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Comparison of Teaching Geometry Between China and USA—From an Oriental Perspective

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Geometry is one of the important parts of Chinese school mathematics. There is a large difference in teaching and contents (standards, curriculum) between the US and China. Many mathematics educators in both countries are trying to reform the instruction of geometry and have made some progress. Close attention has been given to the *Principles and Standards for School Mathematics* (NCTM 2000), in which we have found many good ideas.

In this paper, we introduce new developments of school geometry in China and have made some comparisons between the US and China. The new technology is becoming popular step by step in Chinese high schools. We believe we should learn from each other and exchange the ideas. In doing this mathematics teaching will be improved.

0. INTRODUCTION

Geometry is one of the important parts of Chinese school mathematics. There are large differences in teaching method and content as well as standards and curriculum between the United States and China. Many mathematics educators are trying to reform the instruction of geometry and have made some progress. Wang (2000a; 2000b; 2001c), which have been published in Chinese Journals, has introduced the US school geometry.

In this paper we introduce new developments of the school geometry in China and have made some comparisons between China and the US. The new technology is becoming popular step by step in Chinese high schools. Close attention has been given to the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics 2000), in which we have found many good ideas. We believe we should

learn from each other and exchange our ideas. In doing this, mathematics teaching will be improved.

1. AIM OF TEACHING GEOMETRY

Chinese educators try to help students set up a solid foundation, and American colleagues apply themselves to help students build a wide view of the subject.

Based on the study on both *Chinese Syllabus of Mathematics* (Chinese Ministry of Education 2000a; 2000b) and NCTM (2000), we have found that geometry is an important part of mathematics in both countries, but Chinese students may spend more time studying it. From Junior 1 (Grade 7 in the US) to Senior 3 (Grade 12), Chinese students study geometry for six years and spend two hours on it each academic week.

Under the instructions from Chinese Ministry of Education (2000a; 2000b), Chinese teachers pay attention to cultivate students abilities of spatial imagination that consist of the factors that follow. The students should be able to;

- imagine the objects from their figures and vice versa,
- recognize and consider the movement and change of figures,
- analyze the basic figures from a complex figure,
- find out the basic factors and their relationships from a figure,
- draw the figures according to given conditions, and
- use figures to express the essence of problems in various contexts.

Mathematics Teachers in China ask the students to recognize and understand a large group of concepts, axioms, theorems, and related formulae, and to memorize and master the basic system of knowledge. In China, the learning materials of school mathematics are divided into two subjects: *Algebra and Geometry*. The textbooks of geometry are divided into three parts: *Plane Geometry* (from *Book* 1 to *Book* 4), *Solid Geometry*, and Plane Analytic Geometry. The main ideas of Euclidean geometry are incorporated in the textbooks of geometry.

According to the importance of the knowledge and ability of a student's learning, the requirements of teaching geometry are divided into four levels. We would like to explain these levels by using quadrilaterals as examples from Geometry, Book 3 of Junior 2 (Grade 8):

- Level 1: Know something (elementary level).
  The student should know the concepts of central symmetry, central symmetric transformation, symmetric center, the figures of central symmetry and their characters, application of instability of quadrilaterals, etc.

- Level 2: Correctly understand (developing level).
  The student should understand the following concepts and properties of polygons; vertex, side, the sum of interior angles of a polygon, the sum of exterior angles of a polygon, etc.
- Level 3: Master the topic and be able to apply it (advanced level).
  The student should master the concepts of quadrilateral, parallelogram, rectangle, rhombus, square and trapezium, the theorems of their discriminations and the theorems of their properties, etc.
- Level 4: Master the topic very well, and be able to apply it skillfully (highest level).
  This is the highest requirement of learning mathematics. It is only for particular important concepts or fundamental theorems to be used. For example, Chinese teachers ask the students to master the properties of congruent triangles very well, and be able to apply the related theorems to discriminate congruent triangles skillfully.

According to NCTM (2000), in the US, the instructional goal of geometry is to enable all students to;

- analyze characteristics and properties of 2-dimensional and 3-dimensional geometric shapes and develop mathematical arguments about geometric relationships,
- specify locations and describe spatial relationships using coordinate geometry and other representational systems,
- apply transformations and use symmetry to analyze mathematical situations, and
- use visualization, spatial reasoning, and geometric modeling to solve problems.

Based on the comparison of the curriculum of mathematics between two countries, we have found that Chinese students study geometry more in depth than their American counterparts in some topics; on the other hand, on average, American students may have a wider view of the subject than Chinese students.

