2S(\omega_i)\Delta\omega_i} \tag{26}$$

here $\Delta\omega$ is the sampling frequency. The phases of each harmonic are uniformly distributed on the interval (0.2π) and produced by a random-number generator. For the rest

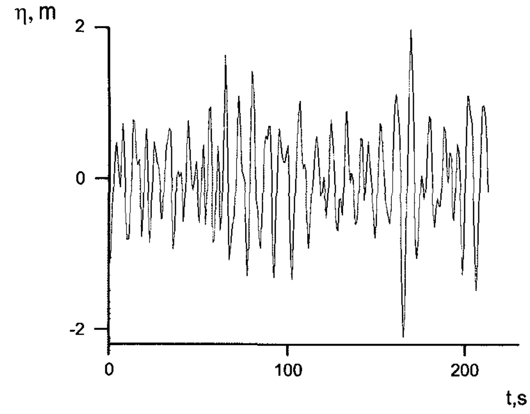

Fig. 8. Wind-generated wave record.

numerical computations a spectrum of wind-generated waves (Fig. 5a) measured in the North Sea (H_S=2.5 m, h=7 m, ε=0.36, μ=0.64, Ur=0.70), is chosen. The dominant frequency is ω=0.74 s^{-1} with sampling frequency equal to 0.03 s^{-1}.

The initial perturbation of 220-s record is presented on Fig. 8.

The corresponding autocorrelation function $K(\tau)$ is given on the Fig. 9, and correlation time is equal to τ= 90 s (about 15 waves). For numerical simulations of waves, the long records ($t \gg \tau$) are considered what implies that surface elevations $\eta(t)$ at times are uncorrelated. Series of 220-s record are took up, each containing 32 waves. The number of realizations is varied in the numerical experiments from 10 to 1000.

It is more reasonable to use wave amplitude $A(x)$ (the maximal wave profile elevation between two zeros) in numerical simulations instead of wave height. To evaluate the distribution probability, first the magnitude P is computed, which is considered as a ratio of number of waves

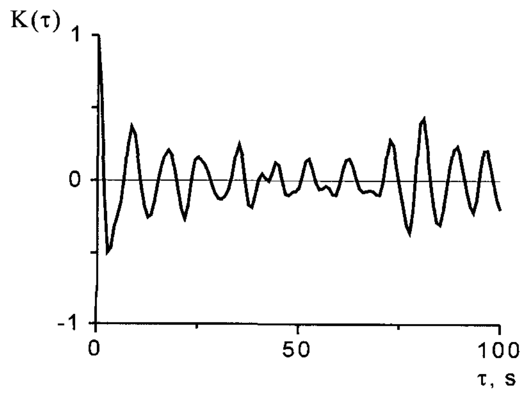

Fig. 9. Correlation function.

with amplitude a to a general number of waves

$$P = \frac{m}{N} \tag{27}$$

then the distribution probability of the wave with amplitude A exceeding a, is

$$F(a) = P(A > a) \tag{28}$$

On Fig. 10 the theoretical Rayleigh probability Eq. (17), marked with the solid line, and numerical experiments obtained from Eq. (28) and averaged over 20, 40, 60 and 80 realizations (a) and over 200, 600, 1000 realizations (b), are presented. The special interest is given to extreme waves with amplitudes exceeded A_s=H_s/2, that is why it is necessary to consider such big number of realization. According to the Rayleigh theory, the probability of exceeding A_f=2A_s is equal $P(A_f)$=0.000336 (this value is marked with dot line). As it could be seen, the computed curve is well approached with the theoretical one only in the case of small number of realizations, but the freak waves are not observed (Fig. 10a). The increasing number of realizations leads to the more stable shape of the computed curve in the range of amplitudes up to 2-2.5. Note, that it moves below the Rayleigh curve. As it was mention the Rayleigh probability is defined for the Gaussian narrow-band random process, which is not corresponding to the spectrum of real wind-generated waves (Massel, 1996).

Freak wave formation is a rare event, and doesn't belong to the averaged dependence. Its probability depends on the number of realizations and, finally, on the number of waves in the record. According to numerical simulations abnormal waves appear in records with 200 and more realizations (the number of local waves is more then 5000). The amplitude of freak wave is growing while increasing number of realizations (Fig. 10).

Increasing number of harmonics, what means the increasing number of random variables, is made in the next series of simulations. We consider twice more harmonics (256) and averaged over 1000 realizations. It leads to the same effect as 2000 realizations contained 128 harmonics were considered (Fig. 10c).

