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A Study on Optimal Design of Single Periodic, Multipurpose Batch Plants

In Hyoung Rhee and Daechul Cho*

요 약 본고는 다종, 소규모 회분식 공정 또는 공장을 수학적 프로그래밍 기법으로 최적 설계하는 방법에 대한 것이다. 제안된 일반 회분식 공장문제는 Papageorgaki와 Reklaitis에 의해 혼합정수비선형식(MINLP)으로 수립된 것인데, 최적해답을 보장하며 공장의 확장 등 불확실성을 고려하여 선형화 한 후(MILP) 푸는 방법론이 제시되었다. Bender식 문제분할 방식을 개조하여 몇가지 예제에 대한 풀이를 제시하였다. IBM의 OSL 최적화 패키지를 이용하였고 MILP를 직접 푸는 경우보다 계산시간을 크게 단축할 수 있었다.

Abstract The purpose of this paper is to describe the design of a general multipurpose batch process or plant in terms of a series of mathematical programing models, and to develop approach solution methodologies. The proposed model for a single period is based on the formulation (MINLP; Mixed Integer Nonlinear Programming) of Papageorgaki and Reklaitis [1], but was linearized (MILP; Mixed Integer Linear Programming) so as to obtain an exact and practical solution, and to allow treatment of uncertainties to be considered in expanding the plant. As a solution strategy a modified Benders' Decomposition was introduced and was tested on three example problems. The optimizing solver, OSL code provided by the IBM Corporation, was used for solving the problems. The solution method was successful in that it showed remarkable reduction in the computing times as compared with the direct solution method.

Key Words : multipurpose batch plant, decomposition, linearization, MILP, single period

1. INTRODUCTION

The recent rapid growth, change of market and industrial needs in specialty chemicals, pharmaceuticals and food products manufactured by batch plants have raised the necessity of research in methods for computer aided design. Especially, multipurpose plants, which handles a variety of production system and changeable scheduling, has been paid more attention for last decade than simple multiproduct plants due to frequent, globalized market change and inevitable competition on cost minimization.

Pioneering research on multiproduct batch plants with single product campaigns was reported by Robinson and Loonkar [2]. That was followed up by Sparrow et al. [3] who developed a model including the optimal selection of the number of parallel units as well as optimization over discrete equipment sizes. Their solution approach was to use heuristics and a branch and bound method. Another MINLP model was presented for the design of multipurpose batch plants with single product campaigns (Grossmann and Sargent) [4]. This model was different from the previous model in that a problem relaxation technique along with the integer enforcing constraints was used and the number of units made integer variables.

Subsequently, a few researchers introduced consideration of semicontinuous units into the design to broaden the scope of the batch plants considered [5, 6]. The resulting problem (MINLP) has the number of batch and semicontinuous units considered as discrete variables to be solved in the way suggested by Grossmann and Sargent. Furthermore, heuristic design principles, such as determination of parallel units, merging or splitting of batch units, were proposed for optimal design.

All the above models are restricted to single product campaigns. Suhami and Mah [7] introduced an MINLP model which yields the optimal selection of configuration of compatible products (campaigns) that can be produced simultaneously. Obtaining the best configuration among all candidates is a combinatorial problem which was solved using a heuristic

*Division of Materials and Chemical Engineering, Soonchunhyang University Asan, 336-745 Korea
Tel. 041-530-1341; HP. 011-9888-5591; e-mail: daechul@sch.ac.kr

rule. This work was extended by the "embedding formulation" proposed by Vaselenak et al. [8]. In that work, a superstructure was proposed that embeds all the grouping of products that are candidates for the optimal schedule. This model reduced the multipurpose design problem to an MINLP which avoids the combinatorial complexity of testing alternative product configurations as in the method of Suhami and Mah. An alternative approach which was very similar to the above, was made by Faqir and Karimi [9]. They developed an efficient way to find a set of dominant horizon constraints for describing the superstructure of campaign.

More recently, a more general problem formulation for multipurpose batch plants was reported. Papageorgaki and Reklaitis [10] employed unit-task-campaign allocation methods in the formulation to identify the design configuration more specifically. Also identical and non-identical parallel units were considered in the MINLP model. All other variables, except for unit-task-campaign allocation variables were treated as continuous variables. To obtain a solution for discrete equipment size, rounding off to nearest discrete size was adopted as a heuristic.

All of the models that have been addressed so far did not use an exact MINLP or MILP formulation to get an optimal solution. Treatment of discrete variables as continuous introduces a gap between the suboptimal solution and the true optimal solution that has not been resolved to date. Therefore, a more rigorous formulation is needed at the expense of greater computing effort, which might be reduced in the near future by exploiting the problem structure. A contribution along this lines was published in 1992 by Voudouris and Grossmann [11] who introduced binary variables for denoting discrete equipment sizes in their linearized MILP formulations. Several cases such as those of single product campaigns, multiple product campaigns, single production routes and multiple production routes were explored, but the results were not compared with previous work.

