Stable Input-Constrained Neural-Net Controller for Uncertain Nonlinear Systems

Jang-Hyun Park and Gwi-Tae Park

Abstract - This paper describes the design of a robust adaptive controller for a nonlinear dynamical system with unknown nonlinearities. These unknown nonlinearities are approximated by multilayered neural networks (MNNs) whose parameters are adjusted on-line, according to some adaptive laws for controlling the output of the nonlinear system, to track a given trajectory. The main contribution of this paper is a method for considering input constraint with a rigorous stability proof. The Lyapunov synthesis approach is used to develop a state-feedback adaptive control algorithm based on the adaptive MNN model. An overall control system guarantees that the tracking error converges at about zero and that all signals involved are uniformly bounded even in the presence of input saturation. Theoretical results are illustrated through a simulation example.
Keywords - input constraint, neural-net control, robust adaptive nonlinear control

1. Introduction

Researching nonlinear control is experiencing rapidly increasing activity. In particular, the theory of explicitly linearizing the input/output response of nonlinear systems to linear systems using the state feedback has received great attention [1,2]. However, this theory relies on an exact cancellation of nonlinear terms to obtain linear input/output behavior. For many highly nonlinear dynamic systems, developing accurate mathematical models is generally difficult, i.e., inevitable uncertainties exist in the constructed models. Therefore, the design of a robust controller that can deal with model uncertainties is very important.

Multilayered neural networks (MNNs) have been successfully applied to many control problems because they need no accurate mathematical models of the system under consideration. Neural networks are well-known for their ability to approximate certain classes of functions to a given accuracy [3,4], and furthermore, the output of the system can be represented by a linear combination of basis functions such as radial basis functions [5]. Based on this property, many researchers presented an adaptive control architecture for uncertain nonlinear systems [6-17]. Compared with conventional adaptive control, needing no linear parameterization condition on the nonlinearities of the system is the key advantage of the control scheme using the NNs is. In the direct adaptive control scheme, the NNs are used to approximate an optimal controller [6,7]. In the indirect one, meanwhile, the MNNs are used to estimate the plant dynamics and these estimates are used to design the controller that achieves asymptotic tracking of a reference input [8-17]. For both schemes, the adjustable parameters are updated on-line by an adaptive law based on a Lyapunov approach.

The purpose of this paper is to develop an indirect adaptive control algorithm using the MNNs for uncertain nonlinear dynamical system under input constraint. As far as we know, no research result exists regarding constrained adaptive control of uncertain nonlinear systems using MNNs. We propose a new control input that can consider input saturation intuitively and analyze the stability of the closed-loop system in the Lyapunov standpoint. We also induce the conditions on the design constants that compensate for the degradation of the tracking performance caused by the saturation of the control input. The proposed controller guarantees that tracking error converges at about zero and that the states and estimated parameters are all bounded even in the presence of the input saturation.

2. Description of Neural Networks

In this paper, the linearly parameterized NNs [16] are used to capture the unknown nonlinearities of the system. In general, the output of the multi-input single-output NN is described by

$$\hat{h}(\mathbf{x}\,|\,\theta)=\sum_{i=1}^{L}\theta_i\xi_i(\mathbf{x})=\theta^{\mathrm{T}}\xi(\mathbf{x}) \tag{1}$$

where $\mathbf{x}\in R^n$ is the input vector to the NNs, $\hat{h}\in R$ is the output, $\theta\in R^L$ is the adjustable weight vector, and $\xi(\cdot):R^n\rightarrow R^L$ is the activation function vector. Among

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the various MNN models, for example, in the case of radial basis function network (RBFN), $\theta_i, i=1,\cdots L$ is the synaptic weight between the ith neuron in the hidden layer and the output neuron. $\xi_i(\mathrm{x})$ is a Gaussian function in the form of

$$\xi_i(\mathbf{x}) = \exp\left(-\frac{|\mathbf{x}-\mathbf{c}_i|}{2\sigma_i^2}\right) \quad (2)$$

$$\xi(\mathbf{x}) = \left[\xi_1(\mathbf{x}) \;\; \xi_2(\mathbf{x}) \;\; \cdots \;\; \xi_L(\mathbf{x})\right]^T \quad (3)$$

where c_i is an n-dimensional vector representing the center of the ith basis function and σ_i is the variance representing the spread of the basis function [5].

