

# Full-Wave Analysis of Microwave Amplifiers with Nonlinear Device by the FDTD Algorithm

Hee-Jin Kang · Jae-Hoon Choi

## Abstract

This paper presents the full wave analysis of microwave circuits with nonlinear device using the finite difference time domain method. The equivalent current source is used to model nonlinear device and all the electric field components at the nonlinear device are updated by FDTD algorithm. The currents and voltages of nonlinear device are calculated by the state equations and iteration method. To validate the proposed method, the S-parameters of NEC NE72089 MESFET in various conditions are analyzed and the results are compared with those of the ADS. The proposed method is applied to the analysis of a microwave amplifier, which includes NEC NE72089 MESFET. The analysis results obtained by the present method show good agreement with those of the ADS.

**Key words :** Amplifiers, Nonlinear Device, FDTD Algorithm, Iteration Method

## I. Introduction

Microwave circuits have become highly integrated system, such as monolithic microwave integrated circuit, including very closely spaced elements, discontinuity structures, and mixture of passive and active devices. The analysis of electromagnetic radiation and coupling effect between circuit elements is the most difficult task in designing such a complex microwave circuit in high frequency. Recently, much attention has been focused on the incorporation of full wave analysis using FDTD algorithm into active device analysis. Chien-Nan Kuo<sup>[1]</sup> analyzed the characteristic of a low noise amplifier using extended FDTD method. He used the forward difference scheme to discretize a set of state equation. Vincent A. Thomas<sup>[2]</sup> established the link between FDTD algorithm and SPICE simulation, in which SPICE is used to analyze the active part of the circuit. Recently, for more stable and accurate analysis, V. S. Reddy<sup>[3]</sup> modified the active sheet between the end of microstrip line and ground plane, in which all the electric field components are updated according to the port termination. Those papers used Taylor series expansion and Jacobian matrix to analyze a nonlinear active device. Unfortunately with this method, to calculate the Jacobian matrix requires a great deal of time and effort and if the equivalent circuit is changed, the Jacobian matrix has to be reevaluated. Therefore this method is not an efficient analysis method.

In this paper, FDTD algorithm using state equations is presented to analyze S-parameters of NEC NE72089 MESFET. To analyze nonlinear state equation, iteration procedure and

central difference approximation are used. Suggested method is verified by calculating the S-parameters in various bias conditions and equivalent circuits. Finally, a microwave amplifier is designed by ADS<sup>[4]</sup> and analyzed by proposed FDTD algorithm.

## II. The FDTD Algorithm to Analysis of Nonlinear Active Device

This algorithm is based on the FDTD representation of Maxwell's equation using a central difference formation and the Yee-cell notation. An active device in a microwave is very small in size and can be modeled by an equivalent circuit for typical operating condition. To analyze a nonlinear device, the transistor is modeled as a current source, and included in the FDTD algorithm<sup>[5]</sup>. The circuit and FDTD equations on the active sheet are solved simultaneously. The active sheet between the end of microstrip line and ground plane is shown in Fig. 1. All the electric field components are updated according to the port termination. This approach provides not only the scattering

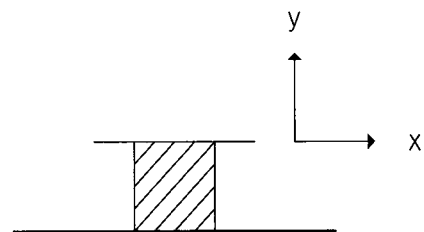


Fig. 1. An active sheet of the microstrip line.

Manuscript received August 1, 2002 ; revised October 28, 2002.

Department of Electrical and Computer Engineering, Hanyang University, 17 Hangdang-dong, Sungdong-gu, Seoul, 133-791, Korea.

properties of a circuit but also the voltage-current relation at each port of an active device under consideration.

### 2-1 Basic FDTD Algorithm

The basic equations for FDTD algorithm<sup>[5]</sup> in time domain are given by

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{J} \quad (2)$$

The inclusion of a microwave active circuit within the FDTD space cell can be implemented by solving equations that govern the voltage-current characteristic of an active device along with Maxwell's equations. The coupling between the circuit-element equation and Maxwell's equation can be established through the independent current term  $\vec{J}$  in equation (2).