As seen so far, a rigorous and general formulation of the multipurpose batch plant design problem based only on a mathematical description excluding heuristics and/or simplifying assumptions has not been given yet.

To achieve this, it is necessary to generate a robust batch plants model with discrete and continuous variables. To guarantee optimality, an MILP model would be preferably used. Because so-called performance enhancing techniques such as SOS, bounding, valid cuts, and so on, along with existing MILP commercial algorithms can be applied. Furthermore, a linear model takes on the role of a stepping stone for a stochastic batch plant model that will be presented in the future.

The formulation to be presented here is a fully and rigorously linearized version of that of Papageorgaki and Reklaitis [1].

2. MATHEMATICAL FORMULATION

2.1 Decomposition Approach to Mixed Integer Problems

The difficulty in the solution of mixed integer problems is that, due to the combinatorial nature of these problems, there are no optimality conditions, such as a Kuhn-Tucker point in the continuous case. That point can be directly exploited for developing efficient solution methods. A relatively well-established method for dealing with this class of problems is decomposition (or partition), whose applicability to a problem strongly depends on the problem structure.

Decomposition was introduced in the linear programming context by Danzig, a pioneer in that area, as the Danzig-Wolfe decomposition [12]. This method was used to handle large scale linear programming problems by splitting the original problem into several subproblems according to the structure of the matrix. The idea was extended and exploited in mixed-variable problems by Benders [13]. Theoretical development of a programming problem (master; which may be discrete, nonlinear etc.) and a linear programming problem (subproblem) from a mathematically complicated original problem was discussed and a computational procedure for solving those problems was presented in his work. This work was extended comprehensively by Geoffrion [14, 15]. Papageorgaki [1]. They employed a modified Benders' decomposition for solving an MINLP problem, in which the master problem was reduced to an MILP by mathematical manipulations and the sub-problem was simply an NLP. Lee [16] also exploited Benders' decomposition to solve a heat exchanger network scheduling problem.

2.2 Original Model Formulation

A general multipurpose batch plant design model has been reported by Papageorgaki and Reklaitis [10]. That formulation is structured as follows;

$$\min \sum_{e=1}^{E} a_e NU_e (V_e)^{be} \tag{1}$$

subject to

$$\sum_{k \in E} \sum_{e \in P_{im}} X_{imek} \ge 1 \quad \forall i \forall m \tag{2}$$

$$X_{imek} + X_{imek} \le 1 \quad e \ne e'; \forall i \forall m \forall k \tag{3}$$

$$X_{i(m-n)e'k} + X_{imek} + X_{i(m+1)e'k} \le 2$$

$$n = 1, m-1 : e \ne e'; \forall i \forall m \forall k \tag{4}$$

$$X_{imek} \le \sum_{e' \in P_{im}} X_{im'e'k} \quad \forall i \forall k \tag{5}$$

$$\sum_{k \in k} n_{ik} B_{ik} \ge Q_i \quad \forall i \tag{6}$$

$$B_{ik} \le \sum_{e \in P_{im}} N_{imek} V_e / S_{ime} \quad \forall i \forall km \in TA_i \tag{7}$$

$$B_{ik} \ge \sum_{e \in P_{im}} X_{imek} \quad \forall i \forall km \in TA_i \tag{8}$$

$$N_e \ge \sum_{(i,m) \in Ue} NG_{imek} N_{imek} \quad \forall e \forall k \tag{9}$$

$$NU_{imek} \le N_e^{\max} X_{imek} \quad \forall i; m \in TA_i; e \in P_{im}; \forall k \tag{10}$$

$$NU_{imek} \ge X_{imek} \quad \forall i; m \in TA_i; e \in P_{im}; \forall k \tag{11}$$

$$NG_{imek} \le NG_{imk}^{\max} \sum_{e \in P_{im}} X_{imek} \quad \forall i; m \in TA_i; \forall k \tag{12}$$

$$TL_{ik} \ge X_{imek} t_{ime} / NG_{imk} \quad \forall i; m \in TA_i; e \in P_{im}; \forall k \tag{13}$$

$$TL_{ik} \le TL_{ik}^{\max} \sum_{m \in TA_i} \sum_{e \in P_{im}} X_{imek} \quad \forall i \forall k \tag{14}$$

$$\sum_{k \in K} T_k \le H \tag{15}$$

$$T_k \ge n_{ik} \quad \forall i \forall k \tag{16}$$

with only integer variables, X_{imek}, and some variable bounds.

