Generalized Directional Morphological Filter Design for Noise Removal

Jinsung Oh, Heesoo Hwang, Changhoon Lee and Younam Kim

Abstract - In this paper we present a generalized directional morphological filtering algorithm for the removal of impulse noise, which is based on a combination of impulse noise detection and a weighted rank-order morphological filtering technique. For salt (or pepper) noise suppression, the generalized directional opening (or closing) filtering of the input signal is selectively used. The detection of impulse noise can be done by the geometrical difference of opening and closing filtering. Simulations show that this new filter has better detail feature preservation with effective noise reduction compared to other nonlinear filtering techniques.
Keywords - directional morphological filter, noise removal

1. Introduction

A morphological filter [1] can analyze the geometrical features of an image by locally comparing it with the structuring element. The shape and size of structuring elements determine the geometrical features that are preserved or removed, thus note the importance of the selection of the structuring element. To remove the noise without detail loss, morphological filters with directional [2,3,4] or multiple structuring elements [5] have been used in image filtering, instead of using single structuring element. Rank-order filtering approaches using a directional support region [6,7] and impulse noise detection [8] have been also used. Instead of using morphological [9,10] or rank-order [11] optimization method, we consider a new adaptive hybrid filter structure for the removal of impulse noise in highly corrupted images.

Based on the concepts of a morphological filter with directional structuring elements [2,3,4] and a rank-order filter [8], we define the generalized directional morphological filters as a weighted rank-order morphological filter with directional structuring elements. First the concepts of directional morphological filters (DMFs) are reviewed. Then weighted rank-order directional morphological filters, i.e., generalized directional morphological filters (GDMFs) are introduced. We then show how to find some of the weighting parameters and its application to the removal of salt and pepper noise. For salt (or pepper) noise suppression, the generalized directional opening (or closing) filter is selected when the salt (or pepper) noise is detected. The noise detection can easily be done by the geometrical difference of opening and closing with a flat structuring element. The performance of the proposed algorithm is compared with that of other nonlinear filtering algorithms under various noise conditions.

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2. Generalized Directional Morphological Filters

The basic idea of directional morphological filtering is to combine the output of basic subfilters such that each subfilter preserves a feature in a specific direction. For a finite number M of structuring elements B_i, the generalized directional opening (o) and closing (•) filters for the given signal $f(\underline{n})$ are given by

$$f_o(\underline{n}) = \phi_{i\in(1,\cdots,M)}[f \circ B_i(\underline{n})] \quad (1)$$

$$f_c(\underline{n}) = \varphi_{i\in(1,\cdots,M)}[f \bullet B_i(\underline{n})] \quad (2)$$

where $\underline{n}$ denotes two-dimensional variables, $\phi(\cdot)$ and $\varphi(\cdot)$ denote nonlinear function, respectively. In Fig. 1, we display as an example four directional structuring elements $\{B_i, i = 1\cdots,4\}$ with a support of 3 points and the union of these structuring elements as B_F.

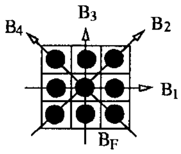


Fig.1 Four directional structuring elements with size 1x3

Letting the rank operator R be the nonlinear function in the above definitions, the outputs $\{r^o(\cdot), r^c(\cdot)\}$ of the directional morphological filters ordered by rank give:

$$R\{f \circ B_i(\underline{n}), i \in (1,\cdots,M)\} = [r_1^o(\underline{n}),\cdots,r_M^o(\underline{n})] \quad (3)$$

$$R\{f \bullet B_i(\underline{n}), i \in (1,\cdots,M)\} = [r_1^c(\underline{n}),\cdots,r_M^c(\underline{n})] \quad (4)$$

where $r_1(\underline{n}) \le r_2(\underline{n}) \le \cdots \le r_M(\underline{n})$ (omitting the subscripts o and c). According to the anti-extensivity of opening and the extensivity of closing, $f \circ B_i(\underline{n}) \le f(\underline{n}) \le f \bullet B_i(\underline{n}), \forall i$, the following relation ship holds:

$$r_1^o(\underline{n}) \le \cdots \le r_M^o(\underline{n}) \le f(\underline{n}) \le r_1^c(\underline{n}) \le \cdots \le r_M^c(\underline{n}). \quad (5)$$