The key advantage of such NNs is their ability to approximate nonlinear mapping to any degree of accuracy, which is summarized in the following theorem.

Theorem 1. (Universal Approximation Theorem): For any given real continuous function h on a compact set $\Omega_x \in R^n$ and an arbitrary $\varepsilon_h > 0$, there exists an NN $\hat{h}$ in the form of Eq. (1) such that

$$\sup_{x\in\Omega_x}\left|\hat{h}(\mathbf{x}\,|\,\theta^*) - h(\mathbf{x})\right| < \varepsilon_h. \quad (4)$$

A proof of this theorem is given in [3-5]. Note that network reconstruction error arises as a result of the inadequacy of the NNs to exactly match an uncertain nonlinear function even if optimal weights to be selected. However, we can make ε_h arbitrarily small by increasing the size of the vector $\xi(\mathrm{x})$.

3. Controller Design and Stability Analysis

3.1 Problem Formulation

In this section, we first set up control objectives and then show how to design an adaptive controller, based on the MNNs, to achieve the objectives.

Consider the nth-order nonlinear systems of the form

$$x^{(n)} = f'(\mathbf{x}) + g(\mathbf{x})u$$
$$y = x \quad (5)$$

where f' and g are unknown continuous functions, $u \in R$ and $y \in R$ are the input and output of the system, respectively, and $\mathrm{x} = [x_1, x_2, \cdots, x_n]^T = [x, \dot{x}, \cdots, x^{(n-1)}]^T$ is the state vector of the system that is assumed to be measurable. For Eq. (5) to be controllable, $g(\mathrm{x})$ is required to be nonzero for all x in the certain controllability region $\Omega_x \in R^n$. Since $g(\mathrm{x})$ is continuous and, without loss of generality, we assume that $g(\mathrm{x})$ is positive and its lower bound g_L exists, i.e., $g(\mathrm{x}) \geq g_L > 0$ for $\mathrm{x} \in \Omega_x$. The control objective is to force the output $y(t)$ to track a given bounded reference signal $y_d(t)$ under the constraint that all signals involved must be bounded and the control input must lie in the prescribed region.

Before preceding, let us rewrite Eq. (5) as

$$y^{(n)} = f(\mathbf{x}) + g(\mathbf{x})u - \mathbf{k}^{\mathrm{T}}\mathbf{x} \quad (6)$$

where

$$f(\mathbf{x}) = f'(\mathbf{x}) + \mathbf{k}^{\mathrm{T}}\mathbf{x}$$

and $\mathrm{k} = [k_n \cdots k_1]^T$ is determined such that the polynomial $h(s) = s^n + k_n s^{n-1} + \cdots + k_1$ is Hurwitz. We approximate the functions $f(\mathrm{x})$, $g(\mathrm{x})$ using the MNN model in the form of Eq. (1) and, in the sequel, denote those terms as $\hat{f}$, $\hat{g}$.

3.2 Controller Design and Stability Proof

A control law is proposed as

$$u = \frac{1}{\varepsilon + \hat{g}(\mathbf{x})}\left(-\hat{f}(\mathbf{x}) + \alpha + \beta\right)$$
$$\alpha = y_d^{(n)} + \mathbf{k}^T\mathbf{x}_d \quad (7)$$

where $\varepsilon \geq 0$ is a positive function and $\mathrm{x}_d = [y_d \;\; \dot{y}_d \cdots y_d^{(n-1)}]$ with y_d being desired output, and β is a robustness term to be defined later. It is easily observed that if $\varepsilon = 0$, the proposed control law is the same as that of the adaptive feedback linearizing control schemes [8-17]. We introduce ε-term in the control to consider input saturation, which will be explained in detail later.