Subscripts i, m, e and k denote product, task equipment type and campaign, respectively. TA_i, P_{im} and U_e are sets related to tasks and types of equipment and K is also a set of campaigns. X_{imek} denotes 0-1 unit-task-campaign assignment variables and V_e, N_e are unit capacity and number of units of type e, respectively. NU_{imek} and NG_{imk} describe the number of units in parallel in phase and the number of parallel groups which are out of phase. TL_{ik}, T_k and n_{ik} denote the limiting cycle time, campaign length, and number of batches, respectively, The two constant parameters, t_{ime} and S_{ime} stand for the processing times and the equipment size factors.

Eq. (1) is a nonlinear objective function consisting of the equipment capital cost. Eq. (2) insures that each task is allocated to at least one unit and one campaign. Eqs. (3) and (4) require that only one type of equipment is allowed at each stage and no unit can be reused in the same production line. In Eq. (5), all the tasks inb the production of a specific product in a specific campaign must be completed in that campaign.

The model has also a constraint that the total production of a product over the entire horizon has to meet the production demand (Eq. (6)). The batch size cannot exceed the minimum capacity of the production line (Eq. (7)). Eq. (9) describes the restriction on the utilization of each type of equipment. In Eq. (13) we can see that the limiting cycle time is the maximum of the possible group processing times. Eq. (15) is the production horizon constraint. Finally Eq. (16) insures that each campaign accommodates the total production time for a product which is assigned to the campaign.

2.3 Modification of the Model

We propose to make this model more rigorous with the conversion of continuous variables to integer ones so as to insure true optimality. We will also attempt to remove the nonlinearity in the model. The following development is based on the original model, but uses linearization introduced via the mathematical treatment shown below.

The first step in the linearization is to discretize the size of the units. That is, the index j was introduced with a new binary variable, Y_{je} to denote possible, discrete unit sizes.

$$V_e = \sum_{j=j^{max}}^{j^{max}} v_{je} * Y_{je} \tag{17}$$

The total number of units of any type, N_e, can be expressed in the same way;

$$N_e = \sum_{p=1}^{p^{max}} p * Z_{pe} \tag{18}$$

Thus, the new objective function of OP becomes as follows:

$$\min \sum_{p=1}^{p^{max}} \sum_{j=j^{min}}^{j^{max}} \sum_{e=1}^{e^{max}} pa_e v_{je}^{be} Y_{je} * Z_{pe} \tag{19}$$

We next apply a well-known mathematical device to the above function. which is nonlinear, to result in a linear formula. Note that the binary product, $Y*Z$ becomes unity only when Y and Z are both unity; otherwise, that product would be zero.

Mathematically this can be stated as follows;

$$Y_{je} + Z_{pe} - \alpha_{pje} \leq 1 \tag{20}$$

$$Y_{je} + Z_{pe} - 2 * \alpha_{pje} \geq 0 \tag{21}$$

where α is a newly introduced binary variable for $Y*Z$. Equations (7) and (9) can be modified in a similar way.

From Eq. (6)-(7), eliminating B_{ik}, we obtain

$$\sum_{k \in K} \sum_{e \in P_{im}} n_{ik} NU_{imek} V_e / S_{ime} \geq \sum_{k \in K} n_{ik} B_{ik} \geq Q_i$$

or

$$\sum_{k \in K} \sum_{e \in P_{im}} n_{ik} NU_{imek} V_e / S_{ime} \geq Q_i$$

By introducing

$$N_{imek} V_e = \sum_{q=1}^{q^{max}} \sum_{j=j^{min}}^{j^{max}} q v_{je} \beta_{qjimek},$$

$$N_{imek} = \sum_{q=1}^{q^{max}} q U_{qjimek} \text{ and } \beta_{qjimek} = U_{qjimek} Y_{je}$$

Eq. (6) is transformed to the following form;

$$\sum_{k \in K} \sum_{e \in P_{im}} \sum_{q=1}^{q^{max}} \sum_{j=j^{max}}^{j^{max}} n_{ik} \beta_{qjimek} V_e q / S_{ime} \geq Q_i \tag{6'}$$

By introducing a new continuous variable, PI_{qjimek} (the product of n_{ik} and b β_{qjimek}), the following three inequalities become equivalent to Eq. (6).

$$\sum_{k \in K} \sum_{e \in P_{im}} \sum_{j=j^{min}}^{j^{max}} \sum_{q=1}^{q^{max}} \frac{qV_{je}}{S_{ime}} PI_{qjimek} \geq Q_i \tag{22}$$

$$PI_{qjimek} \leq n_{ik}^{max} \beta_{qjimek} \tag{23}$$

$$PI_{qjimek} \leq n_{ik} \tag{24}$$

In the same way, Eq. (13) and (16) can be combined into one constraint (elimination of TL_{ik}).

$$T_k \geq \frac{n_{ik} X_{imek}}{NG_{imek}} t_{ime}$$

By introducing a new binary variable, G_{gimek}, the quantity of X_{imek}/NG_{imk}, can be replaced with $\sum_{g=1}^{g^{max}} G_{gimek}/g$.