### 2-2 Equivalent Circuit Model

At each time step, the device voltages of active sheet are evaluated from the state equation<sup>[6]</sup>. The state equation in general can be expressed as

$$A \cdot \frac{dX}{dt} + B \cdot X = F, \quad (3)$$

where the vector  $X$  denotes the state variable, matrix  $A$  and  $B$  are derived from the circuit elements and the variable  $F$  comes from the source. The central-difference approximation is used for the derivative of state variable vector  $X$ . The updating equation for the variable vector  $X$  follows from (3) as

$$\left[ \frac{A}{\Delta t} + \frac{B}{2} \right] X^{n+1} = \left[ \frac{A}{\Delta t} - \frac{B}{2} \right] X^n + F^{n+1/2} \quad (4a)$$

$$A_1 X^{n+1} = B_1 \quad (4b)$$

$$A_1 = A \cdot \Delta t + B \cdot 2 \quad (4c)$$

$$B_1 = (A \cdot \Delta t - B \cdot 2) X^n + F^{n+1/2} \quad (4d)$$

To analyze the nonlinear state equations, the iteration method is used. In nonlinear case  $A$ ,  $B$  and  $F$  are dependent upon  $X^{n+1}$ .

Equation (4b) can be expressed as

$$A_1(X^{n+1})X^{n+1} = B_1(X^{n+1}) \quad (5)$$

The corresponding central finite difference equation can be

written and be solved by following procedure.

$$A_1(X_{p-1})X_p = B_1(X_{p-1}) \quad (6)$$

where,  $p = 1, 2, \dots, k$  with the initial value

$$X_0 = X^n \quad (7)$$

The iteration procedure is used to determine  $X_p$  in (6) until the convergence criteria is satisfied. Then the value of  $X_p$  in (6) becomes  $X^{n+1}$  in (5).

### 2-3 MESFET Analysis Model

Fig. 2 shows the mounting of the MESFET in a stripline circuit. A MESFET transistor is soldered on the substrate in a microstrip line and is connected to the ground plane through vias at source port. The equivalent circuit for MESFET is shown in Fig. 3. The element values of the equivalent circuit are dependent on a transistor size and bias condition.

The state equations for FET can be expressed as

$$-v_{total,g} + L_g \frac{di_g}{dt} + R_g i_g + v_{gd} + v_d + R_s(i_g + i_d) + L_s \frac{d(i_g + i_d)}{dt} = 0 \quad (8a)$$

$$-v_{total,d} + L_d \frac{di_d}{dt} + R_d i_d + v_d + R_s(i_g + i_d) + L_s \frac{d(i_g + i_d)}{dt} = 0 \quad (8b)$$

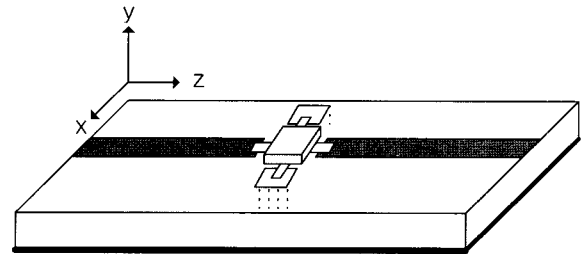


Fig. 2. The MESFET in a stripline circuit.

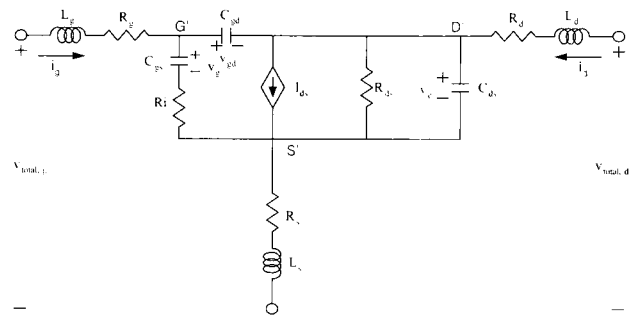


Fig. 3. Equivalent circuit of the MESFET.