Thus, it is necessary to consider rather large number of local waves in the record (about 3-10 thousands of waves) for the numerical modelling of freak waves formation in the irregular wave field. It could be achieved while defining

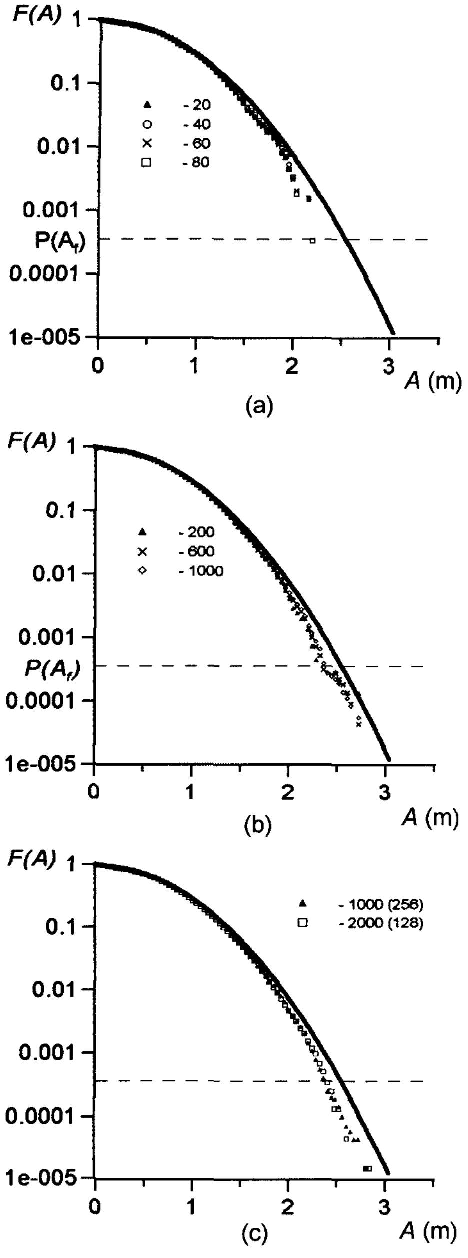

Fig. 10. Distribution functions of wave amplitudes.

frequency spectrum of 128 harmonics with 1000 realizations. Then the probability of the freak wave formation with amplitude, for instance, 2.7 m will be $4 \cdot 10^{-5}$ according to the computing.

As computed distribution function deflects a theoretical one, more detailed data analysis is needed and the mean wave height will be defined not through the dispersion of

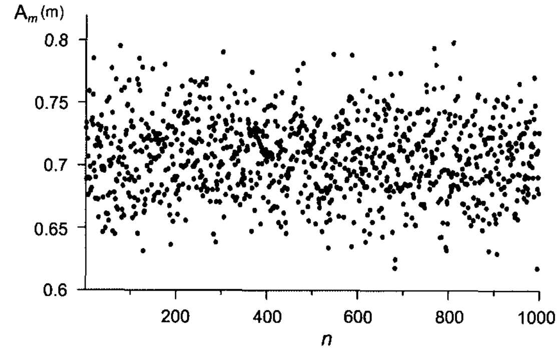

Fig. 11. Realization distribution of the mean amplitudes.

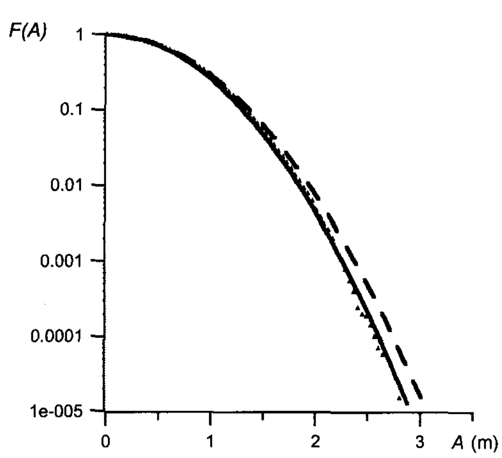

Fig. 12. Rayleigh distribution function (dot line and solid line) and computed curve.

the wave profile ordinates but directly from the wave record.

As it is well observed (Fig. 11), the variance of the mean amplitude values A_m is not so big, and we can take rather small number of realizations to average the mean amplitude over.