Finally, we obtain

$$T_k \geq \sum_{g=1}^{g^{max}} \frac{t_{ime}}{g} PSI_{gimek} \tag{25}$$

where $PSI_{gimek} = n_{ik} G_{gimek}$

$$PSI_{gimek} \leq n_{ik}^{max} G_{gimek} \tag{26}$$

$$PSI_{gimek} \leq n_{ik} \tag{27}$$

$$\sum_{g=1}^{g^{max}} \sum_{e \in P_{im}} PSI_{gimek} \geq n_{ik} \tag{28}$$

Eq. (28) is necessary since T_k otherwise would be reduced to zero.

2.4 Linearized Original Problem (LOP; MILP)

Objective

$$\min \sum_{p=1}^{p^{max}} \sum_{j=j^{min}}^{j^{max}} \sum_{e=1}^{e^{max}} a_e * p * V_{je}^{be} * \alpha_{pje} \tag{29}$$

Assignment and Connectivity

$$\sum_{P=1}^{p^{max}} \sum_{j=j^{min}}^{j^{max}} \sum_{e=1}^{e^{max}} \alpha_{pje} \geq 1 \tag{30}$$

$$\sum_{k \in K} \sum_{e \in P_{im}} X_{imek} \geq 1 \quad \forall i;\forall m \tag{31}$$

$$\sum_{e \in P_{im}} X_{imek} \leq 1 \quad \forall i;\forall m;\forall k \tag{32}$$

$$X_{i(m-n)e'k}+X_{imek}+X_{i(m+1)e'k} \leq 2$$

$$n = 1, m-1; e; \neq e'; \forall i;\forall m;\forall k \tag{33}$$

$$X_{imek} \leq \sum_{e \in P_{im'}} X_{im'e'k} \leq 1 \quad \forall i;\forall m; e \in P_{im};\forall k \tag{34}$$

Equipment Bounds

$$\sum_{P=1}^{P^{MAX}} pZ_{pe} \geq \sum_{(i,m) \in U_e} \sum_{q=1}^{q^{\max}} \sum_{g=1}^{g^{\max}} qgw_{pgimek} \quad \forall e;\forall k \tag{35}$$

Batch Size

$$B_{ik} \leq \sum_{e \in P_{im}} \sum_{j=j^{\min}}^{j^{\max}} \sum_{q=1}^{q^{\max}} \frac{qv_{je}}{S_{ime}} \beta_{qjimek} \quad \forall i;\forall m;\forall k \tag{36}$$

$$B_{ik} \leq B_{ik}^{\max} \sum_{e \in P_{i1}} X_{i1ek} \quad \forall i;\forall k \tag{37}$$

$$B_{ik} \geq B_{ik}^{\min} \sum_{e \in P_{i1}} X_{i1ek} \quad \forall i;\forall k \tag{38}$$

Production Demand

$$\sum_{k \in K} \sum_{e \in P_{im}} \sum_{j=j^{\min}}^{j^{\max}} \sum_{q=1}^{q^{\max}} \frac{qV_{je}}{S_{ime}} PI_{qjimek} \geq Q_i \quad \forall i;\forall m \tag{39}$$

$$PI_{qjimek} \leq n_{ik}^{\max} \beta_{qjimek} \quad \forall i;\forall m; e \in P_{im};\forall k;\forall q;\forall j \tag{40}$$

$$PI_{qjimek} \leq n_{ik} \quad \forall i;\forall m; e \in P_{im};\forall k;\forall q;\forall j \tag{41}$$

