An Approximate Scattering Analysis for Microstrip T-junction

Hyo-J. Eom[1] · Hyun-H. Park[2]

Abstract

An approximate, numerically-efficient solution for a microstrip T-junction is discussed. The microstrip T-junction is modeled as a rectangular waveguide with top/bottom electric walls and side magnetic walls. Comparisons of our solution with others show favorable agreements.

Key words : Microstrip T-junction, Electromagnetic Scattering

Ⅰ. Introduction

A microstrip T-junction is one of basic microstrip passive elements. The problem of microstrip T-junction scattering has been studied by many investigators using approximate and numerical approaches[1]~[6]. A complete full wave analysis for the microstrip T-junction requires much of numerical computations. The purpose of the present paper is to show that an approximate solution exists in rapidly-converging series under a certain assumption. The approximate solution in this paper provides the analytic expressions for scattering from the T-junction. We assume that a microstrip may be modeled as a rectangular waveguide made of top/bottom electric walls and side magnetic walls[7]. We show that our approximate solution for the microstrip T-junction is identical with that for the parallel-plate T-junction of E-plane considered in [8].

Ⅱ. Field Representation and Numerical Results

We wish to model a microstrip (width : W) on a substrate (dielectric constant : ε_r) as a rectangular waveguide (effective width : W_{eff} , effective dielectric constant : ε_{ff}) surrounded by top/bottom electric walls and side magnetic walls. The relationships are referred to [9]

$$\varepsilon_{ff} = \begin{cases} \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[\left(1+12\frac{h}{W}\right)^{-1/2} + 0.04(1-W/h)^2\right] & \text{for } W/h \le 1 \\ \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left(1+12\frac{h}{W}\right)^{-1/2} & \text{for } W/h \ge 1 \end{cases} \tag{1}$$

$$W_{eff} = \frac{377h}{Z_0\sqrt{\varepsilon_{ff}}} \tag{2}$$

where h is a dielectric thickness and Z_0 is the characteristic impedance of microstrip line. Typically the characteristic impedance is 50 Ω.

Consider a microstrip T-junction as shown in Fig. 1. The total E-field in region (I) ($-b<z<0$) consist of the incident and scattered fields.

$$E_y^{iI}(x,z) = a \cos b_s(z+b)\, e^{i\gamma_s x}\,, \tag{3}$$

$$E_y^{I}(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}_y^{I}(\zeta)\cos(\kappa z)\, e^{i\zeta x} d\zeta \tag{4}$$

where

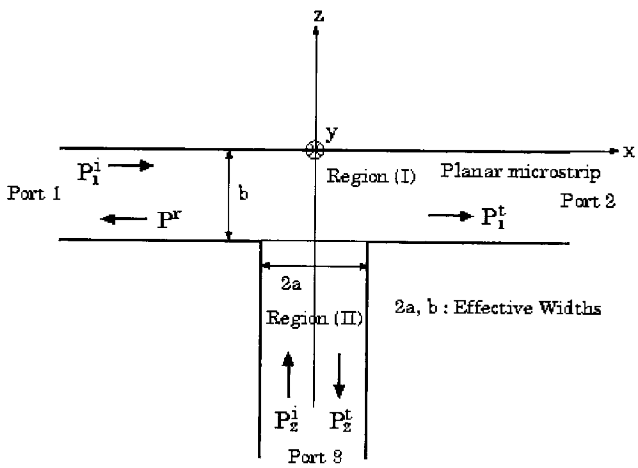

Fig. 1. Geometry of scattering problem ; rectangular waveguide model for microstrip T-junction.

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$b_s = \frac{s\pi}{b}(s=0,1,2,\cdots)$, $\gamma_s = \sqrt{\chi^2 - b_s^2}$, $\chi = \sqrt{k^2 - \zeta^2}$, and $k = \omega\sqrt{\mu\varepsilon_o\varepsilon_{ff}} = 2\pi/\lambda$ is the wave number in substrate.

In region (II) ($z < -b$, $|x| < a$), the incident and transmitted fields are

$$E_y^{iII}(x,z) = \beta\cos a_p(x+a)e^{i\xi_p z}, \quad (5)$$

$$E_y^{II}(x,z) = \sum_{m=0}^{\infty} c_m \cos a_m(x+a)e^{-i\xi_m z} \quad (6)$$

where $\xi_m = \sqrt{k^2 - a_m^2}$ and $a_m = \frac{m\pi}{2a}$. We note that (3), (4), and (6) for $E_y(x,z)$ are identical with (2.1), (2.2), and (2.5) for $H_y(x,z)$ in [8] where the problem of parallel-plate T-junction of E-plane is considered. This means that the mathematical formulation for the microstrip T-junction considered here is similar to that in [8].

