Compensation of Networked Control Systems using LMI-Based H_∞ Optimization Method

Ho-Jun Yoo, Myung-Eui Lee and Oh-Kyu Kwon

Abstract - Delay and noise in networked control systems are inevitable and can degrade system performance or stability. This paper propose a compensation method for networked control systems with network-induced delay and noise using LMI(linear matrix inequality)-based H_∞ optimization. The H_∞ optimization methods have adapted to account for both the time delay and noise effects. Some simulations applied to inverted pendulum with networked control show that the proposed method works well.

1. Introduction

Recently network systems have become widely used, and some considerable attention has been directed the networked control system (NCS). The NCS is defined as the feedback control system that the control loops are closed through a real-time network. In an NCS, the inevitable network-induced delays degrade the system performance and can potentially cause instability. NCSs are used in communication networks and network-induced noise in the communication line is inevitable. Current control research focuses on developing appropriate controllers to compensate for network-induced delay. However, design considerations have been often overlooked as a way of compensating for network-induced noise.

The network-induced delays are time-varying and possibly stochastic dynamics of the traffic. Isle [5] has suggested the use of stochastic Lyapunov functions for stability analysis of systems with random time-varying delays. By making the buffers longer than the worst- case delay, the network-induced delay can be regarded as time-invariant. Luck and Ray [6] regard a network-induced delay as constant via a buffering procedure.

The stability and performance analyses of NCSs are referred to in several papers, but compensatory designs of NCSs are only beginning. Zhang [8] has proposed a current state estimator method to compensate for a stochastic network-induced delay. Luck and Ray [6] present an observer-based compensatory design that is a constant delay method. However, they have considered only delays and no network-induced noises.

In this paper, an LMI-based H_∞ optimization for an NCS is proposed. The control system analysis and design using H_∞ optimizations is robust to external noise or perturbations. Therefore, the main contribution of this paper is to propose a controller design for a networked control system; this design considers both network-induced delays and noise.

This work has been supported in part by Electrical Engineering & Science Research Institute grant (01–042), which is funded by Korea Electric Power Co.

Manuscript received: Nov. 22, 2001 accepted: May 9, 2002

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2. NCS Description

An NCS block diagram with network-induced delay is shown in Fig. 1. In this NCS, the whole closed-loop system consists of a continuous-time plant and a discrete-time controller. A setup is considered that has time-driven sensors that sample the plant outputs periodically at sampling instant. In additions, an event-driven controller and actuator, which can be implemented by an external event interrupt mechanism, calculates the signal as soon as the data arrives.

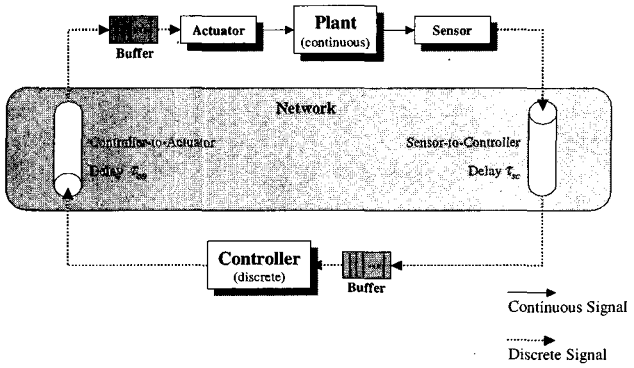

Fig. 1 Block diagram of NCS with network-induced delay

Let us consider a continuous-time plant with measurement noise,

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) + D_w w(t), \quad (2)$$

and a discrete-time controller given as follows:

$$u(kh) = Kx(kh), \tag{3}$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^p$, and A, B, C, D_w, and K are of compatible dimensions.

There are two sources of delay from the network: sensor-to-controller delay τ_{sc} and controller-to-actuator delay τ_{ca}. For time-invariant controllers, the sensor-to-controller delay and controller-to-actuator delay can be lumped together as $\tau = \tau_{sc} + \tau_{ca}$ [4]. In Luck and Ray [6], the networked delay τ is made time-invariant by the introduction of buffers at the controller and actuator nodes. By making these buffers longer than the worst-case delay time, the transfer time can be considered constant.

A periodic sampled signal in a plant sensor is passed through a communication network. This signal arrives at a buffer with a different time due to the networks time-varying delays. Assume that if the sensor-to-buffer delays are measurable and the upper bound $\bar{\tau}$ ($\bar{\tau} = \max \tau_i$) is known beforehand using the time-stamping method [7], then the controller input signal (after buffering) could be made to be a constantly delayed signal. This signal transmission procedure is shown in Fig. 2.

