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Offer Calculus to High School Students: The Use of Technologies Can Clear up People's Doubts

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From the beginning of the 20th century, calculus is gradually offered to high school students in many countries. However, in Chinese high school, the instruction on calculus is nearly an untouched field. Many people don't believe that high school students can study calculus well. They think calculus knowledge in students' brains is likely to become the "half-cooked food", and this can produce a bad effect on the study of formal calculus at university. The authors consider that the emphasis of calculus in high school should be the intuitive understanding of fundamental calculus concepts, and it is also the basis of the understanding of formal concepts. Traditional mathematics course with chalk can't meet the needs of calculus teaching. The use of technologies can enhance the calculus teaching, especially the informal and visual calculus teaching, help students understand the underlying concepts. The authors describe how the use of technologies can improve the calculus teaching and learning, and point out that the use of technologies can clear up people's doubts.

1. INTRODUCTION

At the beginning of the 20th century, calculus, one of the masterpieces of mankind civilization, gradually entered the secondary schools in many countries according to Klein's suggestion. But in China, as calculus is not the content of the *College Entrance Examination*, it's teaching is a blank by and large. So far the teaching of calculus in high school is still an open question.

Among the various views about this phenomenon the most popular statement is that teaching calculus in high school will become "half-cooked rice". Why people have this kind of worry is closely related to the tradition of Chinese teaching. Generally speaking,

Chinese teaching materials are partial to the well-organized formal system and emphasize the logical thinking. And many people cannot accept the informal and intuitive mathematics, they believe that "this kind of mathematics has not been mathematics yet".

At university, calculus teaching is even more based on rigid definition of limit and lays stress on the preciseness of theories. Many university professors worry that the students who study calculus in high schools can hardly understand its basic concepts. They will be apt to form scanty and specious knowledge and the certain knowledge will affect their further study. More teachers believe that not all of the people can understand calculus, because the subject matter itself is beyond their abilities. Unfortunately the past high school calculus teaching materials only simply abridged from the university textbooks, and students' studying effect deepens people's worry further.

If what we teach is just mechanical skills, then what our students learn also is mechanical skills. Similarly if calculus in high school appears in the form of formal structure and is taught as exercises, then the students maybe only remember those formulas by rote but do not really understand the ideas of calculus. To a certain extent, the above worries are reasonable.

In this paper, we do not want to comment whether or not university calculus should be formal. Let's think the proportion of the nationwide university students, especially the university students whose major are science or engineering to the compeer. It's unfair for most students to deprive of their rights to know calculus just in fear of a few students will form "half-cooked rice". For enhancing people's mathematical quality and social competitive ability, we should let everybody know the ideas of calculus and its application. But if calculus not changed is taught in middle schools, the act certainly will end in failure. Teaching calculus in high school should be considered thoroughly the students' age and mental feature, it should be taught in informal and intuitive form. Then calculus will really be accepted by most people.

2. THE CERTAIN RELATIONSHIP BETWEEN MODERN TECHNOLOGY AND HIGH SCHOOL CALCULUS TEACHING

Calculus treated in Chinese high schools, exerting the function of modern technology and strengthening intuition teaching is a breakthrough.

Intuitive teaching bases on not the abstract concepts and phrases but concrete activities, the students can feel directly. Intuitive teaching plays an important role in mathematics teaching. To some extent the first stage of cognition process, vivid intuition, is realized through the intuitive teaching. The intuition ensures the relationship between the concrete and the abstract, ensures the change from "vivid intuition" to "abstract thinking",

thereby becomes the pillar of thinking. Compared with other subjects, mathematics has a more abstract nature and the intuition can improve the development of abstract thinking. So intuitive teaching is more important in mathematics teaching.

In calculus teaching, the chance to use the objective models is seldom, what is widely used is the intuition of signals, especially graphs and tables, they are important intuitive means in calculus teaching. In the course of learning, using these intuitive means properly can help students abstract the nature of the concepts and improve effectively the formation of the basic calculus concepts.

Calculus cannot really enter high school, there are some reasons on the sense level and others related to the technology. The research objects of calculus are function, its nature and expression are the course's core. Differential and integral are the important tools to research function. To recognize function thoroughly, the three different forms of function —symbolic, numeric and graphic must be known well. Traditional teaching lay stress on strictness and abstraction, it's maybe related with the characteristic of the pencil-paper skill. The pencil-paper skill can only show the achievement of mankind thinking, but cannot express the dynamic figure. The symbol is a product of abstract thinking, the pencil-paper skill can express the symbolic form of function and its numeric results well, but there are some defects when the function is expressed in graphic form. At the same time, the pencil-paper skill also cannot replace the human brain to calculate. Modern technology possesses very strong function on the aspects of numeral calculation, signal calculation and graphic expression. It also can combine the three aspects organically, thereby it can offer an environment in which the three aspects can be combined well, so students can certainly gain the best understanding about function.

Modern technologies can show thoroughly the nature of functions and determine the certain relationship between modern technology and calculus teaching.

