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Effects of Dynamic Soil Behaviour on Wave-Induced Seabed Response

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ABSTRACT : *In this paper, an analytical solution for the wave-induced seabed response in a porous seabed is derived. Unlike previous investigations with quasi-static soil behaviour, dynamic soil behaviour is considered in the new solution. The basic one-dimensional framework proposed by Zienkiewicz et al (1980) is extended to two-dimensional cases. Based on the analytical solution derived, the effects of dynamic soil behaviour on the wave-induced seabed response are examined. The boundary of quasi-static soil behaviour and dynamic soil behaviour is clarified, and formulated for engineering practice.*

1. Introduction

Wave-induced seabed response (including pore pressure, effective stresses and soil displacements) is of profound interest to coastal and geotechnical engineers because of its potential effects on sediment transport, on the stability of submarine pipelines and on the performance of coastal-defence structures such as breakwaters. Also, the wave-induced pore pressure has been recognised as a key factor in the estimation of the seepage flow within marine sediments.

Many soil variables affect the wave-induced soil response in a porous seabed. Dynamic soil behaviour is one of important characteristics. However, the fully dynamic soil behaviour form has rarely been considered in the past, because of its complicated mathematical formulations. Most previous researches has considered the quasi-static soil behaviour, due to its simplification (Yamamoto *et al.*, 1978). In such an approximation, the accelerations due to pore fluid and solid are ignored. However, this assumption may be invalid for the seabed under large wave loading.

Mei (1989) derived a set of governing equations for the wave propagating over a saturated seabed. He simplified the complicated mathematical procedure by the boundary-layer approximation. Yuhi and Ishida (1998) further directly solved the boundary value problem proposed by Mei (1989). They considered infinite thickness seabed and discussed the shear waves within the soil column. This framework was extended to the case of finite thickness and examined the wave driven seepage flux in marine sediments (Jeng and Lee, 2001; Jeng *et al.*, 2001).

Based on Biot's poro-elastic theory (Biot, 1956), the influence of dynamic soil behaviour on consolidation problem was discussed by Zienkiewicz *et al.* (1980) through a one-dimensional analysis. They concluded that the dynamic soil behaviour is not important for the under wave loading case. However, in their examples, the speed of compression wave velocity was fixed as 1000m/s, which may not reasonable for some ocean environments.

This paper is aimed to re-examine the effect of dynamic soil behaviour on wave-induced soil response (including pore pressure, effective stresses and soil displacements) with a two-dimensional analysis. A set of fully dynamic analysis will be formulated and represented in non-dimensional parameters. Based on the new solution, the effect of wave and soil characteristics on the wave-induced seabed response will be examined. The applicable range of quasi-static soil behaviour and dynamic soil behaviour will be clarified, from which a formula will be suggested for engineering applications.

2. Boundary Value Problem

In this study, we consider ocean waves propagating over a porous seabed of infinite thickness. The definition of the

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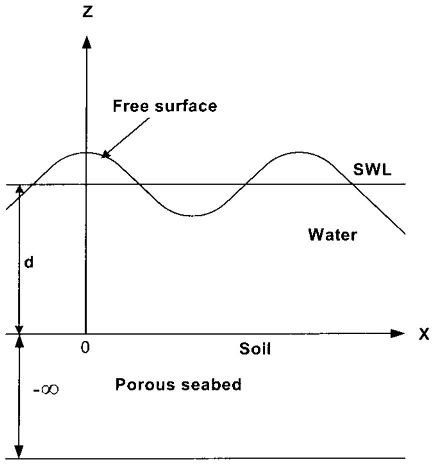


Fig. 1 Definition of wave-seabed interaction

problem is shown in Fig. 1. The ocean wave propagates in the positive x-direction, while the vertical z-axis is upward from the seabed surface.

To derive an analytical solution of the wave-seabed interaction problem (Fig. 1), we consider a horizontal isotropic, homogeneous porous seabed with a high degree of saturation. The soil skeleton and pore fluid are compressible. As a first approximation, the soil skeleton obeys Hooke's law and the flow in the porous bed obeys Darcy's law. The wave pressure on the seabed surface is considered as the only external loading force in the wave-seabed interaction. This implies that the effect of the viscous boundary layer is ignored in this study. To simplify the complicated problem, only linear wave theory is employed in this study.

2.1 Governing equations

For a two-dimensional wave-seabed interaction problem, where the porous seabed is hydraulically isotropic, the governing equations are summarised in tensor form as below (Biot, 1956; Zienkiewicz *et al.*, 1980).

$$\sigma_{ij,j} = \rho\ddot{u}_i + \rho_f \ddot{w}_i \tag{1}$$

$$-p_{,i} = \rho_f \ddot{u}_i + \frac{\rho_f}{n}\ddot{w}_i + \frac{\rho_f g}{K_z}\dot{w}_i \tag{2}$$

$$\dot{u}_{i,i} + \dot{w}_{i,i} = -\frac{n}{K_f}\dot{p} \tag{3}$$

where p is the wave-induced pore pressure, n is soil porosity, ρ is the combined soil and pore fluid density, ρ_f is the fluid density, K_z is soil permeability, u and w are the displacements of solid and relative displacements of solid and pore fluid, respectively.

In Eq.(3), the bulk modulus of pore fluid K_f can be expressed in terms of degree of saturation as

$$\frac{1}{K_f} = \frac{1}{K_{wo}} + \frac{1-S_r}{\gamma_w d} \tag{4}$$

where K_{wo} is the bulk of modulus of pore water, which is 2×10^9 N/m^2, and S_r is the degree of saturation.

The definition of effective stresses, ρ'_{ij}, which are assumed to control the deformation of the soil skeleton, are given in terms of the total stress (ρ_{ij}) and pore pressure (p) as,

$$\sigma_{ij} = \sigma'_{ij} - \delta_{ij} p \tag{5}$$

where, δ_{ij} is the Kronecker Delta denotation. Therefore, the consolidation equation, Eq. (1) becomes

$$\sigma'_{ij} = \delta_{ij} p_{,i} + \rho\ddot{u}_i + \rho_f \ddot{w}_i \tag{6}$$

Under conditions of plane strain, the stress-strain relationship can be expressed as

$$\sigma'_{ij} = D\varepsilon_{ij} \tag{7}$$

where ε_{ij} denote the strain and D is the soil stiffness matrix.

2.2 Boundary conditions

In general, two boundary conditions are required at a rigid impermeable bottom (BBC: Bottom Boundary Condition) and at the seabed surface (SBC: Surface Boundary Condition), respectively.

(a) BBC: boundary condition at the bottom (z - ∞):
For the resting on an impermeable rigid bottom, zero displacements and no vertical flow occurs at the horizontal bottom,

$$u = w = p = 0 \quad as \quad z \rightarrow -\infty \tag{8}$$

(a) SBC: boundary condition at seabed surface (z = 0):
At the surface of the seabed (z = 0), the boundary condition imposed by the wave motion is given by

$$p = \frac{\gamma_w H}{2\cosh kd}\cos(kx - wt) = p_o \cos(kx - wt) \quad at \quad z = 0 \tag{9}$$

where *"cos(kx-wt)"* denotes the spatial and temporal variations in wave pressure within the two-dimensional progressive wave described above. p_o is the amplitude of the wave pressure from the first-order Stokes' wave theory.