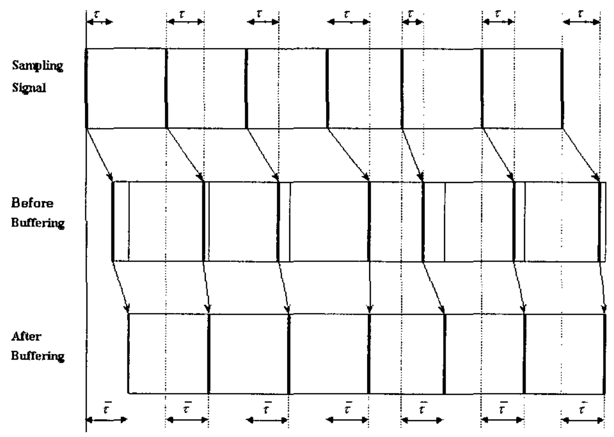

Fig. 2 Timing diagram in NCSs with buffers

If the delay τ is less than one sampling period h, then the network closed-loop system equation can be written as

$$u(t^+) = Kx(t-\tau) + w(t) \tag{4}$$

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t^+) \\ &= Ax(t) + BKx(t-\tau) + Bw(t) \end{aligned} \tag{5}$$

$$y(t) = Cx(t) + D_w w(t) \tag{6}$$

where $u(t^+)$ is piecewise continuous and its value changes only at $kh+\tau$. The effect of network-induced noise is described by $w(t)$, which has unknown, finite-energy, stochastic properties. Assume that the network-induced disturbances and the measurement noises are uncorrelated, i.e. $D_w B^T = 0$.

3. Discrete Time H∞ Controller Design for an NCS

Sampling the network closed-loop system in Eqs.(4) – (6) with period h, the discrete-time linear system with time delay τ is obtained as follows [1]:

$$\begin{aligned} x(kh+h) = \Phi x(kh) + [\Gamma_0(\tau)Kx(kh) \\ + \Gamma_1(\tau)Kx(kh-h)] + \Gamma_w w(kh) \end{aligned} \tag{7}$$

$$y(kh) = Cx(kh) + D_w w(kh) \tag{8}$$

where

$$\Phi = e^{Ah}, \quad \Gamma_w = \int_0^h e^{As} B ds \tag{9}$$

$$\Gamma_0(\tau) = \int_0^{h-\tau} e^{As} B ds, \quad \Gamma_1(\tau) = \int_{h-\tau}^h e^{As} B ds$$
$$x(kh) = 0, \ k < 0, \quad x(0) = x_0. \tag{10}$$

The proposed state feedback law is

$$u(kh) = Kx(kh). \tag{11}$$

When the non-delayed terms Φ and Γ_0 in Eq.(7) are substituted by Φ_K, the closed-loop system from $w(kh)$ to $z(kh)$ is given by

$$x(kh+h) = \Phi_K x(kh) + \Gamma_1(\tau)Kx(kh-h) + \Gamma_w w(kh) \tag{12}$$

$$y(kh) = Cx(kh) + D_w w(kh) \tag{13}$$

where

$$\Phi_K = \Phi + \Gamma_0 K. \tag{14}$$

Theorem 1. Under the discrete-time linear system with delay described by Eqs.(7) – (10), if there exist positive-definite matrices P and R and a given $\gamma > 0$ satisfying the inequality

$$\begin{bmatrix} -P^{-1} & \Phi_K & \Gamma_1 & \Gamma_w & 0 \\ \Phi_K^T & K^T RK - P & 0 & 0 & C^T \\ \Gamma_1^T & 0 & -R & 0 & 0 \\ \Gamma_w^T & 0 & 0 & \gamma^2 I & D_w^T \\ 0 & C & 0 & D_w & -I \end{bmatrix} < 0, \tag{15}$$

then the system in Eqs.(12) – (13) is quadratically stable with an H∞ norm bound γ by the controller (11).

