한국 퍼지 및 지능시스템 학회 2002, Vol. 12, No. 5, pp. 473-480

A DESIGN OF QUASI TIME-OPTIMAL FUZZY CONTROL SYSTEMS

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Abstract

The problems of quasi time-optimal digital control are discussed. A new design methodology of quasi time-optimal fuzzy controllers based on approximation of prototype discrete controller is suggested. Four kinds of practicable structures for fuzzy controllers are considered. Examples of computer design of quasi time-optimal fuzzy control systems are given.

Key Words : Time-optimal control, Quasi time optimal, Fuzzy control systems.

Ⅰ. Introduction

The problem of a nonlinear time-optimal digital control for linear SISO-plant can be stated mathematically as a problem of driving the state vector $X[0]$ to $X[N]=0$ in the minimum number of sampling periods N and finding the scalar control $u[n]$, $n=0,1,\cdots,N-1$, such that minimizes the criterion

$$J=\frac{1}{2}\sum_{n=0}^{N-1} u^T(n)\cdot R\cdot u(n)$$

subject to the dynamic constraint

$$X[n+1]=A\cdot X[n]+B\cdot u[n]$$

and also the amplitude constraint

$$|u[n]|\le U_0$$

where R is symmetric and positive definite.

This problem can be solved by the maximum principle or by the method of discrete dynamic programming [1, 2].

Unfortunately, the control law synthesized by theoretical methods is a nonlinear time-variant function and besides assumes the observation of full state vector of the plant. So its direct implementation as a closed-loop control system is often impossible. For the second order plant an analytical expression of time-optimal digital control in time-invariant form can be derived, but for the high order plants the numerical synthesis is possible only. Therefore, in practice, the synthesis of a quasi time-optimal discrete control is more useful. In this paper, a design methodology of quasi time-optimal fuzzy controllers based on fuzzy approximation of prototype discrete controller is suggested. The nonlinear parts of quasi time-optimal controllers can be approximated as four kinds of different structures. The design examples for four structures are presented.

The paper is organized as follows. In section Ⅱ, the statement of the problem of quasi time-optimal control is reviewed. In section Ⅲ, we propose a design methodology of quasi time-optimal fuzzy controllers based on fuzzy approximation of nonlinear parts of prototype discrete controller. In section Ⅳ, design examples of quasi time-optimal fuzzy control system are given to illustrate effectiveness of the proposed method. Finally, conclusions are included in section Ⅴ.

Ⅱ. Statement of the Problem of Quasi Time-Optimal Control

The synthesized control system will be quasi optimal, that is approximately optimal in sense of main performance criterion, when:

a) The synthesis is carried out mathematically and strictly with use of reduced model of the plant.

b) The optimal control law obtained after synthesis is simplified before its implementation.

c) The modified structure of control algorithm is chosen in the very beginning.

The reduced-order model of the plant and the simplification of optimal control, in which is theoretically exact but not technically realizable, are often used in practice. In the third case the mathematical strictness of a synthesis problem statement is absent and the control

접수일자 : 2002년 5월 13일
완료일자 : 2002년 9월 18일
본 논문은 2000년도 금오공과대학교 교내 연구지원에 의하여 연구되었습니다.

law structure is chosen from the finite set of variants which are technically most expedient.

In regard of digital quasi time-optimal control, the two approaches to its synthesis are possible.

1) The discretization of analytical optimal control law synthesized for reduced continuous plant.

2) The synthesis of discrete control algorithm with simplified structure.

The first approach can be used, when the value of sampling period T_0 is rather small and so after finite-difference approximation of derivatives the control algorithm will ensure processes to be close to optimal.

The block diagram of a time-optimal continuous control system is shown in Fig. 1. In Fig. 1, RU is a relay unit, FC is a functional converter, DU is a differentiating unit, $g(t)$, $e(t)$, $f(t)$ and $y(t)$ are input, error, external disturbance and output coordinate of the plant, respectively.

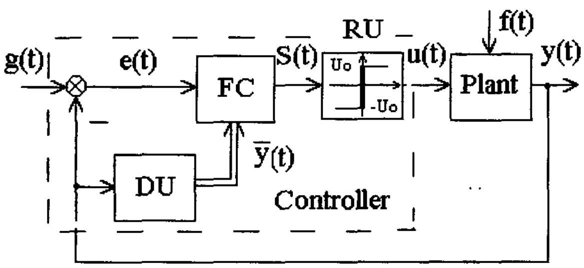


Fig. 1. Block diagram of a time-optimal continuous control system.

The control law is

$$u(t) = U_0 \cdot sign\ S(e(t), \bar{y}(t)) \quad (1)$$

where $S(e(t), \bar{y}(t))$ is a switching surface, $\bar{y} = (\dot{y}, \ddot{y}, \cdots, y^{(m-1)})^T$ is a vector of derivatives of output coordinate and U_0 is constant value.

As well known [2], for linear plant of 2nd order it is possible to derive an analytical expression of switching surface. However its practical implementation may be not simple.

The second approach is more preferable in practice and can be used in case of high order plants. The different modifications of quasi time-optimal discrete control algorithm may be suggested.