In addition, the vertical effective normal stress must vanish at the seabed surface. As for the shear stress in the z-direction, it is known that the shear stress is associated with the oscillatory flow above the seabed. However, the fluid shear exerted at the seabed surface is small and may be neglected (Yamamoto *et al.*, 1978). Thus, it is reasonable to impose the boundary conditions,

$$\sigma'_z = \tau = 0 \quad as \quad z = 0 \tag{10}$$

The boundary value problem, describing the wave-seabed interaction problem, can be solved analytically, based on the governing equations (Eqs.1~3), and boundary conditions (Eqs. 8~10). The analytical solution for the wave-induced soil displacements and relative displacement of pore fluid can firstly be obtained. Then the effective stresses and pore pressure can be calculated from the stress-strain relationship, Eqs. (6) and (7).

3. General Solution

In this section, the analytical solution for the full dynamic soil behaviour will be presented. Re-organising the governing equation, Eq. (3), we have,

$$-p_{,i} = \frac{K_f}{n}(u_{i,i} + w_{i,i}) \tag{11}$$

The substituting Eq. (11) into Eqs (2) and (5), the governing equation can be rewritten as

$$\frac{K_f}{n}(u_{i,i} + w_{i,i})_{,i} = \rho_f \ddot{u}_i + \frac{\rho_f}{n}\ddot{w}_i + \frac{\rho_f}{K_z}\dot{w}_i \tag{12}$$

$$\sigma'_{ij,i} = -\frac{K_f}{n}\delta_{ij}(u_{i,i} + w_{i,i})_{,i} + \rho_f \ddot{u}_i + \rho_f \dot{w}_i \tag{13}$$

Since the wave-induced oscillatory soil response fluctuates periodically, thus all quantities can be replaced immediately by their complex form

$$\bar{x} = kx,\ \bar{z} = kz,\ \bar{t} = \varpi t,\ f = \bar{F}(\bar{z})e^{i(\bar{x}-\bar{t})} \tag{14}$$

where f denotes the wave-induced soil response parameter.

Based on the above transformation, the governing equations can be rearranged in a scalar form by using new parameters as,

$$K_1 = \frac{G}{\dfrac{G}{1-2\mu} + \dfrac{K_f}{n}},\ K_2 = \frac{\dfrac{K_f}{n}}{\dfrac{G}{1-2\mu} + \dfrac{K_f}{n}}$$

$$V_c^2 = \frac{\left(\dfrac{G}{1-2\mu} + \dfrac{K_f}{n}\right)}{\rho},\ \Pi_1 = \frac{K_z V_c^2 k^2}{\rho_f g \varpi} \tag{15}$$

$$\Pi_2 = \frac{\rho\varpi^2}{\left(\dfrac{G}{1-2\mu} + \dfrac{K_f}{n}\right)k^2},\ \beta = \frac{\rho_f}{\rho}$$

In Eq. (15), V_c is the speed of compressive wave, which is directly related to the degree of saturation (S_r), water depth (d), and other soil parameters (such as shear modulus (G), Poisson ratio (μ) and porosity (n)). Since shear modulus, Poisson ratio and porosity are almost constant for most marine sediments, therefore the speed of compressive wave will depend on the degree of saturation and water depth. Fig. 2 illustrates the relationship between V_c and degree of saturation for various water depths. In the example, other soil parameters are taken as G = 5×10^6 N/m^2, μ = 0.35 and n = 0.4. As seen in the figure, the speed of compressive wave (V_c) dramatically increases in near saturated seabed. The value used by Zienkiewicz *et al.* (1980), i.e., V_c = 1000 m/sec, is also indicated in the figure. It is noted that the value used in Zienkiewicz *et al.* (1980) is the case of saturated seabed. However, the degree of saturation varies from 0.9 to 1.0 in marine sediments. Thus, the value of V_c used in Zienkiewicz *et al.* (1980) is invalid for unsaturated marine sediments.

Substituting Eqs. (14) and (15) into Eqs. (12) and (13), and satisfy the bottom boundary condition, Eq. (8), the general form of soil and pore fluid displacements can be written as (the detailed derivation is given in Appendix):

$$\bar{U}_x = a_1 e^{\lambda_1 \bar{z}} + a_3 e^{\lambda_3 \bar{z}} + a_5 e^{\lambda_5 \bar{z}} \tag{16}$$

$$\bar{U}_z = a_1 b_1 e^{\lambda_1 \bar{z}} + a_3 b_3 e^{\lambda_3 \bar{z}} + a_5 b_5 e^{\lambda_5 \bar{z}} \tag{17}$$

$$\bar{W}_x = a_1 c_1 e^{\lambda_1 \bar{z}} + a_3 c_3 e^{\lambda_3 \bar{z}} + a_5 c_5 e^{\lambda_5 \bar{z}} \tag{18}$$

$$\bar{W}_z = a_1 d_1 e^{\lambda_1 \bar{z}} + a_3 d_3 e^{\lambda_3 \bar{z}} + a_5 d_5 e^{\lambda_5 \bar{z}} \tag{19}$$

where λ_i, b_i, c_i and d_i coefficients are given in Appendix.

Then, the wave-induced pore pressure and stresses can be further expressed as,

$$\begin{aligned} p = -\frac{K_f}{n}k[&(i + b_1\lambda_1 + ic_1 + d_1\lambda_1)a_1 e^{\lambda_1\bar{z}} + \\ &(i + b_2\lambda_2 + ic_2 + d_2\lambda_2)a_2 e^{\lambda_2\bar{z}} + \\ &(i + b_3\lambda_3 + ic_3 + d_3\lambda_3)a_3 e^{\lambda_3\bar{z}}] \times e^{i(\bar{x}-\bar{t})} \end{aligned} \tag{20}$$

$$\begin{aligned} \sigma'_x = \frac{2Gk}{1-2\mu}\{&[(1-\mu)i + b_1\lambda_1\mu]a_1 e^{\lambda_1\bar{z}} + \\ &[(1-\mu)i + b_2\lambda_2\mu]a_2 e^{\lambda_2\bar{z}} + \\ &[(1-\mu)i + b_3\lambda_3\mu]a_3 e^{\lambda_3\bar{z}}]\} \times e^{i(\bar{x}-\bar{t})} \end{aligned} \tag{21}$$

$$\begin{aligned} \sigma'_z = \frac{2Gk}{1-2\mu}\{&[i\mu + (1-\mu)b_1\lambda_1\mu]a_1 e^{\lambda_1\bar{z}} + \\ &[i\mu + (1-\mu)b_2\lambda_2\mu]a_2 e^{\lambda_2\bar{z}} + \\ &[i\mu + (1-\mu)b_3\lambda_3\mu]a_3 e^{\lambda_3\bar{z}}\} \times e^{i(\bar{x}-\bar{t})} \end{aligned} \tag{22}$$

$$\begin{aligned} \tau_{xz} = 2G[&(\lambda_1 + ib_1)a_1 e^{\lambda_1\bar{z}} + (\lambda_2 + ib_2)a_2 e^{\lambda_2\bar{z}} + \\ &(\lambda_3 + ib_3)a_3 e^{\lambda_3\bar{z}}] \times e^{i(\bar{x}-\bar{t})} \end{aligned} \tag{23}$$

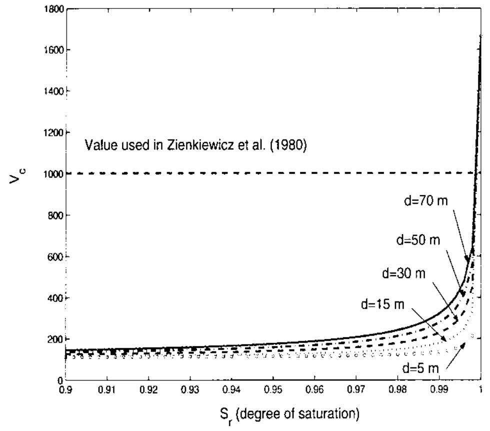


Fig. 2 Relationship between the speed of compressive wave and the degree of saturation for various water depths

Substituting Eqs. (20), (22) and (23) into the boundary conditions at the seabed surface, Eqs. (9) and (10), we can obtain the unknown coefficients *ai*.