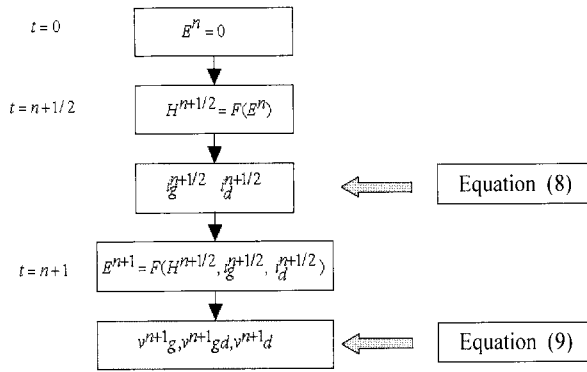


Fig. 4. The procedure of the FDTD algorithm to analyze a nonlinear active circuit.

$$C_{gs} \frac{dv_g}{dt} = \frac{-v_g + v_{gd} + v_d}{R_i} \quad (9a)$$

$$C_{gs} \frac{dv_g}{dt} + C_{gd} \frac{v_{gd}}{dt} = i_g \quad (9b)$$

$$C_{gd} \frac{dv_{gd}}{dt} - C_{ds} \frac{dv_d}{dt} + i_d - I_{ds} - \frac{v_d}{R_{ds}} = 0 \quad (9c)$$

The procedure of FDTD algorithm to analyze a nonlinear active circuit is illustrated in Fig. 4.

As shown in Fig. 4 and the basic FDTD equations, at  $t = n + 1/2$ , the update of  $H^{n+1/2}$  is performed by the values of  $E^n$  and  $H^{n-1/2}$ . The device currents are calculated by the electric fields  $E^n$  and the device voltages. At  $t = n + 1$ , the electric fields  $E^{n+1}$  are updated from  $H^{n+1/2}$  and  $E^n$ . And the device voltages are calculated by the device currents.

#### 2-4 The Analysis of NEC NE72089 MESFET

The parameters of large signal circuit model of The NEC NE72089 MESFET in FDTD simulation are given in Table 1 [7].

Fig. 5 and Fig. 6 show the magnitudes and phases of  $S_{11}$  and

Table 1. Element values of NEC NE 72089 MESFET equivalent circuit model.

Parameter	Value	Parameter	Value
$L_g$	0.906 nH	$R_d$	1.12 $\Omega$
$L_d$	0.937 nH	$R_s$	0.365 $\Omega$
$L_s$	0.320 nH	$R_g$	1.14 $\Omega$
$C_{gd}$	0.086 pF	$R_i$	2.37 $\Omega$
$C_{ds}$	0.378 pF	$g_{ds1}$	4.25 mA/V
$g_{m1}$	48.2 mA/V	$g_{m2}$	11.1 mA/V <sup>2</sup>
$g_{m3}$	-9.13 mA/V <sup>3</sup>	$C_{gs1}$	0.425 pF
$C_{gs2}$	0.138 pF/V	$C_{gs3}$	0.0895 pF/V <sup>2</sup>

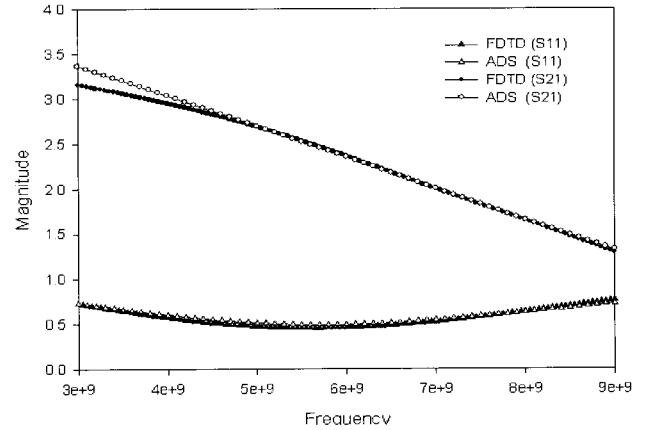


Fig. 5. Magnitudes of  $S_{11}$ ,  $S_{21}$  of the MESFET as a function of frequency.