Production Horizon

$$T_k \geq \sum_{g=1}^{g^{\max}} \frac{t_{ime}}{g} PSI_{gimek} \quad \forall i;\forall m; e \in P_{im};\forall k \tag{42}$$

$$PSI_{gimek} \leq n_{ik}^{\max} G_{gimek} \quad \forall i;\forall m; e \in P_{im};\forall_{im};\forall k;\forall g \tag{43}$$

$$PSI_{gimek} \leq n_{ik} \quad \forall i;\forall m; e \in P_{im};\forall_{im}; \forall k;\forall g \tag{44}$$

$$\sum_{g=1}^{g^{\max}} \sum_{e \in P_{im}} PSI_{gimek} \geq n_{ik} \quad \forall i;\forall m;\forall k \tag{45}$$

$$\sum_{k \in K} T_k \leq H \tag{46}$$

Campaign Ordering

$$\sum_{i \in I} CO_{ik} \geq \sum_{i \in I} CO_{i,k+1} k=1,...k^{\max}; I=A,B,C... \tag{47}$$

$$CO_{ik} \geq X_{i1ek} \quad \forall i;\forall k \tag{48}$$

$$CO_{ik} \leq \sum_{e \in P_{i1}} X_{i1ek} \quad \forall i;\forall k \tag{49}$$

Subsidiary Constraints

$$\sum_{j=j^{\min}}^{j^{\max}} Y_{je} \leq 1 \quad \forall e \tag{50}$$

$$\sum_{j=j^{\min}}^{j^{\max}} Y_{je} \leq \sum_{(i,m) \in U_e} \sum_{k \in K} X_{imek} \quad \forall e \tag{51}$$

$$M* \sum_{j=j^{\min}}^{j^{\max}} Y_{je} \geq \sum_{(i,m) \in U_e} \sum_{k \in K} X_{imek} \quad \forall e \tag{52}$$

where M = Maximum of $\sum_{(i,m) \in U_e} \sum_{k \in K} X_{imek}$

$$\sum_{p=1}^{p^{\max}} Z_{pe} = \sum_{j=j^{\min}}^{j^{\max}} Y_{je} \quad \forall e \tag{53}$$

$$\sum_{q=1}^{q^{\max}} U_{qiimek} = X_{imek} \quad \forall i;\forall m; e \in P_{im};\forall k \tag{54}$$

$$\sum_{q=1}^{g^{\max}} G_{giimek} = X_{imek} \quad \forall i;\forall m; e \in P_{im};\forall k \tag{55}$$

$$Y_{je}+Z_{pe}-\alpha_{pje} \leq 1 \quad \forall p;\forall j;\forall e \tag{56}$$

$$Y_{je}+Z_{pe}-2*\alpha_{pje} \geq 1 \quad \forall p;\forall j;\forall e \tag{57}$$

$$Y_{je}+U_{qimek}-\beta_{qjimek} \leq 1 \quad \forall i;\forall m; e \in P_{im};\forall k;\forall j;\forall q \tag{58}$$

$$Y_{je}+U_{qimek}-2*\beta_{qjimek} \geq 1$$

$$\forall i;\forall m; e \in P_{im};\forall k;\forall j;\forall q \tag{59}$$

$$G_{imek}+U_{imek}-\omega_{qgimek} \leq 1$$

$$\forall i;\forall m; e \in P_{im};\forall k;\forall j;\forall q \tag{60}$$

$$G_{imek}+U_{imek}-2*\omega_{qgimek} \geq 0$$

$$\forall i;\forall m; e \in P_{im};\forall k;\forall j;\forall q \tag{61}$$

For reduction of degeneracy in the campaign-product assignments, the indicator variable CO_{ijt} is introduced to ensure that the lower indexed campaigns involve more products. Eq. (50)-(52) simply state that only one size j of equipment type e must be chosen if the equipment type is used; in the same manner as for Z, U and G in Eq. (53) through (55). Eq. (56) through (61) are just the logical "AND" constraints for the related variables.

It is worth mentioning a minor advantage hidden in the formulation. When alpha represents the cost terms in the model and at least one of Y and Z is zero, the optimizing algorithm will naturally set alpha to zero. Eq. (57), then, can be deleted, because it is redundant.

3. SOLUTION STRATEGY

3.1 Modified Bender's Decomposition

Computing experience with direct solution approach to large MILP's indicates that these kinds of problems normally can require enormous computing effort. As a result we undertook the investigation of Decomposition Methods which are known to yield good approaches to large problems with special structural characteristics [14, 13].

Decomposition usually involves the construction of two problems derived from the original one, namely the master problem (MP) and the sub-problem (SP). The master problem is a broad relaxation of an original one, while the sub-problem is just the original one except that the values of 'complicating' variables are fixed, thus making the sub-problem much easier to solve. Lower bounds on the original problem are given by a properly formulated master problem (in the case of minimization) while an upper bound is provided by the solution of the sub-problem. Solving the two problems repeatedly, and tightening the lower bound given by the master problem will lead to optimality under appropriate termination conditions.

The dual of a mathematical programming problem has been used in decomposition of MILP's because the feasibility of the solution can be double-checked and proper development of the master problem can be achieved by using the duality theorem. However, the application of that principle may be restricted only to a class of MILP's with small number of integer variables; because, otherwise, the derived master problem which should have as much integrality as the original problem can not be handled properly.