3. MODERN TECHNOLOGY PROVIDE FORCEFUL SUPPORT TO INTUITIVE CALCULUS TEACHING

At present, the use of technologies in calculus teaching shows more on intuitive, image level, so it will effect more in informal calculus teaching in middle school than in formal calculus teaching in university especially for the students whose major is mathematics. The use of technologies makes calculus teaching more approach its real goal: not the techniques and skill to calculation, but the understanding of the basic calculus concepts and ideas. Modern technology also assuredly provides forceful support for strengthening intuitive teaching and treasuring the understanding of its nature. It concretely shows:

3.1. Providing the microcosmic environment, strengthening the understanding of the local properties

The differential and integral are one important pair of calculus concepts. The differential shows the local properties of function and involves the change of the points whose distance is very little. Using technology, changing the view window of the graphic or using some special functions, such as the "Zoom in" function of TI-92 graphic calculator which can provide a microcosmic environment, let the students observe this kind of vary. The learner can deal with the particular mathematical concepts or the examples and counterexamples in the correlative conceptual system, exert directly effects to these mathematical objects and relationships and observe the successive variance of these relationships which are expressed by graphic.

3.2. From the view of macroscopic point, deepening the understanding of global properties

Modern technology also can help us to observe the macroscopic phenomenon to deepen the understanding of global properties of function. The relationship between micro and macro, local and global can be shown effectively by using technology. So students can understand the properties of function from different prospective and various sides.

For example, using technology, we can view the complete graph of the function easily, so we can understand it's behavior thoroughly.

If we say when we explore the local properties the modern technology is just like microscope, then when we explore the integral properties the modern technology is like telescope. Through the telescope students can see farther, see deeper and understand the properties of function more profoundly.

3.3. Enriching students' graphic languages, strengthening the graphic analytic methods

Owing to the limitation of the pencil-paper skill, the traditional teaching pays more attention to use the analytic languages such as symbols and analytic formulas, analyze the properties of function from its analytic formula, but pay no attention to its graphic language. In fact, analyzing the properties of function from its graphic form, not only can understand the properties of function more directly, what is more important is that its graph can help us to recognize what we do not find just by using the pure symbol analytic method. The modern technology provides abundant graphs and eliminates the obstacles on technology for understanding the graphic languages, using the graphic analytic

methods and recognizing the properties of function better.

For example, the derivative is a powerful tool to analyze the behaviors of functions. The first and second step derivatives are related to the properties of function. As the graphic language of derivative is the slope of tangent, students can analyze how the property of function changes and how the slope of tangent changes respectively. Thereby students can discover the relation between derivative and some properties of function.

3.4. From different prospective, using various presentations to deepen the understanding of concept

The function of technology also embodies in the reorganization of knowledge; it lets the learners build the relationship between the numeric and graphic presentation. In the process of teaching, we can lead the students to make use of the modern technology to guess the derived function that is based on the definition of derived function. For example, we can let students use the table of TI-92 and its data plotting function to guess the derived function of. The students have no trouble in guessing that the derived function of $\sin x$ maybe $\cos x$.

For another example, to observe

$$\lim_{x\to\infty}\frac{\sin x}{x},$$

in the concrete process of teaching, we can build tables, then observe when the value increases gradually, how it changes, the students can observe and guess the limit of the function through the variation of the values.

Also we can draw the graph of

$$f(x)=\frac{\sin x}{x},$$

observe its trend of variation and guess its limit. Then we calculate its limit and verify the guessed result.

4. REMARKS

In teaching calculus in high school, the most ideal way is to make the technology into the students' handy tools. Only that we can exert better the effect of technology. But according to the situation of China, at present we cannot reach the above level. We only can make the use of technology transit from the teachers' demonstrative tool to students' handy tool. When the technology is used as teachers' demonstrative tool, we must prevent the use of technology to go to the mistake area where the technology turns into an

electric blackboard.

Calculus entering high school, the development of technology provides us one kind of opportunity, the abroad practice provides us the experiences that we can use for reference. However, we need to do lots of work on subject arrangement, content selection, use of technology, etc. Calculus really entering, rooting and blooming in middle school still goes a long way.

REFERENCES

Ellis, W.; Bauldry, B.; Fiedler, J.; Giordano, F.; Lodi, Ed; Vitray, R. & West, R. (1997): *Calculus, Mathematics & Modeling*. Reading, MA: Addison-Wesley Longman Inc.

Tall, David (1996): Information technology and Mathematics education: enthusiasms, possibilities and realities. In: C. Alsina, Jose M. Alvarez, M. Niss, A. Perez, L. Rico, & A. Sfard (Eds.), *Proceeding of the 8th International Congress on Mathematical Education, Jul. 1996* (pp. 14–21). Sevilla, Spain: SAEM. MATHDI **1999b.**00975

Kaput, James J. (1998): Mixing New Technologies, New Curricula and New Pedagogies to Obtain Extraordinary Performance from Ordinary People in the Next Century. In: Han Shick Park, Young H. Choe, Hyunyong Shin & Soo Hwan Kim (Eds.), *Proceedings of the ICMI-EARCOME 1, held at Cheongju, Korea, August 17–21, 1988, Vol. 1* (pp. 141–156). Seoul, Korea: Korea Soc. Math. Edu. MATHDI **1998f.**04629

Zhang, Dianzhou (1994): Reform of Mathematics Curriculum. In: *A Guide to the Research on Mathematical Education* (pp. 514–516). Nanjing: Jiangshu Educational Publishing Company.

Chen, Changping (1996): *Comparative Studies in Mathematics Education*. Shanghai: East China Normal University Publishing Company.