2. DIFFERENT EMPHASES OF STUDENTS' ABILITIES

Most teachers pay attention to improving students mathematical ability, but we have a difference in emphases between two countries.

In China, mathematics educators stress improving students ability in mathematical thinking. Students should be able to;

- observe, compare, analyze, synthesize, abstract and generalize,
- mathematical reasoning using induction, deduction, and analogy methods,

- express their own thoughts or points of view logically and exactly, and
- discover mathematics relationships according to suitable concepts and using methods in their own levels.

In fact, Chinese students begin to study logical reasoning in Junior 2 (Grade 8), and spend much time studying how to prove geometrical propositions. Most teachers and even students see geometry as an exercise in human thought. Designing a chain of mathematical reasoning and expressing the process of a proof logically are great challenges for middle grade students, but Chinese teachers try to develop student's ability to reason mathematically from the beginning of the middle stage. Chinese teachers' effort is also reflected in unified examinations to prove mathematical propositions are always an essential part of examinations.

In the US, mathematics educators also pay attention to developing student's ability to reason and do proof. Compared to Chinese partners, the US requirement of logical reasoning and proof are not as rigorous as that in China. On the other hand, the US pupils study mathematics proof later than Chinese pupils do, but the US educators focus on mathematical conjecture, informal reasoning, and experimental proof.

3. WHAT WE WOULD LIKE TO LEARN FROM US SCHOOL GEOMETRY

We have introduced the main ideas of the US school geometry found in NCTM (2000). Some of the US ideas are warmly welcomed and appreciated by Chinese colleagues. After various discussions, many of Chinese colleagues would like to learn the following from the US school geometry:

3.1. Enhance the position of transformation

Compare with the courses of geometry between two countries, we have found that Chinese students study geometric transformation less than their American partners. For example, they have informally learned some properties of translation, reflection, central symmetry and similarity of figures. In US high schools, teachers help students formally learn many kinds of transformations, such as translations, reflections, rotations, dilations/contractions of objects in the plane and even in 3-dimensional space, and use many representations including sketches, coordinates, vectors, or matrices. US students study not only single transformations but also their combinations and inverses. We have learned some good ideas from NCTM (2000). We will strengthen geometric transformation in the future national curriculum of school mathematics.

3.2. Encourage pupils to make and investigate mathematical conjectures

Conjecture is one of the most important resources of discovering new knowledge of mathematics; it is also a basic method of learning mathematics. Chinese teachers try to help students make conjectures when they study some theorems. Chinese teachers should learn from American colleagues and offer more opportunities for students to make mathematical conjectures.

Example 1. Given ΔABC, suppose that D, E, and F are respectively the feet of the altitudes AD, BF, CE of ΔABC, ΔAB_1C is the reflection map of ΔABC with respected to the segment AC, ΔD_1E_1F is the reflection map of ΔDEF with respected to the segment AC (see Figure 1). What is the relationship between E_1F and DF?

With the help of the TI-92 calculator, students make some investigations and experiments in the context of measurement and of animation, and then make a conjecture: E_1F and DF are concurrent on the same line. This conjecture can be proven without much difficulty.

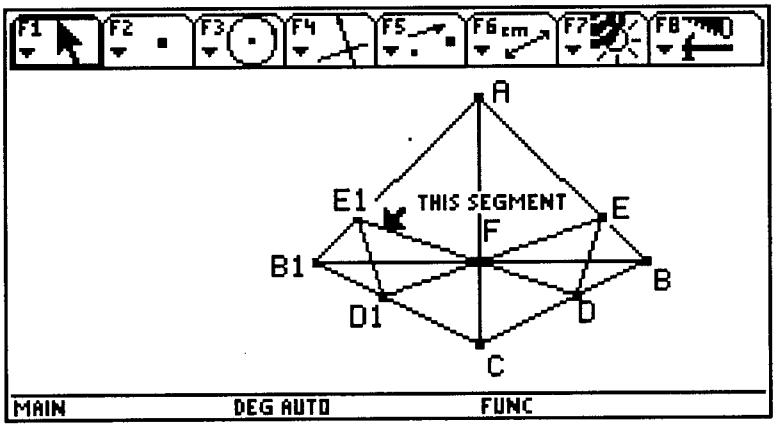

Figure 1. E_1, F, and D are concurrent on the same line.