Using the ranked values, we define a generalized directional morphological filter (GDMF) as a linear combination of the ranked values, i.e.,

$$f_o(\underline{n}) = \sum_{i=1}^{M} \beta_i(\underline{n}) r_i^o(\underline{n}) \quad (6)$$

$$f_c(\underline{n}) = \sum_{i=1}^{M} \gamma_i(\underline{n}) r_i^c(\underline{n}) \quad (7)$$

for M structuring elements with different directions. The weighting coefficients are chosen so that

$$\sum_{i=1}^{M} \beta_i(\underline{n}) = \sum_{i=1}^{M} \gamma_i(\underline{n}) = 1.$$

The simplest case of these GDM filters is when we have two weighting coefficients $\beta(\underline{n})$, $\gamma(\underline{n})$ and $r_M^o(\cdot)$, $r_{M-1}^o(\cdot)$ for the opening and $r_1^c(\cdot)$, $r_2^c(\cdot)$ for the closing so that

$$f_o(\underline{n}) = \beta(\underline{n}) r_M^o(\underline{n}) - (1-\beta(\underline{n})) r_{M-1}^o(\underline{n}) \quad (8)$$

$$f_c(\underline{n}) = \gamma(\underline{n}) r_1^c(\underline{n}) - (1-\gamma(\underline{n})) r_2^c(\underline{n}). \quad (9)$$

When we let $\beta(\underline{n}) = 1$ and $\gamma(\underline{n}) = 1, \forall \underline{n}$, Eqs. (8) and (9) become the conventional directional morphological (DM) filters used in [2,4,5], i.e.,

$$f_o(\underline{n}) = \max_{i \in (1,\cdots,M)} [f \circ B_i(\underline{n})]$$

$$f_c(\underline{n}) = \min_{i \in (1,\cdots,M)} [f \bullet B_i(\underline{n})].$$

3. Weighting Coefficients

We now address the design of the weighting coefficients $\beta(\underline{n})$ and $\gamma(\underline{n})$. Consider the opening case. When the directional opening filter outputs $r_M^o(\underline{n})$ and $r_{M-1}^o(\underline{n})$ are very similar, that is, $r_M^o(\underline{n}) \approx r_{M-1}^o(\underline{n})$, $r_M^o(\underline{n})$ is selected due to $r_M^o(\underline{n}) \geq r_{M-1}^o(\underline{n})$. However, when $r_M^o(\underline{n}) >> r_{M-1}^o(\underline{n})$, $r_M^o(\underline{n})$ might be noisy data due to salt (positive impulse) noise. Thus, in this case, $r_{M-1}^o(\underline{n})$ should be selected. Based on this strategy, the weighting coefficients can be implemented by the switching method depending on the difference of directional morphological filtering outputs:

For $r_M^o(\underline{n}) \approx r_{M-1}^o(\underline{n}), f(\underline{n}) \approx r_M^o(\underline{n})$ by taking $\beta(\underline{n}) \approx 1$, and for $r_M^o(\underline{n}) >> r_{M-1}^o(\underline{n})$, $f(\underline{n}) \approx r_{M-1}^o(\underline{n})$ by taking $\beta(\underline{n}) \approx 0$. To verify this switching method, the real weighting coefficients $\beta(\underline{n})$ and $\gamma(\underline{n})$ are calculated from the upper quarter part of "Barbara" image and its noisy one. From Eqs. (8) and (9), real weighting coefficients $\beta(\underline{n})$ and $\gamma(\underline{n})$ are calculated by