Therefore, we choose a primal (as opposed to a 'dual') method in selecting the so-called "complicating variables" and building master and sub problems. In our formulation, the variables (especially integers) are categorized according to their characteristics. The key to solving the problem lie with the assignment variables since those will define the design of the batch plant and provide tight bounds so that the other integer variables have much fewer structural combinations when searched using the branch and bound technique. Consequently, we found that the assignment variables can serve properly as complicating variables. With those variables fixed we still have an MILP, sub problem but solving it becomes much easier than the initial formulation using the branch and bound node search.

A master problem is a relaxed form of the original formulation. Also it will only produce integer values of the complicating variables. Then, we can take advantage of simplicity of the MINLP formulation, in which the complicating variables, at most, are bounded in integers while the other ones are continuous. This property will save the time for the exhaustive node search required from solving the larger MILP otherwise.

3.2 Master Problem and Integer Cut

As mentioned before, the master-problem is a relaxation of (LOP). The simplest type of relaxation of an MILP is an LP-relaxation, where all binary variables are treated as continuous. But this does not give us a tight lower bound since it results in too many integer variables with fractional values. A modification of (OP), rather than (LOP) was considered to get a better master problem and thus improved lower bound. We next outline the procedure of building a suitable master problem.

From (OP) we know that

$$B_{ik} \le \frac{V_e NU_{imek}}{S_{ime}} \tag{62}$$

$$T_{kn_{ik}} TL_{ik} \ge n_{ik} \frac{t_{ime}}{NG_{imk}} \tag{63}$$

$$\sum_{k \in K} n_{ik} B_{ik} \ge Q_i \tag{64}$$

Combining all of these inequalities into one, we obtain,

$$\frac{T_k V_e NU_{imek} NG_{imk}}{S_{ime} t_{ime}} \geq Q_i \tag{65}$$

Now we define two new variables, $TCAP_{imek}$ and TV_e as follows;

$$TCAP_{imek} = V_e NU_{imek} NG_{imk} \tag{66}$$

$$TV_e = V_e N_e \tag{67}$$

Therefore, Eq. (64) can be written as

$$\frac{T_k TCAP_{imk}}{S_{ime} t_{ime}} \geq Q_i \tag{68}$$

On the other hand, from Eq. (9) we can derive the following equation.

$$V_e N_e \geq \sum_{(i,m) \in U_e} V_e NG_{imk} NU_{imek} \quad \forall e \forall k$$

or

$$TV_e \geq \sum_{(i,m) \in U_e} TCAP_{imek} \quad \forall e \forall k \tag{69}$$

But the above formulation is still not rigorous since many variables that must be integer-valued remain as continuous. Then we need to discretize the variables, TV, N and V in order to prevent the master problem from producing a relatively poor bound. Finally we obtain the following formulation.

$$\frac{T_k TCAP_{imk}}{S_{ime} t_{ime}} \geq Q_i \tag{70}$$

$$\sum_{j=j^{min}}^{J^{max}} \sum_{p=1}^{P^{max}} V_{je} * P * \alpha_{jpe} \geq \sum_{(i,m) \in U_e} TCAP_{imek} \quad \forall e \forall k \tag{71}$$

Also

$$X_{imek} \leq N_{imek} \leq N_e = \sum_{p=1}^{P^{max}} p * Z_{pe} \tag{72}$$

$$TCAP_{mek} \leq V_e^{max} N_e^{max} X_{imek} \tag{73}$$

$$TCAP_{mek} \geq V_e^{max} X_{imek} \tag{74}$$

with Eq. (28)-(33), Eq. (43)-(46) and Eq. (50).

For solving a master problem, two factors are prerequisite. First, integer cuts are needed in order to exclude the previous integer solutions from the next solution sets that will be given by the next master problem. The forms of these cuts are as follows [17]:

$$\sum_{x \in S1} X_{imek} - \sum_{x \in S0} X_{imek} \leq |S_1| - 1 \tag{75}$$

where $S_1 = \{X_{imek} \mid X_{imek} = 1\}$ and $S_0 = \{X_{imek} \mid X_{imek} = 0\}$

The other device is to remove the nonlinearity in Eq. (66) by parameterizing the variables, T_k.

4. RESULTS AND DISCUSSION

The MILP's were solved using OSL (Optimizing Subroutine Library) [18], which is encoded in FORTRAN(C) on an IBM AIX/6000 system. The flow diagram for solving LOP is given in Figure 1.