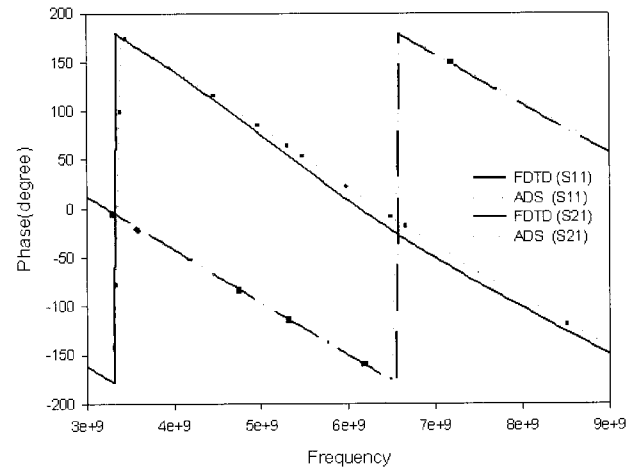


Fig. 6. Phases of  $S_{11}$ ,  $S_{21}$  of the MESFET as a function of frequency.

$S_{21}$ .  $S_{11}$  values are calculated at 8.8 mm away from the gate and  $S_{21}$  are evaluated at 10.0 mm away from the drain. In Fig. 5 and Fig. 6, magnitudes and phases of the S-parameters are compared with those of the ADS. Comparison shows that the proposed FDTD results match very closely to the ADS results.

To verify the usefulness of the proposed method, S-parameters under various bias conditions is evaluated. Also the stability of a transistor with series resistor is investigated.

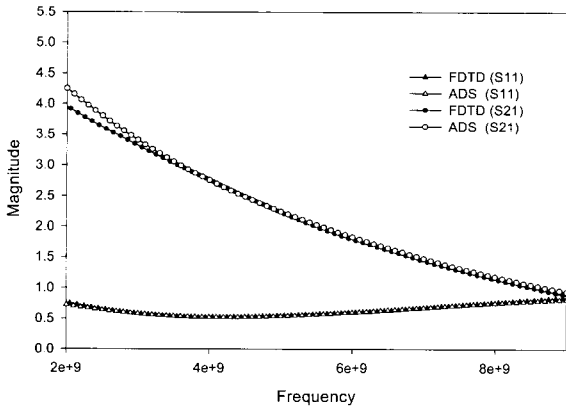
##### 2-4-1 The Variation of Bias Condition

The variation of element values of NEC NE72089 MESFET equivalent circuit model at various bias points are shown in Table 2.

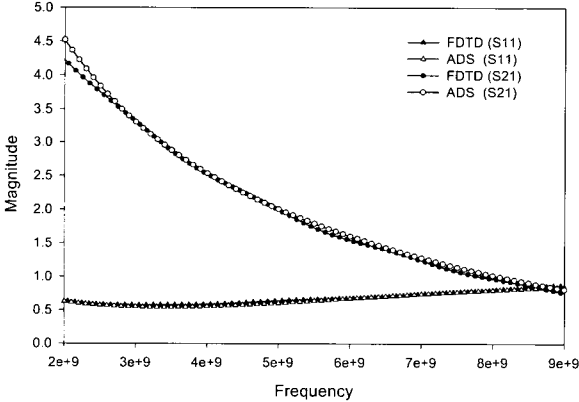
In Table 2, the variation of element values of NEC NE72089 MESFET equivalent circuit model is described. The S-para-

Table 2. Variation of element values of NEC NE72089 MESFET equivalent circuit model.

Parameter	Value(#case 1)	Value(#case 2)
$C_{gd}$	0.15 pF	0.2 pF
$g_{m1}$	70.0 mA/V	100.0 mA/V
$g_{m2}$	20.0 mA/V <sup>2</sup>	40.0 mA/V <sup>2</sup>
$g_{m3}$	-10.0 mA/V <sup>3</sup>	-10.0 mA/V <sup>3</sup>
$C_{gs1}$	0.7 pF	1.0 pF
$C_{gs2}$	0.4 pF/V	0.5 pF/V
$C_{gs3}$	0.1 pF/V <sup>2</sup>	0.2 pF/V <sup>2</sup>