$$\beta(\underline{n}) = \frac{f(\underline{n}) - r_M^o(\underline{n})}{r_M^o(\underline{n}) - r_{M-1}^o(\underline{n})}, \quad \gamma(\underline{n}) = \frac{f(\underline{n}) - r_2^c(\underline{n})}{r_2^c(\underline{n}) - r_1^c(\underline{n})}$$

where $f(\underline{n})$ is the original signal. For $r_M^o(\underline{n}) = r_{M-1}^o(\underline{n})$ and $r_1^c(\underline{n}) = r_2^c(\underline{n})$, the real weighting coefficients are obtained from $\beta(\underline{n}) = f(\underline{n}) / r_M^o(\underline{n})$ and $\gamma(\underline{n}) = f(\underline{n}) / r_1^c(\underline{n})$. Fig. 2 shows its results. Clearly, the mean values of $\beta(\underline{n})$ and $\gamma(\underline{n})$, denoted as $E[\beta(\underline{n})]$ and $E[\gamma(\underline{n})]$, are almost equal to 1 for $r_M^o(\underline{n}) = r_{M-1}^o(\underline{n})$ and $r_2^c(\underline{n}) = r_1^c(\underline{n})$, and to 0 for $r_M^o(\underline{n}) >> r_{M-1}^o(\underline{n})$ and $r_2^c(\underline{n}) >> r_1^c(\underline{n})$. Fig. 2 also verifies the strategy described above. Based on this, the weighting coefficients $\beta(\underline{n})$ and $\gamma(\underline{n})$ can be modeled as

$$\beta(\underline{n}) = 1/\exp\left(\alpha\left(r_M^o(\underline{n}) - r_{M-1}^o(\underline{n})\right)\right) \quad (10)$$

$$\gamma(\underline{n}) = 1/\exp\left(\alpha\left(r_2^c(\underline{n}) - r_1^c(\underline{n})\right)\right) \quad (11)$$

where α is the gain. As shown in Fig. 2, since $\beta(\underline{n}) \approx 1$ for $r_M^o(\underline{n}) \approx r_{M-1}^o(\underline{n})$, $f_o(\underline{n}) \approx r_M^o(\underline{n})$. However, for $r_M^o(\underline{n}) >> r_{M-1}^o(\underline{n})$, $f_o(\underline{n}) \approx r_{M-1}^o(\underline{n})$ due to $\beta(\underline{n}) \approx 0$. These weighting coefficients can be thought of as soft switching depending on the difference of the directional morphological filtering outputs. Setting $\alpha = 16/255$, the error between the original and the generalized directional morphological filtered signal is shown in Fig. 3. As shown in Fig. 3, the error histogram illustrates that the generalized directional opening (or closing) filters can remove salt (or pepper) noise without the bright (or dark) detail-loss.

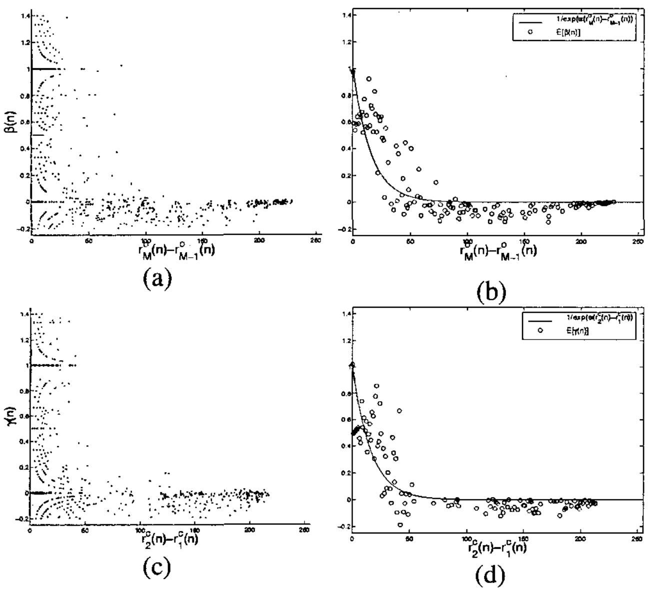
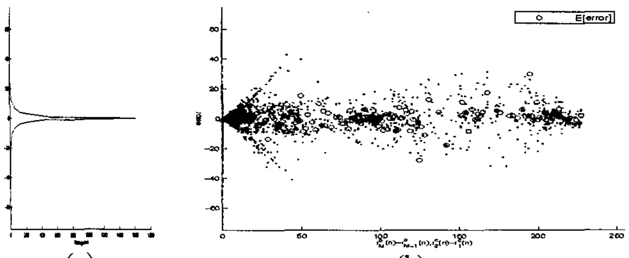