If all acceleration terms are neglected from Eqs. (1)-(3), we will have the quasi-static case equivalent to the simple consolidation problem, which has been done previously (Jeng, 1997).

4. Numerical Results and Discussions

To gain a basic understanding of the mechanism of wave-seabed interaction, results of a parametric study will be presented in this section. As indicated in the analytical solution presented previously, two groups of parameters are defined for the wave-seabed interaction problem: They are:

- Soil parameters: the degree of saturation (S_r), the soil permeability (K_z), Poisson' ratio (μ), soil porosity (n) and shear modulus (G). Among these, the degree of saturation and soil permeability are two dominant factors in the analysis of wave-seabed interaction, while the other soil parameters are almost constant values for most marine sediments, at least for the quasi-static solution (Jeng, 1997).

- Wave parameters: the wave period (T) and water depth (d), which lead to the wavelength through the linear wave dispersion equation. Since we consider only the linear wave loading in this study, the effects of wave height is expectable, if wave period and water depth are given.

4.1 Effects of dynamic soil behaviour

To investigate the influence of dynamic soil behaviour on the wave-induced soil response, two cases are considered as examples. The wave and soil characteristics for the case study are tabulated in Table 1. Among these, wave period of 10 sec and water depth 30 m are used for cases 2, which represents general case of ocean waves (Mei, 1989). It is noted that the velocities of compressive wave V_c for case 1 and 2 are approximately 137 and 127 m/sec, respectively, which is far from the value of 1000 m/sec used in the one-dimensional analysis proposed by Zienkiewicz *et al.* (1980). Referring to Table 1, the values of K_1, K_2 and V_c in both cases are close, only Π_1 and Π_2 have significant differences in both cases.

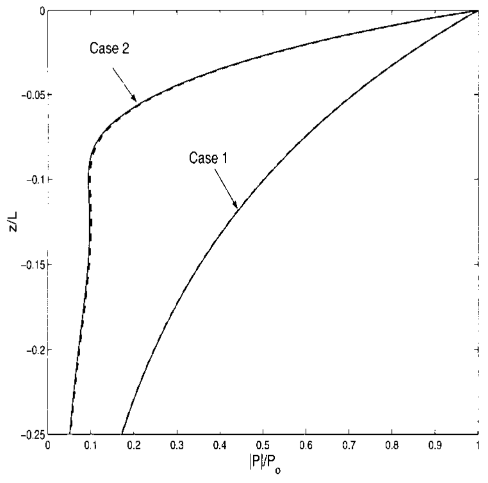
Table 1 Input data for demonstration of effects of dynamic soil behaviour

Parameters	Case 1	Case 2
Wave period T	3.0sec	10 sec
Water depth d	50 m	30 m
Degree of saturation S_r	0.9	0.9
Poisson's ratio μ	0.4	0.4
Soil porosity n	0.35	0.35
Soil permeability K_z	10^{-2} m/sec	10^{-2} m/sec
Dynamic parameters		
K_1	0.12829	0.14973
K_2	0.35855	0.25134
V_c	137.133	126.9343
Π_1	3.7973	0.11366
Π_2	0.0011657	0.011684

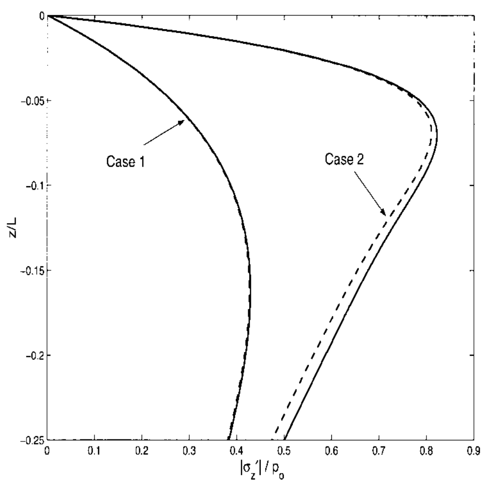
Table 2 Input data for numerical example

Wave Characteristics	
Wave Period T	12 sec or various
Water depth d	30 m or various
Soil Characteristics	
Degree of saturation S_r	0.9 or various
Poisson's ratio μ	0.35
Soil porosity n	0.4
Soil permeability K_z	10^{-2}m/sec 10^{-4}m/sce
Shear modulus G	5.0×10^{6}N/m^2

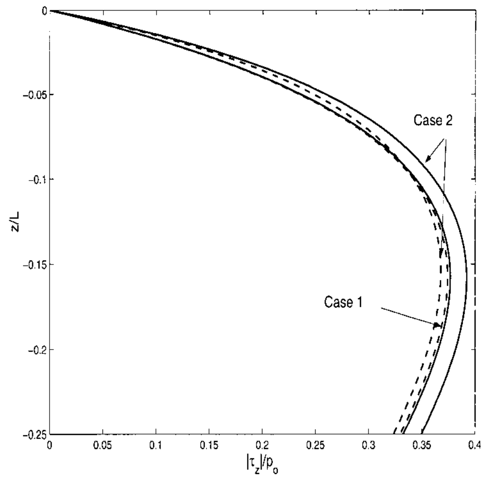
The vertical distributions of the wave-induced pore pressure and effective stresses, shear stress and soil displacements versus soil depths in a porous seabed are presented in Fig 3. In the figure, the solid lines represent the present solution with dynamic soil behaviour, while the dotted lines denote the previous solution with quasi-static soil behaviour (Jeng, 1997). Fig. 3 clearly indicates that the influence of dynamic soil behaviour is insignificant on the wave - induced pore pressure in both cases, but it is



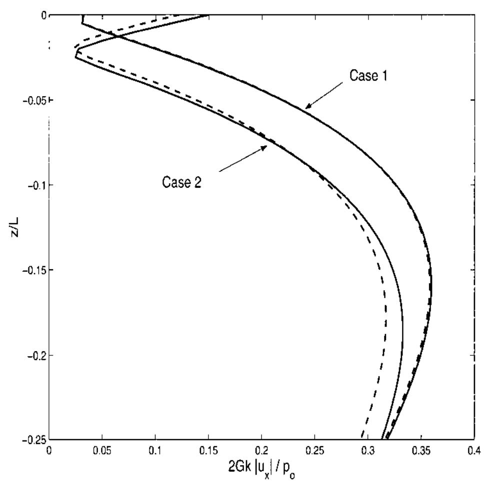
(a) |p| / p_o



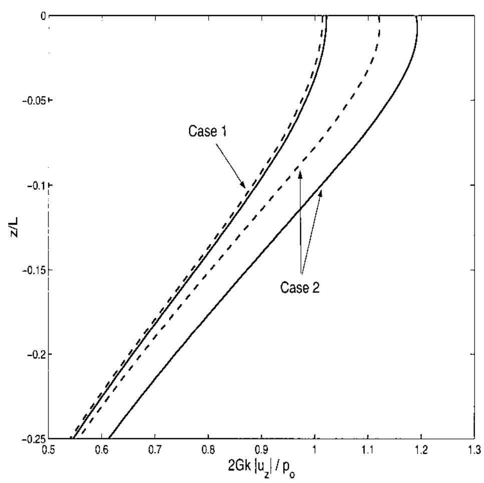
(b) | σ'$_z$| / p_o



(c) | τ_z| / p_o



(d) 2Gk |u_x| / p_o



(e) 2Gk |u_z| / p_o

Fig. 3 Vertical distributions of the wave-induced soil response versus the soil depth (z/L) for case 1 and case 2. Solids lines are dynamic solution and dashed lines are quasi-static solution

significant in other soil response parameters such as effective stresses and soil displacements for Case 2. In general, the effects of dynamic soil behaviour on the wave-induced soil response are insignificant for Case 1. Thus, it can be concluded that dynamic soil behaviour is only important for the wave-induced seabed response for certain combination of wave and soil characteristics with parameters Π_1 and Π_2. The applicable ranges of dynamic soil behaviour and quasi-static solutions will be discussed in detail in later sections.