Substituting Eq. (7) into Eq. (6) can yield the following error dynamics.

$$\begin{aligned} e^{(n)} &= y^{(n)} - y_d^{(n)} \\ &= f(\mathbf{x}) + g(\mathbf{x})u + (\varepsilon + \hat{g}(\mathbf{x}))u - (\varepsilon + \hat{g}(\mathbf{x}))u - y_d^{(n)} - \mathbf{k}^{\mathrm{T}}\mathbf{x} \\ &= -\mathbf{k}^T\mathbf{e} + f(\mathbf{x}) - \hat{f}(\mathbf{x}) + (g(\mathbf{x}) - \hat{g}(\mathbf{x}))u - \varepsilon u + \beta \end{aligned} \quad (8)$$

where $e = y - y_d$ and $\mathrm{e} = [e \;\; \dot{e} \cdots e^{(n-1)}]$. The above equation can be rewritten as

$$\dot{\mathbf{e}} = A\mathbf{e} + b[-(\hat{f} - \hat{f}^*) + (f - \hat{f}^*) - (\hat{g} - \hat{g}^*)u + (g - \hat{g}^*)u - \varepsilon u + \beta]$$
$$= A\mathbf{e} + b[-\tilde{\theta}_f^T \xi_f(\mathbf{x}) + \delta_f - \tilde{\theta}_g^T \xi_g(\mathbf{x})u + \delta_g u - \frac{\varepsilon}{\varepsilon + \hat{g}}(-\hat{f} + \alpha + \beta) + \beta] \tag{9}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \vdots & \\ -k_n & -k_{n-1} & -k_{n-2} & \cdots & -k_1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

and $\tilde{\theta}_f = \theta_f - \theta_f^*$, $\tilde{\theta}_g = \theta_g - \theta_g^*$, $\hat{f}^*$ and $\hat{g}^*$ are shortened denotations of $\hat{f}^*(\mathrm{x}, \theta_f^*)$ and $\hat{g}^*(\mathrm{x}, \theta_g^*)$, respectively. θ_f^* and θ_g^* are optimal approximation parameters and are assumed to exist according to the universal approximation theorem such that $\hat{f}^*$, $\hat{g}^*$ can approximate f, g as best as possible. Therefore, $\delta_f = f - \hat{f}^*$ and $\delta_g = g - \hat{g}^*$ denote the corresponding minimum approximation errors.

Since A is a stable matrix, there exist the positive symmetric matrix P and positive constant q satisfying

$$A^T P + PA = -qI \tag{10}$$

The term εu in Eq. (9) is generated because of the ε -term in the numerator of the control law. This newly-introduced disturbance will later be compensated for by additional conditions on the design constants.

Consider the Lyapunov function

$$V = \frac{1}{2}\mathbf{e}^T P \mathbf{e} + \frac{1}{2\gamma_f}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_g}\tilde{\theta}_g^T \tilde{\theta}_g \tag{11}$$

where $\gamma_f, \gamma_g > 0$ are learning rates. Differentiating V along the solution of Eq. (9), we obtain

$$\dot{V} = -\frac{1}{2}q|\mathbf{e}|^2 + \mathbf{e}^T Pb[-\tilde{\theta}_f^T \xi_f - \tilde{\theta}_g^T \xi_g u + \delta_f + \delta_g u - \frac{\varepsilon}{\varepsilon + \hat{g}^2}(-\hat{f} + \alpha + \beta) + \beta] + \frac{1}{\gamma_f}\tilde{\theta}_f^T \dot{\theta}_f + \frac{1}{\gamma_g}\tilde{\theta}_g^T \dot{\theta}_g . \tag{12}$$