Fig. 2 Calculated weighting coefficient: (a) $\beta(\underline{n})$, (b) $E[\beta(\underline{n})]$, (c) $\gamma(\underline{n})$, (d) $E[\gamma(\underline{n})]$



(a) (b)

Fig. 3 Error between the original and GDM filtered signal: (a) histogram, (b) scatter plot

4. Adaptive Hybrid Filtering Structure

Fig. 4 shows the adaptive hybrid filtering structure for the removal of impulse noise, which is based on a combination of impulse noise detection and GDMF. For salt (or pepper) noise suppression, the generalized directional opening (or closing) is adaptively selected. To do so, the detection of salt and pepper noise is needed, which can easily be done by the geometrical difference of opening and closing with a flat structuring element.

Let $\varepsilon_o(n)$ and $\varepsilon_c(n)$ be the opening and closing distance from the signal $f(n)$, respectively.

$$\varepsilon_o(n) = f(n) - f \circ B_F(n) \tag{12}$$

$$\varepsilon_c(n) = f \bullet B_F(n) - f(n) \tag{13}$$

Table 1 MUX operation

$\varepsilon_o(n)$	$\varepsilon_c(n)$	Output
$\geq T$	0	$f_o(n)$
0	$\geq T$	$f_c(n)$
x	x	$f(n)$
T : threshold, x: don't care		

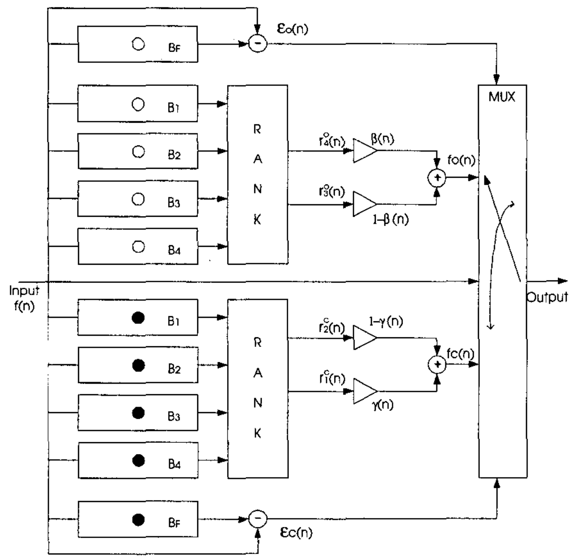


Fig. 4 Adaptive hybrid filtering structure using GDMF with M=4

where $B_F = \bigcup_{i=1}^{M} B_i$ is the flat structuring element. Note that $\varepsilon_o(n) \geq 0$ and $\varepsilon_c(n) \geq 0$. For the salt noise in smooth signal, $f \bullet B_F(n) = f(n)$ but $f \circ B_F(n) < f(n)$. Thus, $\varepsilon_o(n) > 0$ and $\varepsilon_c(n) = 0$. Using this information, the salt noise is easily detected. Likewise, the pepper noise can be also detected. The MUX operation that determines whether to filter and which of the filters to use is defined in Table 1. When the noise is detected, the generalized directional opening or closing is adaptively selected, otherwise the input signal is put forward.