Using the H_x(for $-\infty < x < \infty$) and E_y(for $|x| < a$) continuities at $z = -b$, we obtain the simultaneous equations for c_m as

$$\frac{a\alpha\gamma_s[(-1)^n e^{i\gamma_s a} - e^{i\gamma_s a}]}{(\gamma_s a)^2 - \left(\frac{n\pi}{2}\right)^2} = i\chi_n[c_n e^{i\xi_n b} + \delta_{np}\beta e^{-i\xi_p b}]$$
$$-\frac{a^3}{2\pi}\sum_{m=0}^{\infty}\xi_m I_{mn}[c_m e^{i\xi_m b} - \delta_{mp}\beta e^{i\xi_p b}] \quad (7)$$

where $\chi_0 = 2$, $\chi_1 = \chi_2 = \cdots = 1$ and

$$I_{mn} = \begin{cases} \frac{2\pi\chi_n}{a^3\xi_n\tan(\xi_n b)}\delta_{mn} + \sum_{\nu=0}^{\infty}\frac{-i4\pi\gamma_\nu[1-(-1)^m e^{i2\gamma_\nu a}]}{\chi_\nu a^4 b[\gamma_\nu^2 - a_m^2][\gamma_\nu^2 - a_n^2]} \\ \text{if } m+n \text{ is even} \\ 0 \\ \text{if } m+n \text{ is odd.} \end{cases} \quad (8)$$

The transmitted and reflected fields at $x = \pm\infty$ are

$$E_y^I(\pm\infty, z) = \sum_\nu K_\nu^{\pm}\cos b_\nu(z+b)e^{\pm i\gamma_\nu x} \quad (9)$$

where

$$K_\nu^{\pm} = \sum_{m=0}^{\infty}\frac{\pm i\xi_m[\delta_{mp}\beta e^{-i\xi_p b} - c_m e^{i\xi_m b}][(-1)^m e^{\mp i\gamma_\nu a} - e^{\pm i\gamma_\nu a}]}{\chi_\nu b(\gamma_\nu^2 - a_m^2)}, \quad (10)$$

$$0 \le m < \frac{2ak}{\pi} \quad (m:\ integer).$$

When $\beta = 0$ (no incidence from port 3), the simultaneous equations (7) reduce to (2.13) in [8].

Although the final expressions for the reflection(ρ) and transmissions(τ_1 and τ_2) coefficients with $\beta = 0$ are identical with those in [8], we repeat here for the sake of convenience.

$$\tau_1 = \frac{P_1^t}{P^i} = |1 + K_s^+|^2 + \sum_{\nu\neq s}\frac{\chi_\nu\gamma_\nu^*}{\chi_s\gamma_s^*}|K_\nu^+|^2, \quad (11)$$

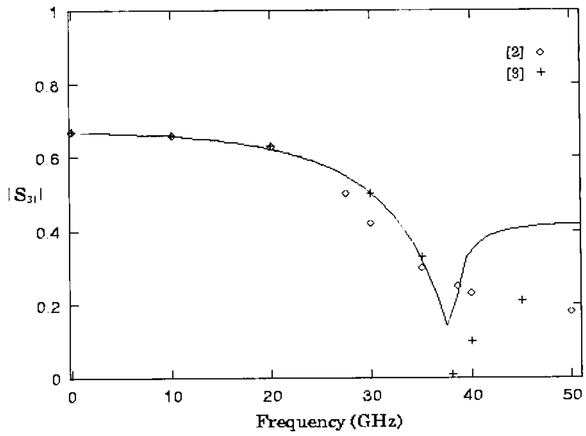

Fig. 2. $|S_{31}|$ comparison with [2] and [3] ; $s=0$, $\varepsilon_r=9.7$, $h=0.635$ mm, $W=0.56$ mm, $\varepsilon_{ff}=6.49$, $2a = b = 1.54$ mm.

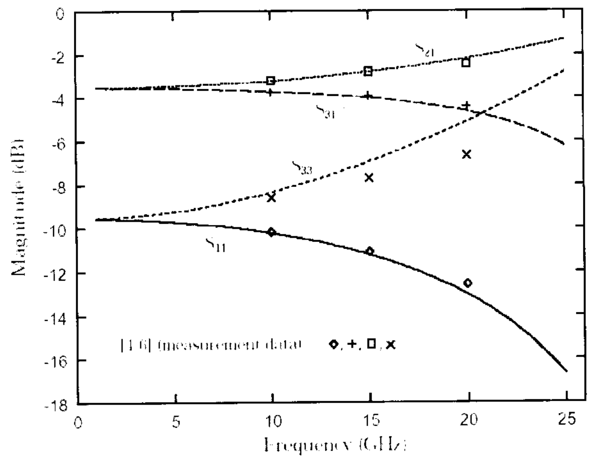

Fig. 3. Magnitude of S-parameters ; $s=0$, $\varepsilon_r=9.9$, $h=0.635$ mm, $W=0.6096$ mm, $\varepsilon_{ff}=6.66$, $2a = b = 1.855$ mm.

$$\rho = \frac{P^r}{P^i} = \sum_\nu\frac{\chi_\nu\gamma_\nu^*}{\chi_s\gamma_s^*}|K_\nu^-|^2, \quad (12)$$

$$\tau_2 = \frac{P_2^t}{P^i} = \frac{2a}{b}\sum_m\frac{\chi_m\xi_m^*}{\chi_s\gamma_s^*}|c_m|^2. \quad (13)$$

Figs. 2 and 3 show comparisons between ours and other existing solutions[2]~[6]. Our solution agrees favorably for $f < 40\,GHz$ although our approximate solution ignores the effects of radiation or surface wave generation. The number of modes used in (7) is 6, indicating a rapidly-converging rate of our solution.

Ⅲ. Conclusion

The microstrip T-junction problem is shown to be mathematically equivalent to the parallel-plate T-junction of E-plane. The solution agrees well with other existing solution in low frequency regime where the radiation effects are not significant. The series solution in this paper is easy to use in low-frequency regime.

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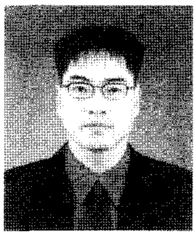

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