We now propose a gradient update law with constant σ -modification [19] for adjusting θ_f and θ_g as

$$\dot{\theta}_f = \gamma_f(\mathbf{e}^T Pb\xi_f - \sigma_f \theta_f) \tag{13}$$

$$\dot{\theta}_g = \gamma_g(\mathbf{e}^T Pb\xi_g u - \sigma_g \theta_g) \tag{14}$$

where $\sigma_f, \sigma_g > 0$ are the design constants. Substituting Eq. (13) and Eq. (14) into Eq. (12), we have that

$$\dot{V} = -\frac{1}{2}q|\mathbf{e}|^2 - \sigma_f \tilde{\theta}_f^T \theta_f - \sigma_g \tilde{\theta}_g^T \theta_g - \mathbf{e}^T Pb\left(\frac{\varepsilon}{\varepsilon + \hat{g}}(-\hat{f} + \alpha + \beta)\right) + \Lambda \tag{15}$$

where

$$\Lambda = \mathbf{e}^T Pb(\delta_f + \delta_g u) + \mathbf{e}^T Pb\beta \tag{16}$$

We need the following assumptions for the stability proof.

Assumption 1: There exists the known constant $\psi > 0$ such that

$$|\delta_f + \delta_g u_\alpha| < \psi \tag{17}$$

where

$$u_\alpha = \frac{1}{\varepsilon + \hat{g}(\mathbf{x})}(-\hat{f}(\mathbf{x}) + \alpha) \tag{18}$$

for all $\mathrm{x} \in \Omega_x$.

Assumption 2: There exists a constant γ such that

$$0 < \frac{|\delta_g|}{\varepsilon + \hat{g}} < \gamma < 1 \tag{19}$$

for all $\mathrm{x} \in \Omega_x$.

Assumption 2 is reasonable since, as already mentioned in Section 2, $|\delta_g|$ can be made arbitrarily small according to the universal approximation theorem and ε is the positive function.

By using Eq. (17) and Eq. (19), determining the robustifying term β yields

$$\beta = -\frac{\psi}{1-\gamma}\mathrm{sgn}(\mathbf{e}^T Pb). \tag{20}$$

By using Eq. (20), we obtain

$$
\begin{aligned}
\Lambda &= \mathbf{e}^T P\mathbf{b}\left(\delta_f + \delta_g\left(u_\alpha + \frac{\beta}{\varepsilon+\hat{g}}\right)\right) + \mathbf{e}^T P\mathbf{b}\beta \\
&= \mathbf{e}^T P\mathbf{b}\left(\delta_f + \delta_g u_\alpha\right) + \mathbf{e}^T P\mathbf{b}\left(1+\frac{\delta_g}{\varepsilon+\hat{g}}\right)\beta \\
&\le \left|\mathbf{e}^T Pb\right|\left|\delta_f + \delta_g u_\alpha\right| + \mathbf{e}^T Pb\left(1+\frac{\delta_g}{\varepsilon+\hat{g}}\right)\beta \\
&\le \left|\mathbf{e}^T Pb\right|\psi - \mathbf{e}^T Pb\,\mathrm{sgn}\left(\mathbf{e}^T Pb\right)\frac{\left(1+\frac{\delta_g}{\varepsilon+\hat{g}}\right)}{1-\gamma}\psi \\
&\le \left|\mathbf{e}^T Pb\right|\psi - \left|\mathbf{e}^T Pb\right|\frac{1-\gamma}{1-\gamma}\psi \\
&= 0.
\end{aligned} \tag{21}
$$

Combining this result with Eq. (15) and using the relations

$$\tilde{\theta}_f\theta_f = \frac{1}{2}\left|\tilde{\theta}_f\right|^2 + \frac{1}{2}\left|\theta_f\right|^2 - \frac{1}{2}\left|\theta_f^*\right|^2 \tag{22}$$

$$\tilde{\theta}_g\theta_g \ge \frac{1}{2}\left|\tilde{\theta}_g\right|^2 - \frac{1}{2}\left|\theta_g^*\right|^2, \tag{23}$$

it can be seen that Eq. (12) becomes

$$
\begin{aligned}
\dot{V} \le& -\frac{1}{2}q|\mathbf{e}|^2 - \mathbf{e}^T Pb\left(\frac{\varepsilon}{\varepsilon+\hat{g}}\left(-\hat{f}+\alpha+\beta\right)\right) \\
&-\sigma_f\left(\frac{1}{2}\left|\tilde{\theta}_f\right|^2 + \frac{1}{2}\left|\theta_f\right|^2 - \frac{1}{2}\left|\theta_f^*\right|^2\right) \\
&-\sigma_g\left(\frac{1}{2}\left|\tilde{\theta}_g\right|^2 - \frac{1}{2}\left|\theta_g^*\right|^2\right).
\end{aligned} \tag{24}
$$