4.2 Effects of soil characteristics

Generally speaking, it is common to observe gas in marine sediments (Okusa, 1985). It has been reported that the degree of saturation is an important factor in the evaluation of the wave-induced seabed response (Okusa, 1985; Jeng, 1997). In this sub-section, the effects of degree of saturation on the pore pressure, effective stress, and shear stress are re-examined here. Results from both solutions with dynamic (the present solution) and quasi-static soil behaviour (Jeng, 1997) will be included here. The input data of wave and soil properties for the following numerical examples are given in Table 2.

The distributions of the wave-induced soil response in coarse and fine sands are illustrated in Figs. 4 and 5, respectively. As shown in Fig. 4(a), the degree of saturation significantly affects the wave-induced pore pressure. Its influence will increase as the degree of saturation changes. The distribution of pore pressure in a seabed with K_z=10^{-2} m/sec changes smoothly, while it changes dramatically near the seabed surface in a seabed with K_z=10^{-4} m/sec (Fig. 5(a)). However, the effects of degree of saturation (S_r) on the relative difference of the wave-induced pore pressure are almost identical for both dynamic and quasi-static solutions. Figs. 4(b) and 5(b) illustrate the comparison between dynamic and quasi-static analytical solutions on the vertical effective normal stress for various degrees of saturation and soil permeability. Figs. 4(b) and 5(b) show the distribution of the vertical effective stress in the seabed with both soil permeability. The influence of permeability will increase as the degree of saturation decreases. It also indicates that a more significant difference between dynamic and quasi-static solutions has been observed, when soil depth increases.

The distribution of shear stresses for various values of degree of saturation and soil permeability are presented in Figs. 4(c) and 5(c). It can be seen that the degree of saturation does not significantly affect the shear stresses (at least from the examples presented here). Dynamic solutions for each value of degree of saturation are almost identical. Similar trends can be found in the quasi-static analytical solutions. However, differences between dynamic and quasi-static analytical solutions are apparent.

4.3 Effects of wave characteristics

Wave period (T) and water depth (d) are two important wave parameters, which directly affect the wavelength and other wave characteristics. Thus, it is of interest to examine the effects of wave period, and water depth on the wave-induced seabed response.

Figs. 6 and 7 present the vertical distributions of the wave-induced pore pressure ($|p|$ / p_o), effective normal stress ($|\sigma'_z|$ / p_o) and shear stress ($|\tau_z|$ / p_o) for various values of the wave period (T) in seabeds with different permeabilities. In general, the maximum amplitude of the wave-induced pore pressure decreases as the wave period increases, as shown in Fig. 6. However, dynamic soil behaviour is not important in estimation of the wave-induced soil response with varying wave periods (at least from the examples presented in Fig. 6).

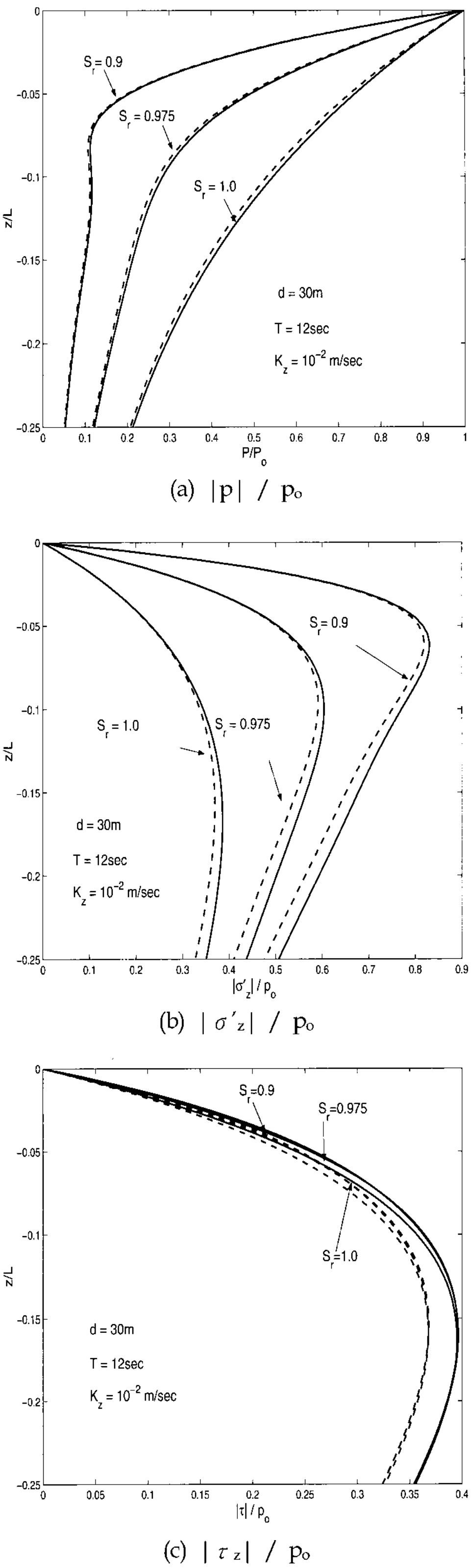


Fig. 4 Vertical distributions of the wave-induced soil response versus the soil depth for various values of degree of saturation (S_r) in coarse sand. Solids lines are dynamic solution and dashed lines are quasi-static solution

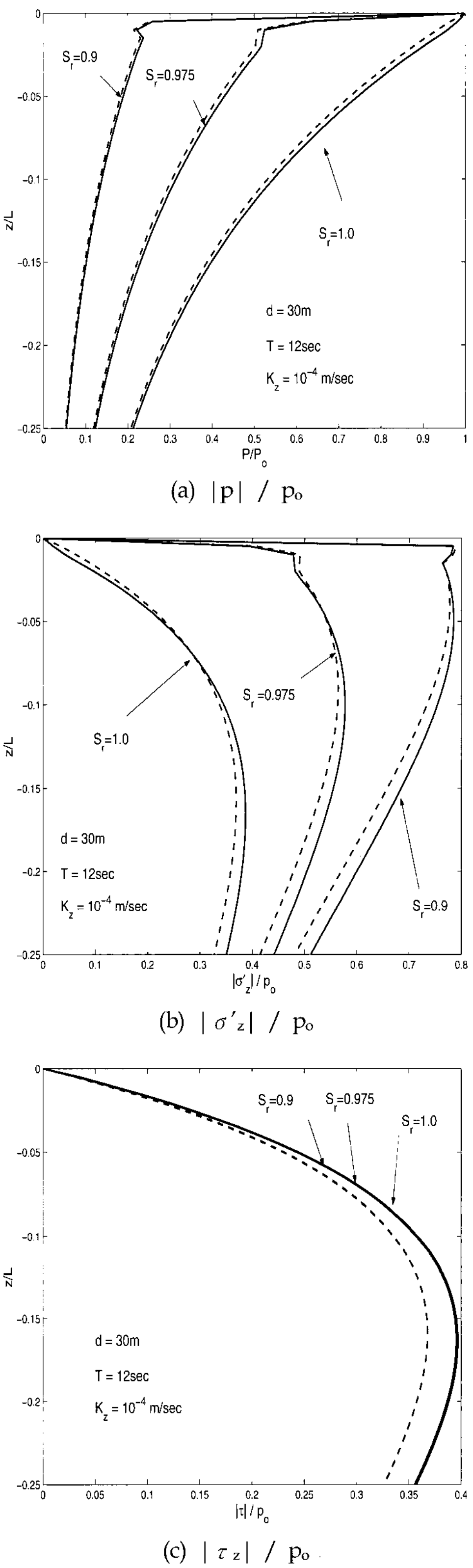


Fig. 5 Vertical distributions of the wave-induced soil response versus the soil depth for various values of degree of saturation (S_r) in fine sand. Solids lines are dynamic solution and dashed lines are quasi-static solution