(a) Magnitudes of  $S_{11}$ ,  $S_{21}$  at case 1



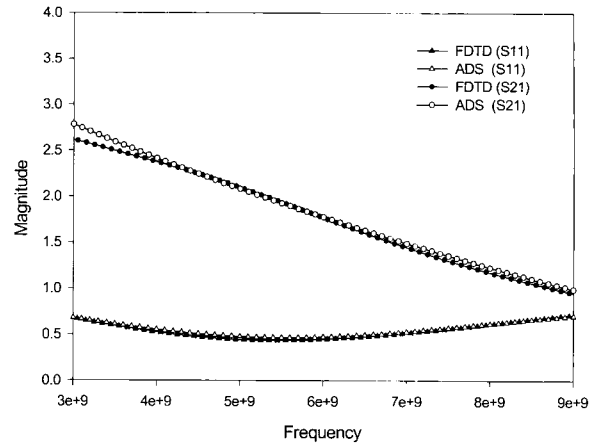
(b) Magnitudes of  $S_{11}$ ,  $S_{21}$  at case 2

Fig. 7. Magnitudes of  $S_{11}$ ,  $S_{21}$  of the MESFET at various bias points.

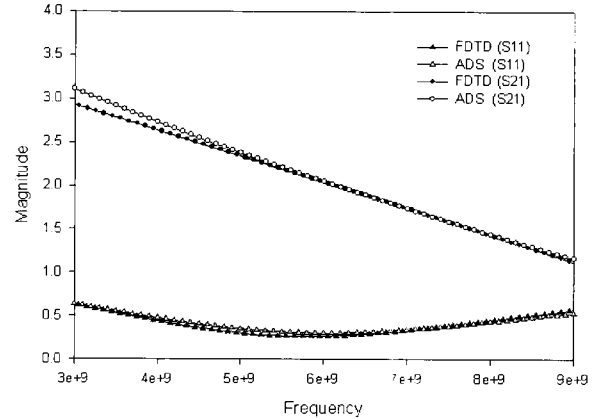
meters of the transistors are calculated by taking the Fourier transform of the time response at two bias points. In Fig. 7, the results of proposed FDTD method are compared with those of ADS. The agreement is good between the two data sets and the usefulness of this method is verified.

#### 2-4-2 Transistor with Resistive Loading

In many cases, a series and/or shunt resistive loading of



(a) An output series resistive loading transistor ( $R=25 \Omega$ )



(b) An input series resistive loading transistor ( $R=10 \Omega$ )

Fig. 8. Magnitudes of  $S_{11}$ ,  $S_{21}$  of the MESFET with series resistors.

transistor are used to improve stability. When output and input of a transistor are loaded with series resistors S-parameters are calculated. In Fig. 8, ADS and FDTD results are compared and their results agree very well. Therefore the proposed FDTD method is useful in the analysis of active circuits with resistors to improve stability or to compose matching circuits.

### III. Analysis of Microwave Nonlinear Amplifier

Nonlinear microwave amplifier configuration using the NEC NE72089 MESFET is shown in Fig. 9. The line lengths of microstrip matching circuits are given in Table 3. Output series resistor ( $R=25 \Omega$ ) is used to improve the stability over a wide bandwidth. This circuit was simulated on substrates with  $\epsilon_r = 2.2$  and thickness of 0.795 mm. The space steps used are  $\Delta x = 0.44$  mm,  $\Delta y = 0.265$  mm and  $\Delta z = 0.4$  mm.

In Fig. 10, FDTD analysis results and those of ADS are compared. The center frequency of designed amplifier is 3.8

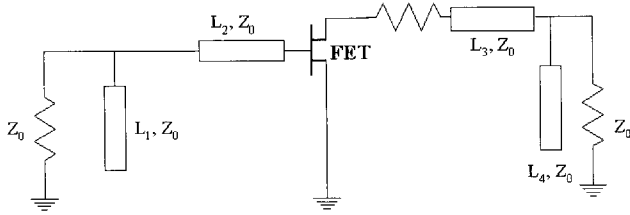


Fig. 9. The nonlinear microwave amplifier configuration.

Table 3. The microstrip line lengths of amplifier.