At this point, it is necessary to explain why we introduce ε in the control input in Eq. (7). In most practical applications, the controller must satisfy a specific control authority, i.e., the control input must satisfy the constraint of the form

$$|u| \le \bar{u} \tag{25}$$

where $\bar{u}$ is a positive constant. The inequality in Eq. (25) defines the control authority constraint by constraining the magnitude of the control input $u(t)$. From Eq. (7), it can be easily observed that we can interpret the control saturation as ε having the value of

$$\varepsilon = -\hat{g} + \frac{\left|-\hat{f}+\alpha+\beta\right|}{\bar{u}} \tag{26}$$

while no saturation means $\varepsilon = 0$. Note that ε is a virtual term assumed to have the value of Eq. (26) in the case of input saturation. In the sequel, we will induce the conditions under which the effect of the nonzero ε (that is, in the case of input saturation) can be compensated for.

Take

$$q = 2(q_1 + q_2) \tag{27}$$

with $q_1, q_2 > 0$. Then Eq. (24) becomes

$$
\begin{aligned}
\dot{V} \le& -(q_1+q_2)|\mathbf{e}|^2 + |\mathbf{e}||Pb|\left(\left|\hat{f}\right| + |\alpha| + |\beta|\right) \\
&-\sigma_f\left(\frac{1}{2}\left|\tilde{\theta}_f\right|^2 + \frac{1}{2}\left|\theta_f\right|^2 - \frac{1}{2}\left|\theta_f^*\right|^2\right) - \sigma_g\left(\frac{1}{2}\left|\tilde{\theta}_g\right|^2 - \frac{1}{2}\left|\theta_g^*\right|^2\right) \\
\le& -(q_1+q_2)|\mathbf{e}|^2 + c_1|\mathbf{e}|\left(c_2\left|\theta_f\right| + c_3 + c_4\psi\right) \\
&-\sigma_f\left(\frac{1}{2}\left|\tilde{\theta}_f\right|^2 + \frac{1}{2}\left|\theta_f\right|^2 - \frac{1}{2}\left|\theta_f^*\right|^2\right) - \sigma_g\left(\frac{1}{2}\left|\tilde{\theta}_g\right|^2 - \frac{1}{2}\left|\theta_g^*\right|^2\right) \\
\le& -\left(q_1 - \frac{1}{4}\right)|\mathbf{e}|^2 - \frac{\sigma_f}{2}\left|\tilde{\theta}_f\right|^2 - \frac{\sigma_g}{2}\left|\tilde{\theta}_g\right|^2 - q_2|\mathbf{e}|^2 + c_1c_2|\mathbf{e}|\left|\theta_f\right| \\
&-\frac{\sigma_f}{2}\left|\theta_f\right|^2 + \frac{\sigma_f}{2}\left|\theta_f^*\right|^2 + \frac{\sigma_g}{2}\left|\theta_g^*\right|^2 + c_1^2\left(c_3 + c_4\psi\right)^2
\end{aligned} \tag{28}
$$

where $c_1 = |Pb|$, $c_4 = \dfrac{1}{1-\gamma}$. Also used,

$$0 < \frac{\varepsilon}{\varepsilon+\hat{g}} < 1 \tag{29}$$

$$\left|\xi_f(\mathbf{x})\right| \le c_2$$

$$|\alpha| \le c_3$$

$$|\mathbf{e}|c_1\left(c_3 + c_4\psi\right) \le \frac{1}{4}|\mathbf{e}|^2 + c_1^2\left(c_3 + c_4\psi\right)^2 \tag{30}$$

with $c_2, c_3 > 0$ being computable constants. Choose the design constants q_1 and σ_f such that

$$
\begin{aligned}
q_1 &> \frac{1}{4} \\
\sigma_f &> \frac{c_1^2c_2^2}{2q_2}.
\end{aligned} \tag{31}
$$