3.3. Enrich the content of learning geometry and introduce useful tools from other aspects of mathematics as much as possible

There is a large part of classical geometry in school mathematics; it is useful for training students thought. If so much knowledge of classical geometry stays in secondary school mathematics, students will not have enough time to learn the other branches of mathematics that are also important for their further study. We should learn from NCTM (2000) and enrich the content of school geometry. In fact, some important concepts,

principles and methods such as vector, matrix, transformations, combination, and graph theory have been integrated into the senior high school mathematics. The experimental textbooks have been used in ten provinces or large cities of China.

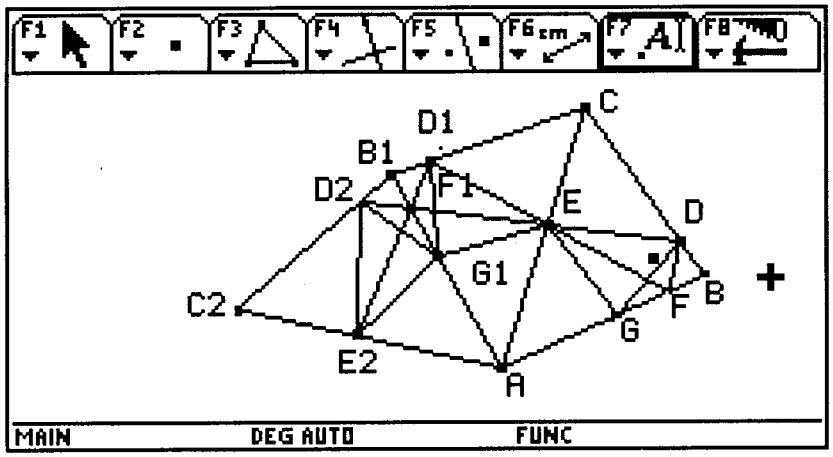

Figure 2. Which inscribed triangle has the shortest perimeter?

Example 2. Suppose ΔDEF is an inscribed triangle of ΔABC, D, E, and F are respectively the feet of altitudes of ΔABC. Prove that ΔDEF have minimum perimeter among all described triangles of ΔABC (Figure 2).

Consider two times consecutive reflections with respect to the sides of AC, AB_1, the maps of ΔABC respectively are ΔAB_1C, ΔAB_1C_2, the maps of ΔDEF respectively are ΔD_1EF_1, $\Delta D_2E_2F_1$ just as in Figure 2, according to the properties of reflection, $F_1E=FE$, $D_2F_1=D_1F_1=DF$,

On the other hand, from example we can see that DE, EF_1, and F_1D_2 are concurrent on a line. So that

$$ED+EF_1+F_1D_2 = DE+EF+FE = \text{perimeter of } \Delta DEF$$
$$= \text{length of segment } DEF_1D_2 = \text{segment } DD_2$$

Compared with other inscribed triangles of ΔABC, for example, if at least one point among G is not the foot of the altitude of ΔABC, DEG_1D_2 become broken lines. Of all these cases, the shortest perimeter is equal to segment DD_2.

Organize vivid and colorful activities to attract students interest of shape and space; using and applying technology in geometry class.

We appreciate that American teachers try to design a variety of activities for attracting student's desire of learning mathematics. NCTM (2000) offer us many excellent examples of teaching geometry. Principle of technology is becoming well known all over

the world.

In China, the *Geometer's Sketchpad* is popular for teaching geometry in developed districts. On the other hand, we are also interested in TI graphing calculators. Three *Centers of Using Technology of Teaching Mathematics* have been set up in Beijing, Shanghai, and Guangzhou. With the help of dynamic software, geometry class can be improved.

Example 3 (Nine points circle). Any three collinear points are contained on a circle. However, a remarkable result concerning nine points in a triangle, known as the nine-point circle theorem, states that the following nine points all lie on a single circle determined by ΔPQR (Figure 3).

A, B, C: the midpoints of the sides.
D, E, F: the feet of the altitudes.
G, H, I: the midpoints of the segments jointing the vertices (*P, Q, R*) to the orthocenter, *O*.