Proof. First, we define a Lyapunov candidate as

$$V(x(kh)) \equiv x^T(kh)Px(kh) + x^T(kh-h)K^T RKx(kh-h). \tag{16}$$

Assuming the zero-input system, i.e., $w(kh) = 0$, the difference of the Lyapunov functional is

$$\Delta V_k = V(x(kh+h)) - V(x(kh)) \tag{17}$$

$$= \begin{bmatrix} x(kh) \\ Kx(kh-h) \end{bmatrix}^T H \begin{bmatrix} x(kh) \\ Kx(kh-h) \end{bmatrix} \tag{18}$$

where

$$H = \begin{bmatrix} \Phi_K^T P \Phi_K + K^T RK - P & \Phi_K^T P \Gamma_1 \\ \Gamma_1^T P \Phi_K & \Gamma_1^T P \Gamma_1 - R \end{bmatrix}, \tag{19}$$

which ensures the quadratic stability of the closed-loop system in Eqs.(12) – (13). Next, an initial condition of zero and take the following performance index:

$$J = \sum_{k=0}^{\infty} [\, y^T(kh)y(kh) - \gamma^2 w^T(kh)w(kh)]. \tag{20}$$

For any non-zero $w(kh) \in L_2[0,\infty)$,

$$J \le \sum_{k=0}^{\infty} [\, y^T(kh)y(kh) - \gamma^2 w^T(kh)w(kh) + \Delta V_k\,].$$

Let $\delta(kh) = [x^T(kh) \quad x^T(kh-h)K^T \quad w^T(kh)]^T$; then

$$J \le \sum_{k=0}^{\infty} \delta^T \Omega \delta$$

where

$$\Omega \equiv \begin{bmatrix} \Phi_K^T P \Phi_K + K^T RK - P + C^T C & \Phi_K^T P \Gamma_1 & C^T D_w + \Phi_K^T P \Gamma_w \\ \Gamma_1^T P \Phi_K & \Gamma_1^T P \Gamma_1 - R & \Gamma_1^T P \Gamma_w \\ D_w^T C + \Gamma_w^T P \Phi & \Gamma_w^T P \Gamma_1 & D_w^T D_w - \gamma^2 I + \Gamma_w^T P \Gamma_w \end{bmatrix} < 0. \tag{21}$$

This inequality (21) implies that $\|y(kh)\|_2 \le \gamma \|w(kh)\|_2$ for any non-zero $w(kh) \in L_2[0,\infty)$. Therefore, the system is quadratically stable with H_∞ norm bound γ by the controller (11). Using Schur complements [2], the inequality is shown to be equivalent to Eq.(15).

In Theorem 1, the inequality in Eq.(15) is not an LMI structure due to the $K^T RK$ term in the 2 × 2 element. So this inequality needs an augmented linear structure.

Theorem 2. For the discrete time-delay system Eq.(7) – (10), there exist symmetric positive-definite matrices P, R_1, and R_2 such that the matrix inequality in Eq.(15) holds if and only if there exist positive-definite matrices Q and S and a matrix M such that the following LMI is satisfied:

$$\begin{bmatrix} -Q + \Gamma_1 S \Gamma_1^T & \Phi Q + \Gamma_0 M & \Gamma_w & 0 & 0 \\ Q\Phi^T + M^T \Gamma_0^T & -Q & 0 & QC^T & M^T \\ \Gamma_w^T & 0 & -\gamma^2 I & D_w^T & 0 \\ 0 & CQ & D_w & -I & 0 \\ 0 & M & 0 & 0 & -S \end{bmatrix} < 0. \tag{22}$$

Furthermore, if the matrix inequality in Eq.(22) has a feasible solution, then the system in Eqs.(12) – (13) is quadratically stable with an H_∞ norm bound γ. Let

$$M = KP^{-1}, \quad Q = P^{-1}, \quad S = R^{-1}. \tag{23}$$

Proof. It follows from the Schur complements [2] and some change of variables that Eq.(15) is equivalent to,

$$\begin{bmatrix} -P^{-1} & \Phi_K & \Gamma_1 & \Gamma_w & 0 & 0 \\ \Phi_K^T & -P & 0 & 0 & C^T & K^T \\ \Gamma_1^T & 0 & -R & 0 & 0 & 0 \\ \Gamma_w^T & 0 & 0 & -\gamma^2 I & D_w^T & 0 \\ 0 & C & 0 & D_w & -I & 0 \\ 0 & K & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0 \tag{24}$$

$$\Leftrightarrow \begin{bmatrix} -P^{-1} + \Gamma_1 R^{-1} \Gamma_1^T & \Phi_K & \Gamma_w & 0 & 0 \\ \Phi_K^T & -P & 0 & C^T & K^T \\ \Gamma_w^T & 0 & -\gamma^2 I & D_w^T & 0 \\ 0 & C & D_w & -I & 0 \\ 0 & K & 0 & 0 & -R^{-1} \end{bmatrix} < 0. \tag{25}$$