Thus, if we consider the distribution function obtained through the mean amplitude value (solid line on Fig. 12) and compare it to the distribution function calculated through the dispersion (dot line on Fig. 12), it is well noticed that the first one better conforms to the numerical simulation results (star symbol on Fig. 12).

As it was demonstrated, that amplitude probability distribution is less then Rayleigh statistics, it is interesting how the spectrum shape influences on this characteristic. We consider JONSWAP spectrum with different parameters of γ and α, which characterize the spectrum width and height. And following to this parameters we try to find if the distribution function calculated through the dispersion would be

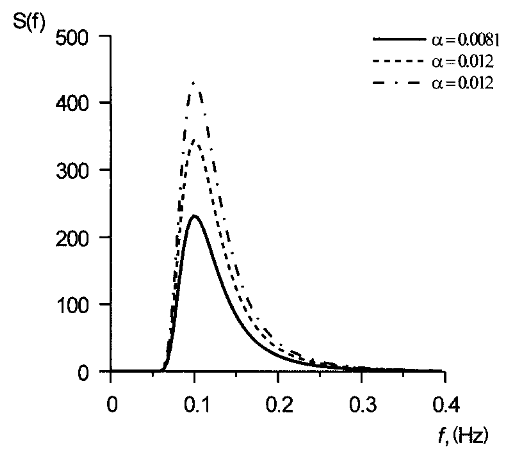

Fig. 13. JONSWAP spectrum for γ=1.

more or less then Rayleigh distribution.

The numerical experiments of JONSWAP spectrum modelling with different values of parameters are set. To increase the number of random variables series we consider 6 samples of time, each of them contains 1000 realizations of 250 seconds wave records.

γ=1, α=0.0081, α=0.012, α=0.015, α=0.018, α=0.02;
γ=4, α=0.0081, α=0.012, α=0.015, α=0.018, α=0.02;
γ=10, α=0.0081, α=0.012, α=0.015, α=0.018, α=0.02.

The initial shape of JONSWAP spectrum depending on the parameter α is represented in Fig. 13. Using different shapes of initial spectrum theoretical and computed distribution functions (Fig. 14) are calculated. The obtained results are the same like in previous numerical experiments: the experimental curves move below the theoretical one, what does not depend on the spectrum shape. The Rayleigh distribution, calculated through the mean averaging amplitudes of the wave records, shows the best correlation with computed results.

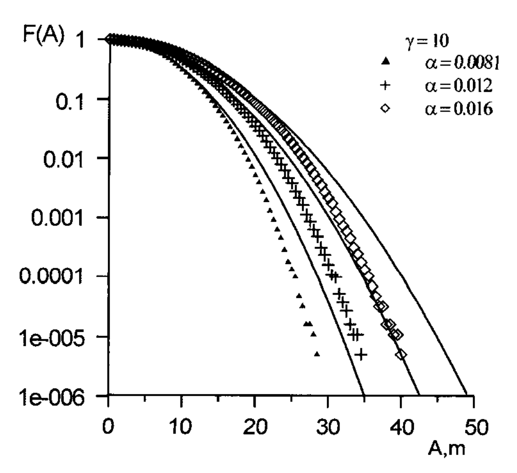

Fig. 14. Computed distributional functions (marked with dots) and Rayleigh curves (solid lines). JONSWAP parameters: γ=10, (a) α=0.0081, (b) α=0.012, (c) α=0.016.

6. Conclusion

Data of freak wave observation in shallow water is given. The applicability of the Korteweg - de Vries equation to describe freak wave phenomenon in the random shallow sea is discussed. First, the weakness of nonlinearity and dispersion parameters is considered for different types of wind-generated water waves (fully developed sea, non-stationary wind waves and swell), including in-situ measured spectra. It is shown that the Korteweg - de Vries equation could be applied for the description of wind-generated water waves in the coastal zone at some limitations on wind speed, fetch, basin depth and swell parameters. The numerical modelling of the Korteweg-de Vries equation is applied to obtain detail characteristics of the freak wave phenomenon. Due to rare character of the freak waves, numerical simulation requires large computed domain, large computation time and large number of realizations; corresponding requirements are discussed. The numerical modelling of the wave records with given spectrum confirmed that the amplitude distribution function is less then the Rayleigh statistics in large-amplitude range. For stable evaluation of the amplitude distribution of the freak waves with heights equal to 2–2.3 significant wave height it is necessary to have about 10 thousands of local waves in the wave record according to our numerical simulations.

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