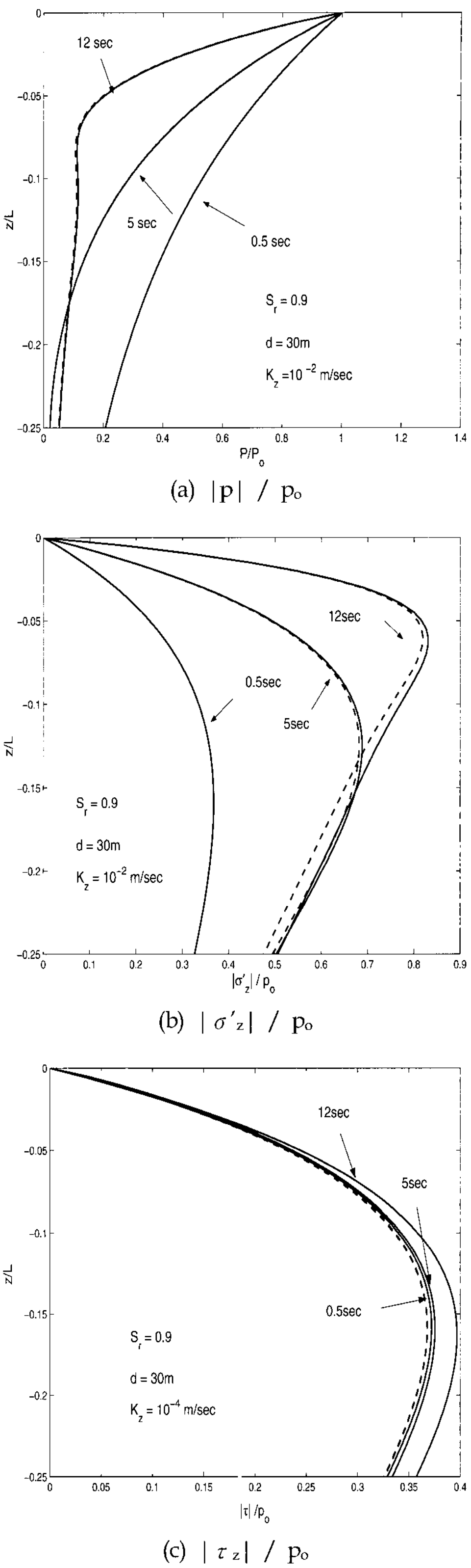


Fig. 6 Vertical distributions of the wave-induced soil response versus the soil depth for various values of wave periods (T) in coarse sand. Solids lines are dynamic solution and dashed lines are quasi-static solution

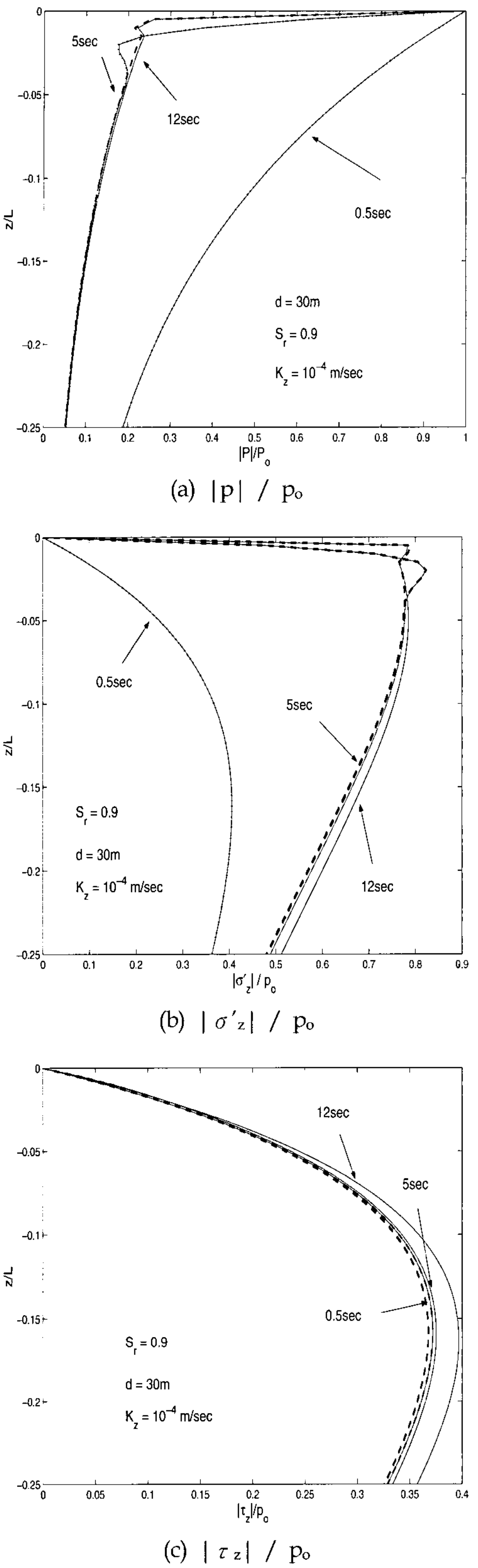


Fig. 7 Vertical distributions of the wave-induced soil response versus the soil depth for various values of wave periods (T) in fine sand. Solids lines are dynamic solution and dashed lines are quasi-static solution

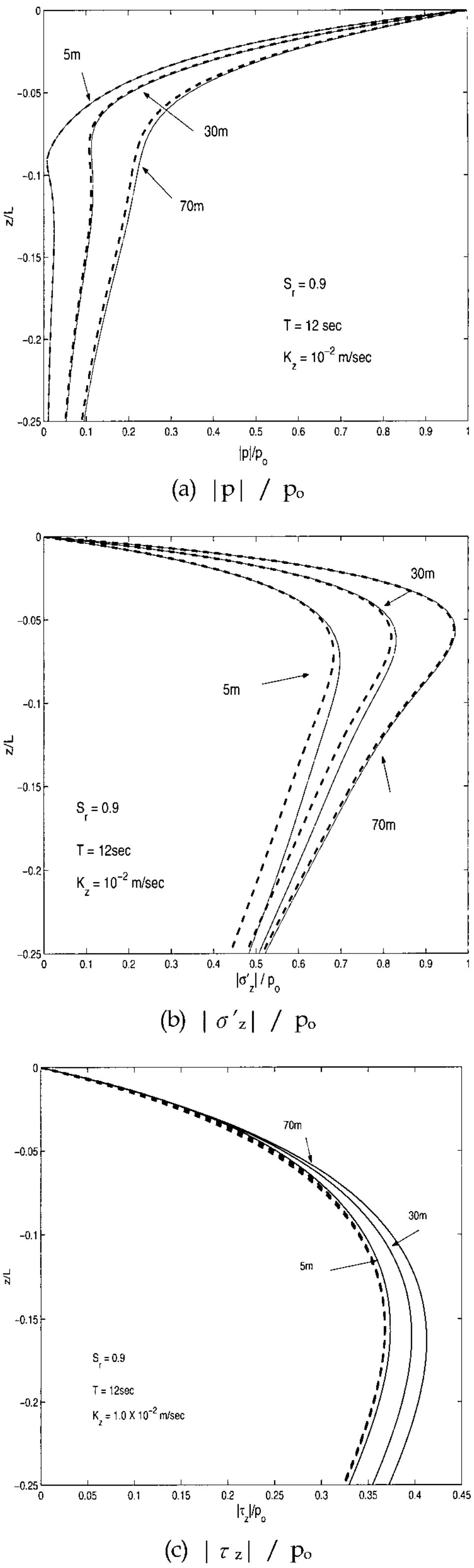


Fig. 8 Vertical distributions of the wave-induced soil response versus the soil depth for various values of water depth (d) in coarse sand. Solids lines are dynamic solution and dashed lines are quasi-static solution