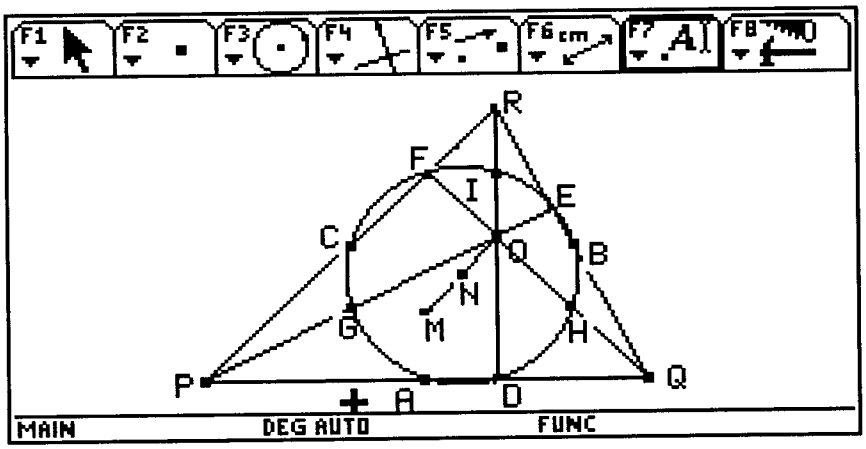

Figure 3. Nine points circle

For the first step, we should determine the center and the radius of the circle. We choose three points from the above nine points, if these three can be vertices of a right angle triangle, therefore, the hypotenuse of the small triangle would be the diameter of the nine points circle, and midpoint of the hypotenuse would be the center of that nine points circle.

We can easily see, $CF \perp QF$, so that, ΔHFC is a right angle triangle, CH is its hypotenuse, similarly, we can show that ΔHAC is also a right angle triangle, now, we have found four points *H, F, A, D* are on the same circle. Similarly, we can find, other points of *B, E, I, G*, and *C* are concurrent on the circle.

Suppose O and M are respectively orthocenter and circumcenter of ΔPQR, N is the midpoint of segment OM. In this case, N should be the center of the nine points circle, the segment NH should be the radius of the nine points circle. If the students notice that H, O, C, and M can be the vertices of a parallelogram, the proposition of nine points can be proven completely.

With the help of technology, the restriction of the traditional compass and ruler method of drawing a figure can be broken. For instance, in Example 2, if we use the traditional method to draw Figure 2, the accumulated error will become large, and the sum $DE+EF_1+F_1D_2$ will not be easily shown as a segment DD_2, DEF_1D_2 would be drawn as broken lines. Similarly, the students can not easily draw a nine points circle by the use of traditional methods.

4. NATIONAL CHARACTERISTICS OF TEACHING MATHEMATICS

What national characteristics of teaching mathematics that should be kept in the geometry class?

We have paid close attention to good ideas from developed counties, but if the exterior considerations are not appropriate for us, we would like to keep our own ideas. We think the following traditions of teaching geometry are still suitable for the current situation of China.

4.1. Teachers should play a leading role in the process of instruction

The teacher is the educator in the class; he/she should play a leading role in the process of student learning. Because classes in China are very large, on average (there are sometimes more than 50 students in a class), discipline and order are necessary to guarantee student learning. The activities of learning geometry should be carried through following the teacher's suggestions. If everyone can do everything that they want to do in the class, the process of teaching would not be easily carried through.

The main tasks of a teacher in the instruction of mathematics are;

- design some meaningful questions to introduce the topics,
- give suitable explanations of the concepts, theorems, axioms, and formulae, explain their background, properties, and applications,
- organize a variety of activities, lead students to practice the theory and method that they have learned, and
- put forward some challenging tasks which relate to the topics and help students deepen their understanding.

4.2. Students should be helped to set up a solid foundation

Students should be helped to form · · ·

- a correct motivation for learning geometry. They should recognize that geometry is a precious heritage of human beings, a useful tool of daily life and further study, and an important subject for training thought processes so that students enjoy learning the course.
- a correct attitude toward the subject. The students should recognize that to study geometry is not always very fun, sometime they will think it hard. In fact, they may often meet challenges from geometry. They should have enough confidence and willingness to overcome the difficulties.
- good ability to read the textbook, finding key words, asking and answering questions, understanding the implications of some key words, expressing their opinions clearly and correctly, spotting inconsistencies in the process of an argument of mathematics.
- solid foundation of the subject. They should understand the basic knowledge, master some useful methods, and memorize some important properties of figures. In our opinion, understanding, memorizing, and applying knowledge are all important for student development.
- favorable behavior, good habit, and appropriate method of learning the subject. For example, when they present their results orally, written, or visually to express their process of reasoning, they should master normative format and check that there is a reason in each step.

In our opinion, we should encourage students to learn mathematics and take it very seriously. Sometimes the activities of learning mathematics are vivid and interesting. On the other hand, rigorous training is also necessary for student development.

5. CONCLUSION

We have learned many good ideas of teaching geometry from NCTM Standards. In fact, the Chinese national standards of school mathematics are being revised and some of the US ideas have been absorbed in the standards for Chinese mathematics education. On the other hand, we pay close attention to stressing the Chinese characteristics as well. We believe learning from each other will enhance the level of teaching geometry in the new century.

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