Then $\dot{V}$ becomes

$$
\begin{aligned}
\dot{V} \le& -\left(q_1 - \frac{1}{4}\right)|\mathbf{e}|^2 - \frac{\sigma_g}{2}\left|\tilde{\theta}_g\right|^2 - q_2\left(|\mathbf{e}| - \frac{c_1c_2}{2q_2}\left|\tilde{\theta}_f\right|\right)^2 \\
&+\left(\frac{c_1^2c_2^2}{4q_2} - \frac{\sigma_f}{2}\right)\left|\tilde{\theta}_f\right|^2 + \frac{\sigma_f}{2}\left|\theta_f^*\right|^2 + \frac{\sigma_g}{2}\left|\theta_g^*\right|^2 + c_1^2\left(c_3 + c_4\psi\right)^2 \\
\le& -\left(q_1 - \frac{1}{4}\right)|\mathbf{e}|^2 + \left(\frac{c_1^2c_2^2}{4q_2} - \frac{\sigma_f}{2}\right)\left|\tilde{\theta}_f\right|^2 - \frac{\sigma_g}{2}\left|\tilde{\theta}_g\right|^2 \\
&+\frac{\sigma_f}{2}\left|\theta_f^*\right|^2 + \frac{\sigma_g}{2}\left|\theta_g^*\right|^2 + c_1^2\left(c_3 + c_4\psi\right)^2.
\end{aligned} \tag{32}
$$

Let

$$c=\min\left\{\frac{2\left(q_1-\frac{1}{4}\right)}{\lambda_{\max}(P)},\gamma_f\left(\frac{\sigma_f}{2}-\frac{c_1^2c_2^2}{4q_2}\right),\gamma_g\sigma_g\right\}$$

$$\lambda=\frac{\sigma_f}{2}\left|\theta_f^*\right|^2+\frac{\sigma_g}{2}\left|\theta_g^*\right|^2+c_1^2\left(c_3+c_4\psi\right)^2. \tag{33}$$

Then Eq. (33) can be written as

$$\dot{V}\le -cV+\lambda. \tag{34}$$

From Eq. (34), we have $\dot{V}\le 0$ provided that $V>\lambda/c$. Thus, we can prove the uniform ultimate boundedness of V with respect to the set

$$\Omega_{\lambda/c}=\left\{V(t):V\le\frac{\lambda}{c}\right\}. \tag{35}$$

The following theorem summarizes our main result.

Theorem 2. Consider the nonlinear system Eq. (5) and assume that Assumptions 1 and 2 hold. The control law of Eq. (7) together with the learning laws of Eqs. (13) and (14) guarantee that the Lyapunov function of Eq. (11) is uniformly ultimately bounded with respect to the set $\Omega_{\lambda/c}$ defined by Eq. (35). From the definition of the Lyapunov functtion of Eq. (11), it can be easily seen that the same property holds for the signals $|\mathbf{e}|, |\theta_f|$ and $|\theta_g|$.

Remark 1. Regardless of the magnitude of ε, the condition of Eq. (29) is always satisfied. Thus, the stability of the closed-loop system is guaranteed as long as the parameters q_1 and σ_f are selected such that the condition of Eq. (31) holds. That is, the price paid for nonzero ε is that the constants q_1 and σ_f must satisfy the condition of Eq. (31). Note also that with the appropriate choice of the parameters, the value of λ/c can be rendered to be arbitrarily small.