First, we start by solving a master problem, and solve a sub-problem (LOP) which matches with fixed X-values (equipment assignment to tasks and campaigns). Then a master-problem that will produce another set of X-values with a reasonable lower bound, excluding the former set of X's (manifested as integer cuts) is solved. Next termination conditions are tested, namely, whether the current lower bound exceeds the current upper bound or not, or the master problem is eventually infeasible. When the termination conditions are satisfied, the iterations are stopped because the optimal solution is found or the problem is proved to be infeasible/unbounded. Otherwise, we return to solve the next sub-problem and

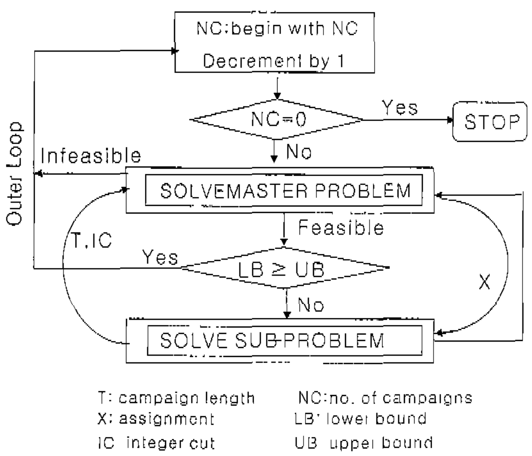

Figure 1. The Flow Diagram of the Bender's Decomposition Algorithm for a Single Period Model

Table 1. Data for calculation of Equipment Cost

Equipment Type	Cost Coeff.	Cost Exponent
E1	200	0.6
E2	220	0.6
E3	280	0.6
E4	300(360)	0.6
E5	350	0.6

Table 2. Equipment and operation information in test problem 1

Max. No. of Groups (out of phase)	1
Max. No. of Unit (in phase)	2
Allowable Unit Sizes (kg)	2000, 3000, 4000
Time Horizon (hr)	6000
Demand in product A (kg/horizon)	600000
Demand in product B (kg/horizon)	450000
Demand in product C (kg/horizon)	600000

processing times and size factors (parenthesis)

Product. Task	Equipment Type			
	E1	E2	E3	E4
A.T1		3.5(2.0)		2.5(1.2)
A.T2	2.5(1.5)		2.0(1.2)	
B.T1	3.5(1.2)		3.9(1.6)	
B.T2		4.1(1.8)		
B.T3			3.0(1.2)	3.2(3.2)

Table 3. Equipment and operation information in test problem 2

Max. No. of Groups (out of phase)	2
Max. No. of Unit (in phase)	2
Allowable Unit Sizes (kg)	2000, 3000, 4000
Time Horizon (hr)	1000
Demand in product A (kg/horizon)	50000
Demand in product B (kg/horizon)	60000

processing times and size factors (parenthesis)

Product.Task	Equipment Type			
	E1	E2	E3	E4
A.T1	5.0(1.2)		4.5(1.25)	
A.T2		3.0(1.3)		2.5(1.2)
A.T3	4.0(1.1)		4.5(1.1)	
B.T1		6.0(1.4)	5.5(1.2)	
B.T2	4.0(1.15)			3.0(1.2)
C.T1			7.5(1.5)	
C.T2		6.5(1.2)		
C.T3	6.0(1.1)			5.0(1.2)

repeat the procedure.

Note that the integer cuts are accumulated and the values of the parameterized variables, T_k are replaced with updated ones from the previous sub- problem whenever a new master-problem is created. By treating T_k as a parameter, which is the only nonlinear element in the master-problem, we can remove all nonlinearity from the master problem.

Table 4. Equipment and operation information in test problem 3

Max. No. of Groups (out of phase)	1
Max. No. of Unit (in phase)	2
Allowable Unit Sizes (kg)	2000, 3000, 4000
Time Horizon (hr)	6000
Demand in product A (kg/horizon)	600000
Demand in product B (kg/horizon)	500000
Demand in product C (kg/horizon)	600000
Demand in product D (kg/horizon)	600000

processing times and size factors(parenthesis)

Product. Task	Equipment Type				
	E1	E2	E3	E4	E5
A.T1			4.5(1.25)		
A.T2		3.0(1.3)			
A.T3	4.0(1.1)				
B.T1		6.0(1.4)	5.5(1.2)		
B.T2	4.0(1.15)				
C.T1			7.5(1.5)		
C.T2		6.5(1.2)			
C.T3			5.0(1.2)	6.0(1.2)	
D.T1	5.0(1.2)				
D.T2		3.0(1.3)		2.5(1.2)	
D.T3					6.5(1.1)

Cost coefficients of all equipment are given in Table 1. Operation information inputs for three example problems are shown in Tables 2 through 4.