Pre- and post-multiplying both sides of the inequality in Eq.(25) by

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & P^{-1} & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix},$$

we have

$$\begin{bmatrix} -P^{-1} + \Gamma_1 R^{-1} \Gamma_1^T & \Phi_K P^{-1} & \Gamma_w & 0 & 0 \\ P^{-1}\Phi_K^T & -P^{-1} & 0 & P^{-1}C^T & P^{-1}K^T \\ \Gamma_w^T & 0 & -\gamma^2 I & D_w^T & 0 \\ 0 & CP^{-1} & D_w & -I & 0 \\ 0 & KP^{-1} & 0 & 0 & -R^{-1} \end{bmatrix} < 0. \tag{26}$$

Changing the variables to $M = KP^{-1}$, $Q = P^{-1}$, and $S = R^{-1}$, the inequality (26) is converted to Eq.(22).

Inequality (22) is an LMI in terms of Q, M, and S. The state feedback controller can be calculated after finding the LMI solution. The proposed quadratically stable controller is

$$K = MQ^{-1}. \tag{27}$$

Using MATLAB's LMI Toolbox, the solutions can be easily obtained because Eq.(22) is an LMI in terms of variables.

Theorem 2 provides an LMI condition in Eq.(22) for the H_∞ controller, which guarantees the H_∞ norm bound γ of the transfer function T_{zw}. Solving the following minimization problem, we can obtain the H_∞ controller, which minimizes the H_∞ norm bound :

$$\underset{M,Q,S}{Minimize}\ \gamma \tag{28}$$

subject to

$$\begin{bmatrix} -Q+\Gamma_1 S\Gamma_1^T & \Phi Q+\Gamma_0 M & \Gamma_w & 0 & 0 \\ Q\Phi^T+M^T\Gamma_0^T & -Q & 0 & QC^T & M^T \\ \Gamma_w^T & 0 & -\gamma^2 I & D_w^T & 0 \\ 0 & CQ & D_w & -I & 0 \\ 0 & M & 0 & 0 & -S \end{bmatrix} < 0. \quad (29)$$

The solution of the minimization problem in Eqs.(28) – (29) gives the delay independent H_∞ controller that guarantees the stability and the H_∞ norm bound γ.

4. Simulation for Inverted Pendulum in NCSs

Let us consider the simplified inverted pendulum experiment model with a network delay τ. The linearized equations of the inverted pendulum dynamics are derived as follows :

$$\begin{bmatrix} \dot{x} \\ \dot{\alpha} \\ \ddot{x} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{m_p g}{m_c} & 0 & 0 \\ 0 & \frac{(m_p+m_c)g}{m_c l_c} & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \alpha \\ \dot{x} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_c} \\ -\frac{1}{m_c l_p} \end{bmatrix} F$$

where x is the position of the cart, α is the angle of the rod, F is the input force to the cart, m_p is the mass of the rod, m_c is the mass of the cart, and l_p is the center of gravity of the rod. To convert to a voltage input, we derive the relationship

$$F = \frac{K_m K_g}{Rr} V_{in} - \frac{K_m^2 K_g^2}{Rr^2} \dot{x}.$$

Substituting this into the matrix equation, we have

$$\begin{bmatrix} \dot{x} \\ \dot{\alpha} \\ \ddot{x} \\ \ddot{\alpha} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -4.5 & -16.8 & 0 \\ 0 & 46.9 & 55.3 & 0 \end{bmatrix} \begin{bmatrix} x \\ \alpha \\ \dot{x} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3.8 \\ -12.4 \end{bmatrix} V_{in}.$$

So, the values of matrices are given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -4.5 & -16.8 & 0 \\ 0 & 46.9 & 55.3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 3.8 \\ -12.4 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad D_w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

An LQR controller that fails to consider delay τ may result in an unstable closed-loop system due to the effect of the network-induced delay.

The conventional LQR controller is

$$K_{LQR} = [17.1696 \quad 68.1903 \quad 19.6588 \quad 8.1552], \quad (30)$$

which was obtained with a choice of the weighting matrices

$$Q = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R = 0.0003.$$

There is no doubt that this LQR controller stabilizes the system without delay. However, when applied to a system delay, the closed-loop system is unstable as shown in Fig. 3. In this simulation, sampling time and network delay are assumed to be $h = 0.03$ and $\tau = 0.015$, respectively.