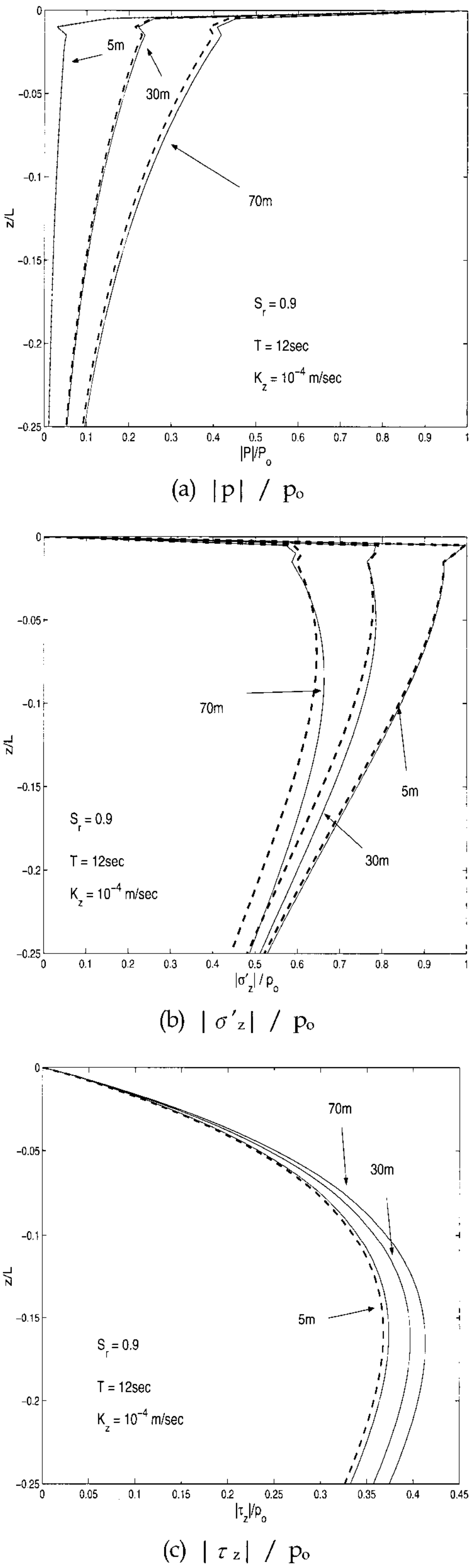


Fig. 9 Vertical distributions of the wave-induced soil response versus the soil depth for various values of water depth (d) in fine sand. Solids lines are dynamic solution and dashed lines are quasi-static solution

Figs. 8 and 9 illustrate the vertical distributions of the wave-induced pore pressure for various water depths and soil permeability. The magnitude of the wave-induced pore pressure increases as water depth (*d*) increases, but the distribution is significantly affected by permeability. Only a slightly difference of pore pressure between dynamic and quasi-static analytical solutions is observed. Figs. 8(b) and 9(b) present the vertical distribution of effective stress for various water depths in seabeds with different soil permeability. The figures clearly indicate that water depth significantly affects the vertical effective normal stress. The vertical effective stresses increase as water depth decreases. It is also observed that there are more significantly differences between dynamic and quasi-static solutions in deeper region of soil column.

The vertical distributions of the wave-induced shear stress for various water depth with different soil permeability are presented in Figures 8(c) and 9(c). As seen from the figures, significant difference of the shear stress between dynamic and static solution is only observed in water depth of 5 m.

4.4 When should dynamic soil behaviour be considered?

As demonstrated in previous numerical examples, it can be concluded that the effects of dynamic soil behaviour on the wave-induced soil response is important under certain combination of wave and soil parameters. Thus, it may be helpful to know, When should we consider dynamic soil behaviour? This is also particularly important for engineers to know when the conventional quasi-static solution is applicable.

As shown in the analytical solution derived previously, wave and soil characteristics have been involved in four coefficients, K_1, K_2, Π_1 and Π_2. As shown in Table 1, the changes of wave and soil characteristics are insensitive to the values of K_1 and K_2 while it is sensitive to Π_1 and Π_2. Thus, it will be easier to use these two parameters to investigate the applicable range of quasi-static solution. The Π_1 and Π_2 parameters were firstly suggested by Zienkiewicz *et al.* (1980) in their one-dimensional analysis.

Fig. 10(a) illustrates the boundary between dynamic and quasi-static solution in terms of Π_1 and Π_2. In the examples, if the relative difference of soil response is greater than 3% of dynamic solution, it is declared that dynamic solution must be used. Various values of soil permeability are used in the example. Herein, we use Cases 1 and 2 as an example to explain Fig. 10. In Fig. 10(a), point "•"indicates Case 1, and point "△"is for Case 2. Note that the soil permeailit for both cases is 10^{-2} m/sec, as mentioned previously. For the point of both cases with given Π_1 and Π_2 located above the curve given in Figure 10, the quasi-static soil behaviour must be considered (e.g., Case 1). For the point below the curve, dynamic soil behaviour is acceptable (e.g., Case 2).

Although the degree of saturation plays an important role in the analysis of the wave-induced soil response, its influences on the boundary between dynamic and quasi-static soil behaviour is insignificant (see Fig. 10(b)). After preliminary tests for various ranges of water depths and wave periods, we found that the boundary between dynamic and quasi-static soil behaviour can be considered as a constant with same permeability.

Based on Fig. 10(a), we learn that when the point located below a curve with particular value of soil permeability, we can use quasi-static solution with less than 3% relative differences. To further help practical engineers in determining the boundary between dynamic and quasi-static soil behaviour, we expressed the boundary as

$$\Pi_2 = C\Pi_1^{\,m} \tag{24}$$

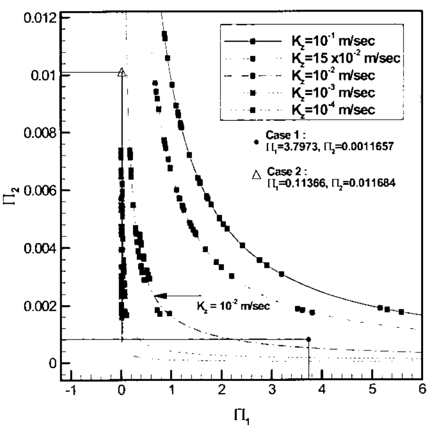
with correlation coefficients are greater than 97% for the curves presented in Figure 10(a).

In equation (24), the coefficients C and m can be represented in terms of soil permeability, as shown in Fig. 11. However, according to the parametric study, the coefficients m is almost constant value (Fig. 11(a)).