Figures 2 to 4 show three illustrative results for the test problems. In Figure 3, we can see that due to the common usage of most equipment in every production line, the maximum number of campaigns is arranged for production and product B needs the larger units of E1 and E3 (all the others are smallest

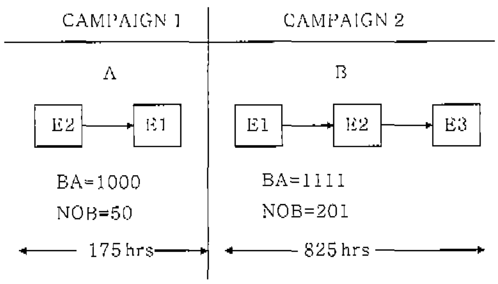

Figure 2. Optimal Configuration of Test Problem 1 (BA: batch size; NOB: number of batches; A and B: products)

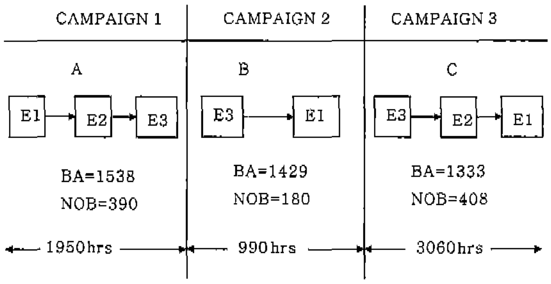

Figure 3. Optimal Configuration of Test Problem 2 (BA: batch size; NOB: number of batches; A, B and C: products)

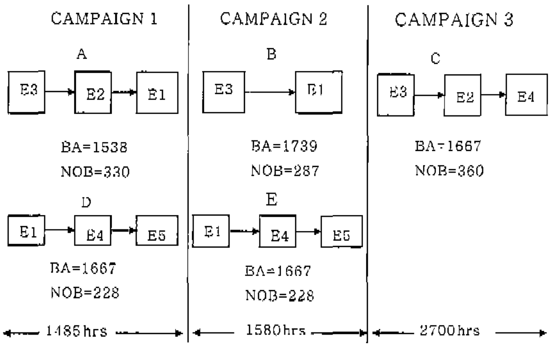

Figure 4. Optimal Configuration of Test Problem 3 (BA: batch size; NOB: number of batches; A, B, C and D: products)

in size) to meet the demand within the limited time of campaign 2. Products (A and D) and (B and D), in Figure 4, are shown to be allocated in campaigns 1 and 2, respectively, allowing the cheapest equipment, E1, all in use. However, the cost minimization objective function chose another campaign 3 (2700 hrs) rather than dual use of expensive equipment, E3 and E4 for even shorter production period.

Three examples tested were computationally intractable in a sense that they cannot be solved directly with the number of integer variables presented in Table 8. Even the smallest problem (test 1) with 288 integers took 3701.76 seconds to be completed; the others were beyond reasonable computing capability.

By the decomposition explained in the previous section, tremendous benefit in reducing computing time was achieved (see Table 5). The integer cuts, Eq. (75), were estimated not to be so effective in these sample problems because they were not closely related to some other key integer variables, α_{pje}, β_{qjimek}, and ω_{qgimek} except for X_{imek}. The results are shown in

Table 5. Computing results of test problems (NP: No. of Products; NI: No. of Integer variables; NC: not computable, MP: master, SP: sub problem, Direct: original MILP)

problem	NP	NI	CPU(s)	MP	SP	Direct
Test 1	2	288	55.18	3.84	46.08	3701.76
Test 2	3	500	1270.13	338.55	479.75	NC
Test 3	4	727	13080.11	3213	6664	NC

Table 8. The problems, however, still exhibit the exponential increase of computing efforts needed under the decomposition method as the problem grows in size.

5. CONCLUSIONS

This research is about the optimal design in terms of units-campaigns arrangement and the corresponding unit size and the item numbers (parallel groups) in general multipurpose batch plants. Exact, optimal solution to the design problems was sought through a modified Bender's Decomposition algorithm with breaking an integer-flooded problem into a relaxed master problem and a sub-problem, which were connected with the unit-task-campaign assignment variables serving as the complicating variables. The plant was assumed to be operated under NIS (no intermediate storage) policy and each task to be performed with a constant processing time in its assigned unit.

The decomposition was successful in the sense that it converted an originally intractable problem to two problems of manageable size that required reasonable computing times, providing that the number of integer variables involved in the subproblems is not too large. However, the loosely embedded integer cuts should be extended to the other integer variables which are critically related to the complicating variables and most probably contribute to reduce the number of branches and bounds being searched. Also, further algorithmic improvement should be attempted to downsize the master problem in more relaxed form in order to obtain reasonable computation times even with expansion of design problem size.

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