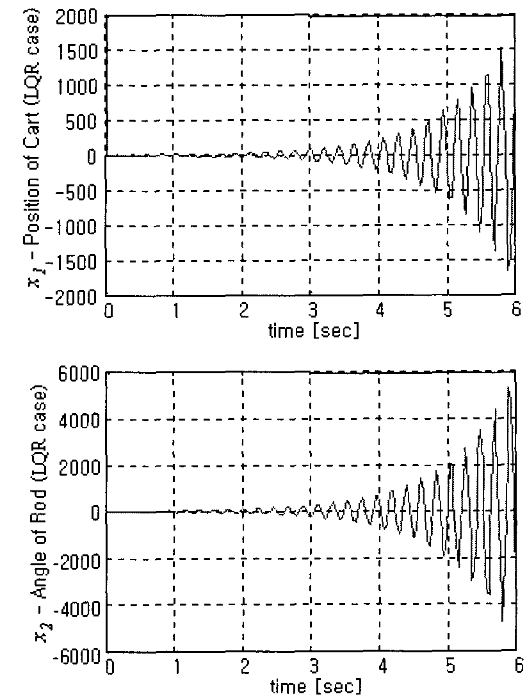

Fig. 3 Response of the conventional LQR controller

On the other hand, our objective is the design of a discrete-time H_∞ controller for this NCS such that the corresponding bound is minimized. Such a design is found using the LMI-toolbox in MATLAB [3]. The optimal solution is given by

$$P = \begin{bmatrix} 126.9271 & 210.3351 & 86.9217 & 31.0241 \\ 210.3351 & 1601.3880 & 506.7649 & 197.1137 \\ 86.9217 & 506.7649 & 175.7100 & 66.5726 \\ 31.0241 & 197.1137 & 66.5726 & 26.8768 \end{bmatrix}$$

$R = 0.0614$.

By Theorem 2, the proposed H_∞ controller is given by

$$K = [4.3914 \quad 48.5885 \quad 16.5855 \quad 7.4715]. \quad (31)$$

Using the control law in Eq.(31), we have the simulation results shown in Fig.4, which indicates that the proposed method works well.

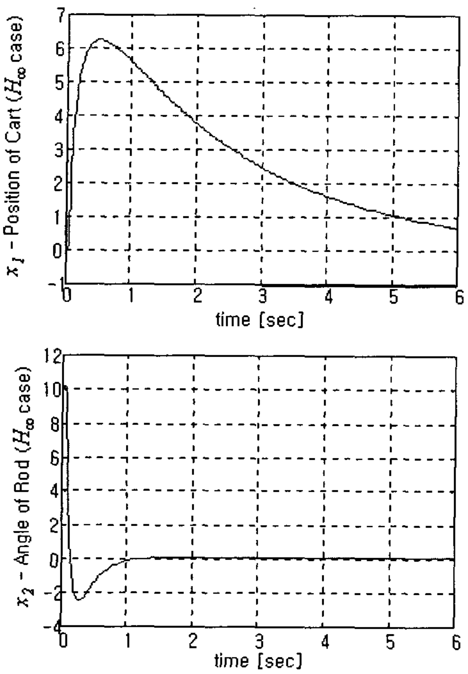

Fig. 4 Response of the proposed H_∞ controller

5. Conclusion

An LMI-based H_∞ controller for an NCS with networked-induced delay and noise has been proposed. In the proposed method, the controller is designed to account for both delays and noises. The network-induced delays are assumed to be constant using buffers. To show the effectiveness of the proposed control scheme, some simulations are performed and application to the inverted pendulum model with an NCS shows that it works well.

In this paper, delays are assumed to be less than the sampling period. This short-delayed NCS can be solved using the proposed delay-independent H_∞ controller. However, if we have a control delay larger than the sampling period, the proposed methods cannot compensate for the NCS. One possibility could be to use a stochastic delayed model. A proper delay-dependent H_∞ controller can be designed for these systems.

Acknowledgment

This work has been supported in part by Electrical Engineering & Science Research Institute grant (01–042), which is funded by Korea Electric Power Co.

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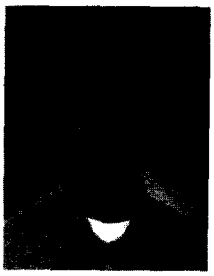

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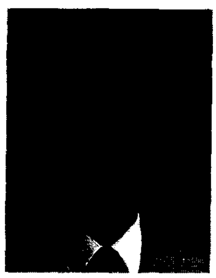

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