Parameter	Length(mm)	Parameter	Length(mm)
$L_1$	9.2	$L_2$	10.0
$L_3$	13.4	$L_4$	6.2

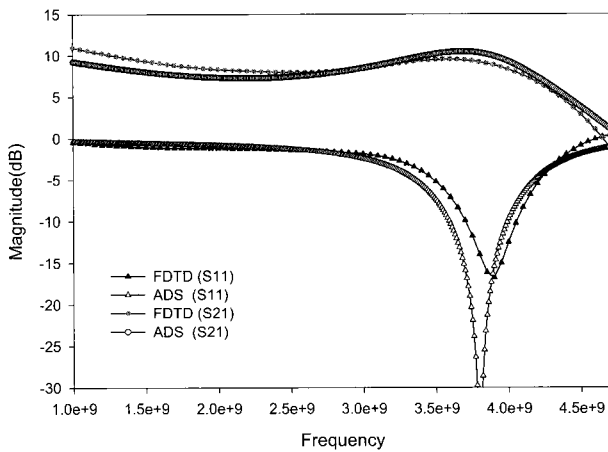


Fig. 10. Comparison between the FDTD analysis and the ADS.

GHz. To calculate S-parameters of a nonlinear amplifier in Fig. 10, 2500 time steps were used. The computational time is 43 minutes 30 seconds on Pentium IV personal computer. The calculated s-parameter values are in good agreement with those of the ADS.

#### IV. Conclusion

This paper presents the full wave analysis of microwave circuits with nonlinear device using the finite difference time domain method. For more stable and accurate analysis, we incorporate the active sheet between the end of microstrip line and ground plane, in which all the electric field components are

updated according to the port termination. The equivalent current source is used to model nonlinear device and all the electric field components at the nonlinear device are updated by FDTD algorithm. The currents and voltages of nonlinear device are calculated by the state equations and iteration method.

To verify the utility and efficiency of the proposed method, the S-parameters of NEC NE72089 MESFET at various bias points are calculated and the results are compared with those of ADS. The proposed method is applied to the analysis of a microwave amplifier. The calculated S-parameter values are in good agreement with those of the ADS.

The method suggested in this paper can be extended to analyze the characteristics of more conventional active circuits (or devices).

This work was supported by Korea Research Foundation under the program BK21.

#### References

- [1] C. N. Kuo, B. Houshmand and T. Itoh, "Full-Wave Analysis of Packaged Microwave Circuit with Active and Nonlinear Device: An FDTD Approach", *IEEE Transaction on Microwave Theory and Techniques*, vol. 45, no. 5, pp. 819-826, May 1997.
- [2] V. A. Thomas, M. E. Jones, M. Picket-May, A. Taflove and E. Harrigan, "The Use of SPICE Lumped Circuits as Sub-grid Models for FDTD Analysis", *IEEE Microwave and Guided Wave Letters*, vol. 4, no. 5, pp. 141-143, May 1994.
- [3] V. S. Reddy, R. Garg, "An Improved Extended FDTD Formulation for Active Microwave Circuits", *IEEE Transaction on Microwave Theory and Techniques*, vol. 47, no. 9, pp. 1603-1608, September 1999.
- [4] *Advanced Design System version 1.5*, Agilent Technologies, 2000.
- [5] A. Taflove, *Computational Electromagnetics : the finite-difference time domain method*, Artech House, 1995.
- [6] C. A. Desoer, E. S. Kuh, *Basic circuit theory*, New York: McGraw-Hill, 1969.
- [7] C. C. Huang, T. H. Chu, "Analysis of MESFET Injected-Locked Oscillator in Fundamental Mode of Operation", *IEEE Transaction on Microwave Theory and Techniques*, vol. 42, no. 10, pp. 1851-1857, October 1994.

Hee-Jin Kang



was born in Jeju, Korea. She received the B.S degree from Jeju University, Jeju, Korea, in 1996, and received the M.S. and Ph. D. degrees from Hanyang University, Seoul, Korea, in 1998 and 2002, respectively. Her current research interests include computational electromagnetics and microwave device design.

Jae-Hoon Choi



received the B.S. degree from Hanyang University, Seoul, Korea, in 1980, and the M.S. and Ph. D. degrees from the Ohio State University in 1986 and 1989, respectively. From Sept. 1989 to Feb. 1991, he was an research analyst with Telecommunications Research Center at Arizona State University, Tempe, Arizona. He had been with the Korea Telecom as a team leader of Satellite Communications Division from 1991 to 1995. In 1995, he joined the Department of Electrical and Computer Engineering, Hanyang University as an assistant professor and is now an associate professor at the same university. His research interests are computational electromagnetics, satellite and mobile communication antenna design, and microwave device design.