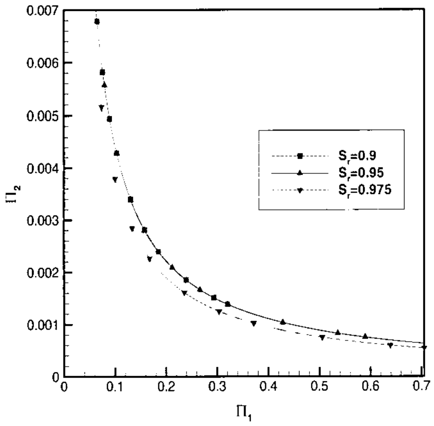
We can apply Fig. 11(a) and equation (24) to determine the boundary between dynamic and quasi-static soil behaviour with the following steps:

(1) Calculate the dynamic constants, Π_1 and Π_2, with given wave and soil parameters.

(2) With the soil permeability, determine C and m coefficients from Fig 11, respectively. Then, substituting the C and m coefficients into equation (24), and determine the boundary of the dynamic and quasi-static soil behaviour.

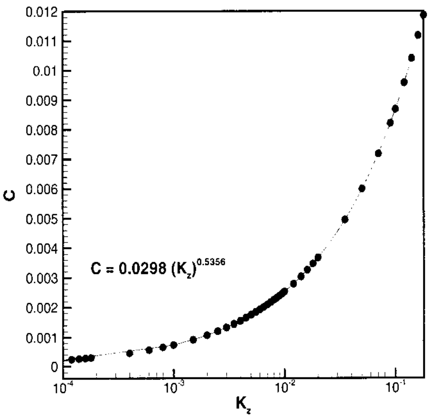


(a) K_z

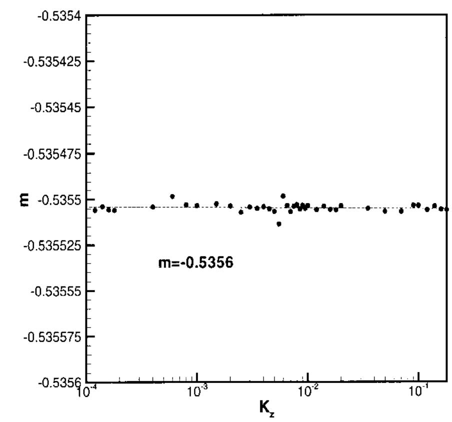


(a) S_r

Fig. 10 The relationship between Π_1 and Π_2 for various (a) soil permeability and (b) degree of saturation



(a) C



(b) m

Fig. 11 The relationship of coefficients (a) C and (b) m and permeability.

(3) Based on the calculated Π_1 and Π_2. For point falling above the boundary, the dynamic soil behaviour must be considered, otherwise, the quasi-static solution is sufficient.

5. Conclusions

In this study, an analytical solution for wave-induced seabed response which includes dynamic soil behaviour is derived. In the model, the accelerations due to both pore fluid and soil are included. Based on the new analytical solutions, the effects of dynamic soil behaviour on wave-induced seabed response are found to be important with respect to the vertical effective stresses under certain combinations of wave and soil conditions.

With the analytical solution proposed in this study, two parameters, Π_1 and Π_2, are used to determine the boundary between quasi-static (Jeng, 1997) and dynamic soil behaviour (the present solution). For point falling above the boundary, dynamic soil behaviour must be considered. Otherwise, the previous solution with quasi-static soil behaviour (Jeng, 1997) is sufficient for engineering practice with less than 3% of relative error. An approximate formula is given in Eq. (24) for the determination of the boundary of two solutions.

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Appendix

In this appendix, we will derive the general foam of the wave-induced seabed response by including the dynamic soil behaviour. Herein, we write Eq. (6) in scalar form as

$$\sigma'_x = 2G\left[\frac{\partial u_x}{\partial x} + \frac{\mu}{1-2\mu}\varepsilon\right] \tag{25}$$

$$\sigma'_z = 2G\left[\frac{\partial u_z}{\partial z} + \frac{\mu}{1-2\mu}\varepsilon\right] \tag{26}$$

$$\tau_{xz} = G\left[\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right] \tag{27}$$

where, the volumetric strain ε is defined by

$$\varepsilon = \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \tag{28}$$

Therefore Eqs. (11) and (12) can be re-organised in a scalar form as

$$\frac{K_f}{n}\left(\frac{\partial \varepsilon}{\partial x} + \frac{\partial \xi}{\partial x}\right) = \rho_f \ddot{u}_x + \frac{\rho_f}{n}\ddot{w}_x + \frac{\rho_f g}{n}\dot{w}_x \tag{29}$$

$$\frac{K_f}{n}\left(\frac{\partial \varepsilon}{\partial z} + \frac{\partial \xi}{\partial z}\right) = \rho_f \ddot{u}_z + \frac{\rho_f}{n}\ddot{w}_z + \frac{\rho_f g}{n}\dot{w}_z \tag{30}$$

$$\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = -\frac{K_f}{n}\left(\frac{\partial \varepsilon}{\partial x} + \frac{\partial \xi}{\partial x}\right) + \rho \ddot{u}_x + \rho_f \ddot{w}_x \tag{31}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma'_z}{\partial z} = -\frac{K_f}{n}\left(\frac{\partial \varepsilon}{\partial z} + \frac{\partial \xi}{\partial z}\right) + \rho \ddot{u}_z + \rho_f \ddot{w}_z \tag{32}$$

in which

$$\xi = w_{i,i} = \frac{\partial w_x}{\partial x} + \frac{\partial w_z}{\partial z} \tag{33}$$

Introducing K_1, K_2 Π_1 and Π_2 into Eqs (29)-(32), we have

$$(\beta\Pi_2 - K_2)\overline{U}_x + iK_2 D\overline{U}_z + \left(\frac{\beta\Pi_2}{n} - K_2 + \frac{i}{\Pi_1}\right)\overline{W}_x + iK_2 D\overline{W}_z = 0 \tag{34}$$

$$iK_2D\bar{U}_x + \left(K_2D^2+\Pi_2\beta\right)\bar{U}_z + iK_2D\bar{W}_x + \left(K_2D^2+\frac{\beta\Pi_2}{n}+\frac{i}{\Pi_1}\right)\bar{W}_z = 0 \tag{35}$$

$$K_1D^2\bar{U}_x - \frac{2(1-\mu)}{1-2\mu}K_1\bar{U}_x + \Pi_2\bar{U}_x - K_2U_x + iD\bar{U}_z + \left(\Pi_2\beta - K_2\right)\bar{W}_x + iK_2D\bar{W}_z = 0 \tag{36}$$

$$\left[\frac{2(1-\mu)}{1-2\mu}K_1D^2 + K_2D^2 - K_1 + \Pi_2\right]\bar{U}_z + iD\bar{U}_x + iK_2D\bar{W}_x + \left(K_2D^2+\Pi_2\beta\right)\bar{W}_z = 0 \tag{37}$$

Where D denotes ∂/∂_z.
Eqs. (34)- (37) can be written in a matrix form as

$$\begin{bmatrix} K_1D^2+A_{11} & iD & A_{12} & iK_2D \\ iD & A_{21}D^2+A_{22} & iK_2D & K_2D^2+\Pi_2\beta \\ A_{31} & iK_2D & A_{32} & iK_2D \\ iK_2D & K_2D^2+\Pi_2\beta & iK_2D & K_1D^2+A_{41} \end{bmatrix} \begin{bmatrix} \bar{U}_x \\ \bar{U}_z \\ \bar{W}_x \\ \bar{W}_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{38}$$