Remark 2. To prohibit $\hat{g}$ from being zero, we adopt the update scheme for θ_g in [20]. That is, Eq. (36) is used whenever an element $\theta_{gi}=\varepsilon'$

$$\dot{\theta}_{gi}=\begin{cases}\gamma_g\left(\mathbf{e}^T Pb\xi_{gi}u-\sigma_g\theta_{gi}\right) & \text{if } \mathbf{e}^T Pb\xi_{gi}u\ge\sigma_g\theta_{gi}\\ 0 & \text{if } \mathbf{e}^T Pb\xi_{gi}u<\sigma_g\theta_{gi}\end{cases} \tag{36}$$

where ξ_{gi} is the ith component of ξ_g.

4. Simulation Example

To illustrate the control procedure and performance, we apply the proposed robust adaptive controller to control the inverted pendulum to track a sinewave trajectory. The dynamic equations of the system are given by [2]

$$\dot{x}_1=x_2$$
$$\dot{x}_2=\frac{g\sin(x_1)-\frac{mlx_2^2\cos(x_1)\sin(x_1)}{m_c+m}}{l\left(\frac{4}{3}-\frac{m\cos^2(x_1)}{m_c+m}\right)}+\frac{\frac{\cos(x_1)}{m_c+m}}{l\left(\frac{4}{3}-\frac{m\cos^2(x_1)}{m_c+m}\right)}u \tag{37}$$

where $x_1=\theta$ represents the angle of the pendulum, x_2 represents the angular velocity, $g=9.8\ \mathrm{m/s^2}$ is the acceleration due to gravity, m_c is the mass of cart, m is the mass of pole, l is the half length of pole, and u is the applied force (control). We choose $m_c=1$ kg, $m=0.1$ kg, and $l=0.5$ m in the following simulations. We also choose the reference signal $y_m(t)=\frac{\pi}{30}\sin(t)$ in the following simulations.

The design parameters are specified as follows. Let $k_1=2, k_2=1$ (so that $s^2+k_1s+k_2$ is stable), and $q=10$, then we have the Lyapunov equation (11) and obtain

$$P=\begin{bmatrix}15 & 5\\ 5 & 15\end{bmatrix} \tag{38}$$

which is positive-definite with $\lambda_{\min}=2.9289$. We also choose $\sigma_g=0.001$ and $\gamma=0.1$.

The compact set Ω_x is chosen to be $|x_j|\le\frac{\pi}{6}$ for both $j=1,2$. We have chosen the RBFNs as approximators for unknown nolinearities whose approximation capability has been proven in [5]. The radial basis functions for $\mathbf{x}$ are described by

$$\xi_i(\mathbf{x})=\exp\left(-\frac{|\mathbf{x}-\mathbf{c}_i|}{2\sigma_i^2}\right), i=1,\cdots,25$$

$$\sigma_i=\sqrt{\frac{\pi}{12}},\quad \mathbf{c}_i=\{(x_1,x_2)|x_1\in S_1, x_2\in S_2\}, i=1,\cdots,25$$
$$S_j=\{-0.2\pi,\ -0.1\pi,\ 0,\ 0.1\pi,\ 0.2\pi\}, j=1,2.$$

The adaptation rate constants γ_f and γ_g are set to 2000 and 10, respectively, and γ is chosen to be 0.1. The initial values are $\theta^0_{fk}=0$, $\theta^0_{gk}=1.5$ for all $k=1,\cdots,25$ where θ^0_{fk} and θ^0_{gk} denote the kth element of the initial vector θ^0_f and θ^0_g, respectively, that is, no *a priori* information for θ_f and θ_g is assumed to be known. The initial state is $x(0)=[-0.1\ 0]^T$.