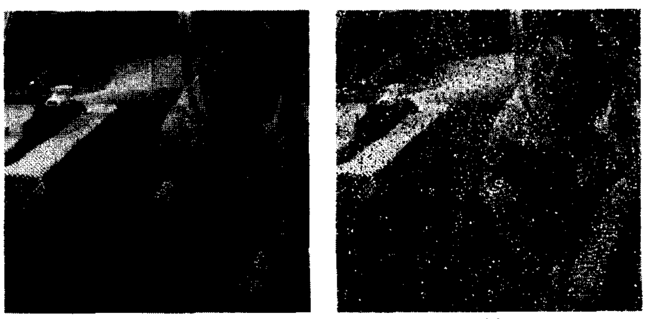
5. Simulation Results

We have compared the performance of the proposed algorithm with other nonlinear filtering techniques, i.e., median filter with 3x3 window and SD-ROM filter proposed in [8]. It is shown that the SD-ROM filter outperforms a number of existing nonlinear filters. The performance of the hybrid filtering structure using both GDM filter and conventional DM filter where M=4, $\alpha = 16/255$ and $T = 18$ is also evaluated. To improve the performance, the recursive implementation [8] is used for both the SD-ROM filter and the proposed filter. For the simulation, we generate salt and pepper noise with equal probabilities, which are set to 255 and 0, respectively. The restoration results in PSNR (dB) under 10 % and 20 % impulse noise are listed

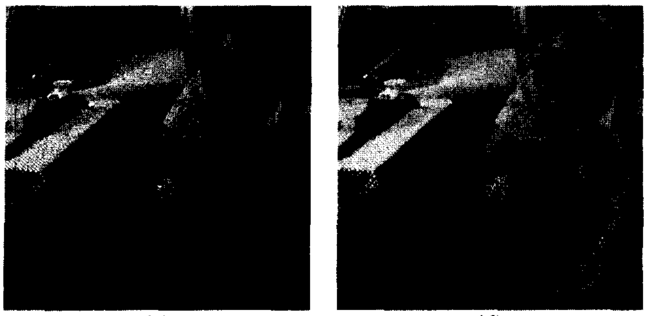
Table 2 Comparative restoration results in PSNR (dB) for impulse noise

Algorithm	10% Impulse Noise		
	Bridge	Barbara	Baboon
Median Filter	25.55	24.80	20.58
SD-ROM Filter	29.08	27.11	22.11
DM Filter	26.84	26.95	24.42
GDM Filter	28.93	28.43	25.15

Algorithm	20% Impulse Noise		
	Bridge	Barbara	Baboon
Median Filter	24.02	23.60	19.97
SD-ROM Filter	26.79	25.95	21.64
DM Filter	20.79	20.77	19.80
GDM Filter	26.47	26.61	23.59

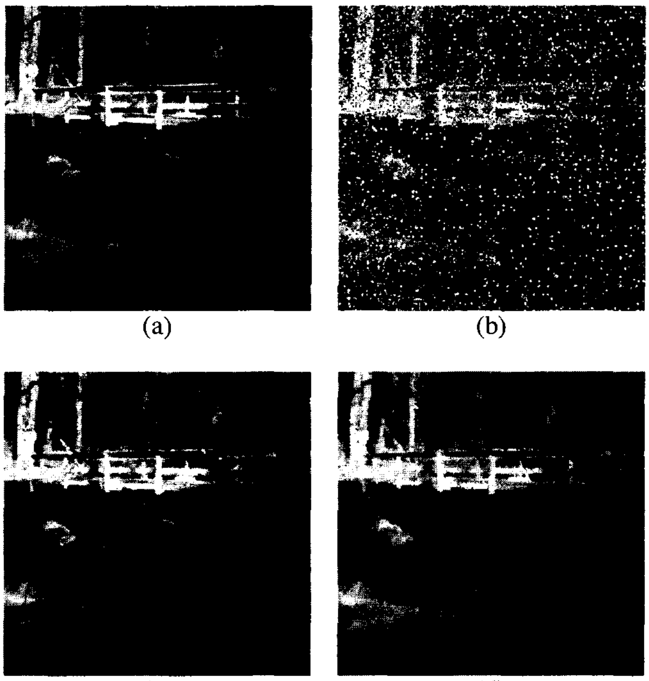


(a) (b)