In (38), the A_{ij} coefficients are given by

$$A_{11} = \Pi_2 - \frac{2(1-\mu)}{1-2\mu}K_1 - K_2 \tag{39}$$

$$A_{12} = \beta\Pi_2 - K_2 \tag{40}$$

$$A_{21} = \frac{2(1-\mu)}{1-2\mu}K_1 \tag{41}$$

$$A_{22} = K_2 - K_1 + \Pi_2 \tag{42}$$

$$A_{31} = \beta\Pi_2 - K_2 \tag{43}$$

$$A_{32} = \frac{\beta\Pi_2}{n} - K_2 + \frac{i}{\Pi_1} \tag{44}$$

$$A_{41} = \frac{\beta\Pi_2}{n} + \frac{i}{\Pi_1} \tag{45}$$

The characteristic equation of Eq. (38) can be expressed as

$$\left(\alpha_4D^6 + \alpha_3D^4 + \alpha_2D^2 + \alpha_1\right)\bar{U}_x = 0 \tag{46}$$

where

$$\alpha_1 = -\left(A_{12}A_{31} - A_{11}A_{32}\right)\left(A_{22}A_{41} - \Pi_2^2\right) \tag{47}$$

$$\begin{aligned}\alpha_2 = {} & [K_2(-A_{31}A_{41} + A_{11}A_{22}K_2 - A_{22}A_{31}K_2 \\ & + A_{11}A_{41}K_2 + A_{31}\Pi_2 - 2A_{11}K_2\Pi_2 + A_{31}K_2\Pi_2) \\ & + A_{32}(A_{41} + A_{11}A_{21}A_{41} + A_{22}A_{41}K_1 + A_{11}A_{22}K_2 \\ & + A_{22}K_2^2 - 2K_2\Pi_2 - 2A_{11}K_2\Pi_2 - K_1\Pi_2^2) \\ & - A_{12}(A_{21}A_{31}A_{41} + K_2(A_{22}A_{31} + A_{41} + A_{22}K_2 \\ & - \Pi_2 - 2A_{31}\Pi_2 - K_2\Pi_2))]\end{aligned} \tag{48}$$

$$\begin{aligned}\alpha_3 = {} & [A_{21}(-K_2(A_{12}A_{31} - A_{11}K_2 + A_{12}K_2 + A_{31}K_2) \\ & + A_{32}(A_{41}K_1 + A_{11}K_2 + K_2^2)) \\ & + K_2(A_{32}(1 + A_{22}K_1 - 2K_2 - A_{11}K_2 + 2K_1\Pi_2) \\ & + K_2(1 + A_{12}A_{31} + A_{22}K_1 + A_{41}K_1 - 2K_2) \\ & - A_{11}K_2 + A_{12}K_2 + A_{31}K_2 + K_2^2 - 2K_1\Pi_2))]\end{aligned} \tag{49}$$

$$\alpha_4 = K_1\left(A_{21} - K_2\right)K_2\left(A_{32} + K_2\right) \tag{50}$$

The roots of the characteristic Eq. (46) can be expressed as

$$\lambda_1 = -\frac{\alpha_3}{3\alpha_4} + \frac{\sqrt[3]{2}\left(\alpha_3^2 - 3\alpha_2\alpha_4\right)}{3\alpha_4\alpha_5^{\frac{1}{3}}} + \frac{\alpha_5^{\frac{1}{3}}}{3\sqrt[3]{2}\alpha_4} \tag{51}$$

$$\lambda_2 = -\frac{\alpha_3}{3\alpha_4} + \frac{(1+i\sqrt{3})\left(\alpha_3^2 - 3\alpha_2\alpha_4\right)}{3\sqrt[3]{4}\alpha_4\alpha_5^{\frac{1}{3}}} - \frac{(1+i\sqrt{3})\alpha_5^{\frac{1}{3}}}{6\sqrt[3]{2}\alpha_4} \tag{52}$$

$$\lambda_3 = -\frac{\alpha_3}{3\alpha_4} + \frac{(1-i\sqrt{3})\left(\alpha_3^2 - 3\alpha_2\alpha_4\right)}{3\sqrt[3]{4}\alpha_4\alpha_5^{\frac{1}{3}}} - \frac{(1-i\sqrt{3})\alpha_5^{\frac{1}{3}}}{6\sqrt[3]{2}\alpha_4} \tag{53}$$

$$\alpha_5 = -2\alpha_3^3 + 9\alpha_2\alpha_3\alpha_4 - 27\alpha_1\alpha_4^2 + \sqrt{4\left(-a_3^2 + 3\alpha_2\alpha_4\right)^3 + \left(9\alpha_2\alpha_3\alpha_4 - 2\alpha_3^3 - 27\alpha_1\alpha_4^2\right)^2} \tag{54}$$

Thus, the general solution of the soil displacement from Eq. (46) can be written as

$$\bar{U}_x = a_1e^{\lambda_1\bar{z}} + a_2e^{-\lambda_1\bar{z}} + a_3e^{\lambda_2\bar{z}} + a_4e^{-\lambda_2\bar{z}} + a_5e^{\lambda_3\bar{z}} + a_6e^{-\lambda_3\bar{z}} \tag{55}$$

To satisfy the bottom boundary condition, Eq. (8), the coefficient a_2, a_4 and a_6 are zero in Eq. (55). Thus, the general solution for the soil and pore fluid displacements can be expressed as

$$\bar{U}_x = a_1e^{\lambda_1\bar{z}} + a_3e^{\lambda_2\bar{z}} + a_5e^{\lambda_3\bar{z}} \tag{56}$$

$$\bar{U}_z = a_1b_1e^{\lambda_1\bar{z}} + a_3b_3e^{\lambda_2\bar{z}} + a_5b_3e^{\lambda_3\bar{z}} \tag{57}$$

$$\bar{W}_x = a_1c_1e^{\lambda_1\bar{z}} + a_3c_3e^{\lambda_2\bar{z}} + a_5c_3e^{\lambda_3\bar{z}} \tag{58}$$

$$\bar{W}_z = a_1d_1e^{\lambda_1\bar{z}} + a_3d_3e^{\lambda_2\bar{z}} + a_5d_3e^{\lambda_3\bar{z}} \tag{59}$$

Where b_i , c_i and d_i coefficients are given by

$$b_i = \begin{bmatrix} -i\lambda_i & iK_2 & K_2\lambda_i^2 + \Pi_2 \\ -A_{31} & A_{32} & iK_2\lambda_i \\ -iK_2\lambda_i & iK_2\lambda_i & K_2\lambda_i^2 + A_{41} \end{bmatrix} / \Delta_i \tag{60}$$

$$c_i = \begin{bmatrix} A_{21}\lambda_i^2 + A_{22} & -i\lambda_i & K_2\lambda_i^2 + \Pi_2 \\ iK_2\lambda_i & -A_{31} & iK_2\lambda_i \\ K_2\lambda_i^2 + \Pi_2 & -iK_2\lambda_i & K_2\lambda_i^2 + A_{41} \end{bmatrix} / \Delta_i \tag{61}$$

$$d_i = \begin{bmatrix} A_{21}\lambda_i^2 + A_{22} & iK_2 & -i\lambda_i \\ iK_2\lambda_i & A_{32} & -A_{31} \\ K_2\lambda_i^2 + \Pi_2 & iK_2\lambda_i & -iK_2\lambda_i \end{bmatrix} / \Delta_i \tag{62}$$

$$\Delta_i = \begin{bmatrix} A_{21}\lambda_i^2 + A_{22} & iK_2 & K_2\lambda_i^2 + \Pi_2 \\ iK_2\lambda_i & A_{32} & iK_2\lambda_i \\ K_2\lambda_i^2 + \Pi_2 & iK_2\lambda_i & K_2\lambda_i^2 + A_{41} \end{bmatrix} \tag{63}$$

Then, the solution from the wave-in duced pore pressure and effective stresses can be obtained as shown in Eqs. (20)-(23).

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