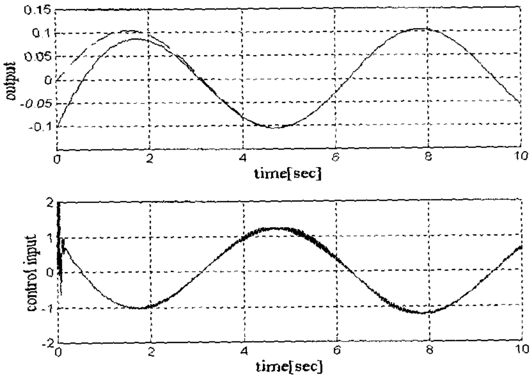

Fig. 1 In the case of $q_1=0.2, \sigma_f=0.5$ without input saturation: (a) the system output (line) and desired output (dashed line) (b) control input

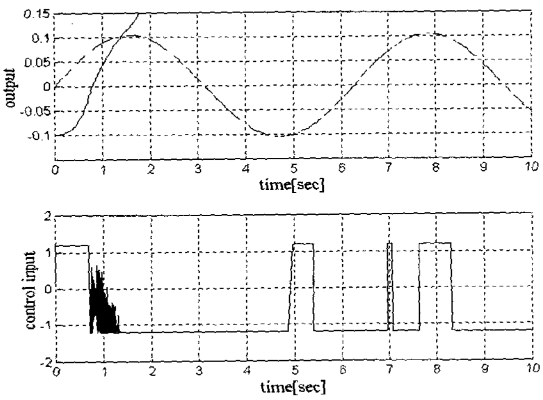

Fig. 2 In the case of $q_1=0.2, \sigma_f=0.5$ and occurring input saturation with $\bar{u}=1.2$: (a) the system output (line) and desired output (dashed line) (b) control input

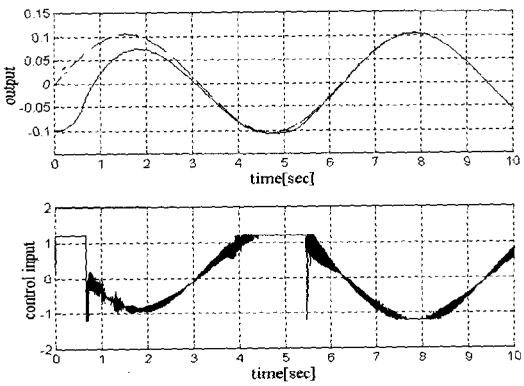

Fig. 3 In the case of $q_1=4, \sigma_f=4.2$ and occurring input saturation with $\bar{u}=1.2$: (a) the system output (line) and desired output (dashed line) (b) control input

For comparison, we simulate the three controllers for this inverted pendulum tracking problem. First, we use the proposed controller with $q_1=0.2$ (thus, $q_2=9.8$), $\sigma_f=0.5$ and no input saturation occurs. Secondly, we use the same values for q_1 and σ_f as the former case, and this does not satisfy the condition of Eq. (31) but input saturation with $\bar{u}=1.2$ occurs. The last simulation is conducted using the same controller but with $q_1=4$ (thus, $q_2=6$), $\sigma_f=4.2$ and the same input saturation occurs. It can be easily checked that they satisfy the condition of Eq. (31). The system output, reference output, and control input of the three controllers are illustrated in Figures 1 through 3.

From the results, it can be inferred that the system output tracks the desired output well with the third controller, while the system with the second controller has lost the stability.

5. Conclusions

In this paper, we apply MNNs to model uncertain or ill-defined feedback linearizable nonlinear systems having input saturation. Adaptive laws are developed to adjust uncertain parameters that are linear in the output of the neural networks. The proposed control input has an additional term (ε) that can consider the input saturation efficiently. Also, the proposed control input also contains the robustifying control term that adaptively compensates for the reconstruction errors. The adaptive laws and control input are established to stabilize the closed-loop system from the Lyapunov standpoint. It has been shown that additional conditions on some design constants are needed to guarantee the stability of the closed-loop system under input constraint. Simulations for the inverted pendulum system have demonstrated that the proposed control architecture can maintain stability under input saturation and provides satisfactory tracking performance.

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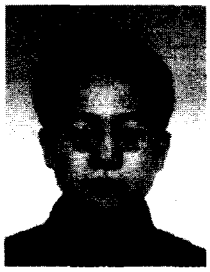

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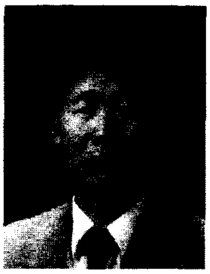

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