(c) (d)

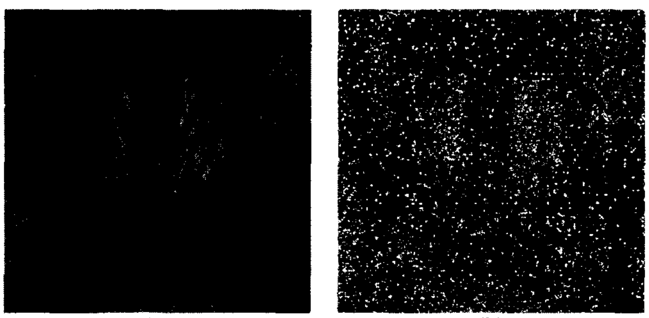
Fig. 5 Noise removal of corrupted "Barbara" image: (a) original image, (b) noisy image corrupted by 10 % impulse noise, (c) restored image by SD-ROM filter (d) restored image by GDM filter



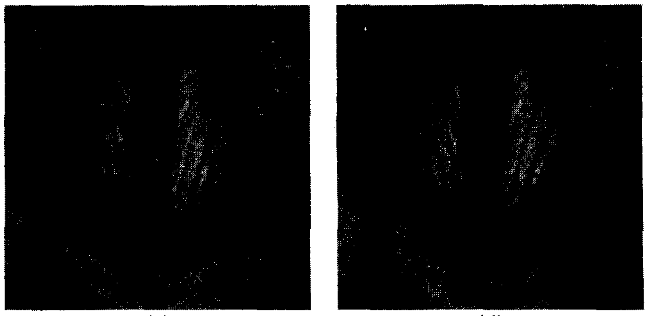
(a) (b)

(c) (d)

Fig. 6 Noise removal of corrupted "Bridge" image: (a) original image, (b) noisy image corrupted by 20 % impulse noise, (c) restored image by SD-ROM filter (d) restored image by GDM filter



(a) (b)



(c) (d)

Fig. 7 Noise removal of corrupted "Baboon" image: (a) original image, (b) noisy image corrupted by 20 % impulse noise, (c) restored image by SD-ROM filter (d) restored image by GDM filter

in Table 2. The proposed filter overall outperforms other nonlinear filters as shown in Table 2. In particular, the hybrid filtering structure using GDM filter gives considerably better performance than that using DM filter. Fig. 5 shows filtering of "Barbara" image corrupted by 10 % impulse noise. Fig. 6 and 7 show filtering of the "Bridge" and "Baboon" images corrupted by 20 % impulse noise, respectively. As shown in Fig. 5 and 7, the hybrid filtering structure using GDM filter effectively suppresses the impulse noise with the detail features preservation compared to other nonlinear filters. In case of "Bridge" image, the

SD-ROM filter has a slightly better performance than the proposed method. By comparing the restored images, it is observed that the generalized directional morphological filter is very useful for the filtering of the image that has the detail features.

6. Conclusions

In this paper we have generalized directional morphological opening and closing filters. A hybrid filtering structure using impulse noise detection and GDMF was proposed. The generalized directional opening (or closing) filter effectively removes salt (or pepper) noise with the bright (or dark) detail-preservation. The adaptive hybrid filtering structure performs a very efficient impulse noise removal. The simulation results indicate that this proposed filter performs better than other nonlinear filters in preserving desired image features while effectively reducing impulse noise.

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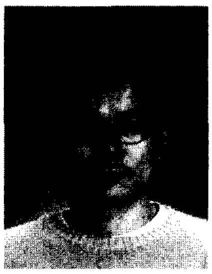

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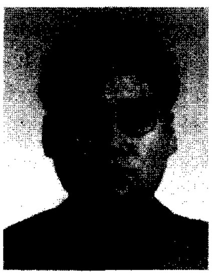

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