The general structure of a digital system with quasi time-optimal discrete controller can be represented as shown in Fig. 2. In Fig. 2, H is a zero-order holder, RU is a relay unit that value of $U_0[n]$ is variable, DU is a difference unit, FC is a functional converter, and T_0 is a sampling period.

The control law has the following structure

$$u(n) = U_0[n] \cdot sign\ S_a(e(n), \bar{y}(n)) \quad (2)$$

where $S_a(e[n], \bar{y}[n])$ is an approximated switching surface and $\bar{y}[n] = (\nabla y[n], \cdots, \nabla^{(m-1)} y[n])^T$ is a vector of output coordinate differences.

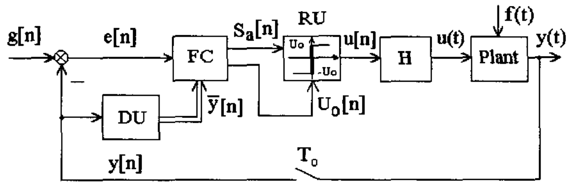


Fig. 2. Block diagram of quasi time-optimal digital control system.

For relay digital control, when U_0 is constant value, the undesirable plant's oscillations appear at intersections of switching surface and in neighborhood of phase variables origin. So using the quasi-relay control that U_0 is variable, it is possible to eliminate such oscillations.

Ⅲ. A Design of Quasi Time-Optimal Fuzzy Controllers

The main idea of design of quasi time-optimal fuzzy controllers proposed here consists in fuzzy approximation of nonlinear parts of prototype discrete controller.

The four stages shown in Fig. 3 can be carried out in design process using a computer.

On the first stage the synthesis of quasi time-optimal discrete controller is carried out. The methodology of synthesis which is recommended to use on the first stage is suggested in section 3.1. In general, the switching surface are determined in tabular form by simulation and then can be approximated by quadratic, cubic or other sort of nonlinear functions.

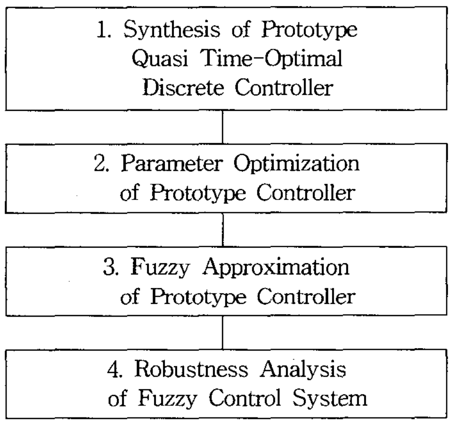


Fig. 3. Stages of design of quasi time-optimal fuzzy controllers.

On the second stage, the parameter optimization of prototype controller is carried out. The error of switching surface approximation and the elimination of undesirable sliding mode may require the optimization of discrete controller parameters by numerical methods. The integral criterion minimization or the multi-criterion optimization can be used.

On the third stage, some of nonlinear parts of prototype controller are approximated using fuzzy implementation. In section 3.2 the four kinds of practicable structures for quasi time-optimal fuzzy controllers has been proposed.

In case of the plant with uncertainties the robustness analysis has to be carried out on the fourth stage.

3.1 Synthesis of Quasi Time-Optimal Discrete Controllers

The synthesis of prototype discrete controllers includes the next five steps.

1. The discretization of state-space model of continuous plant.

The use of plant's discrete model eliminates the necessity in finite-difference approximation of derivatives during definition of desirable switching surface.

2. The calculation of desirable switching surface in tabular form.

As desirable switching surface the braking phase trajectory of discrete plant can be used, which may be found by simulation with non-zero initial conditions.

3. The approximation of switching surface.

From point of view of controller implementation, the quadratic approximation represents the greatest interest.

4. The modification of structure of quasi time- optimal controller.

The different variants of discrete control law that value of U_0 is variable can be proposed and chosen in practice.

5. The parameter optimization of quasi-optimal control system.

The optimization of controller's parameters may be carried out at first with discrete, and then with continuous plant's model.

As preliminary optimization the integral criterion minimization can be applied. The final values of prototype controller parameters have to be chosen in Pareto-domain using multi-criterion optimization.

The general expression of quadratic switching surface is as follows

$$S_a(e[n],\bar{y}[n]) = e[n] - \beta_I^T\bar{y}[n] - \bar{y}^T[n]\beta_{II}\bar{y}[n] \quad (3)$$

where β_I is $(m-1)$-vector and β_{II} is $(m-1)\times(m-1)$ -matrix of coefficients, which can be determined by the regression method.

In particular, concerning the origin of coordinates $e[n]$ and $\nabla y[n]$, the quadratic switching line is

$$S_a(e[n],\nabla y[n]) = e[n] - \beta_1\cdot\nabla y[n] - \beta_2\cdot(\nabla y[n]\cdot|\nabla y[n]|) \quad (4)$$

and parameters β_1 and β_2 can be found as a solution of the system of linear algebraic equations

$$A_{\nabla y}\cdot X = B_e \quad (5)$$

where $A_{\nabla y} = \begin{pmatrix} \nabla y[1] & \nabla y[1]\cdot|\nabla y[1]| \\ \vdots & \vdots \\ \nabla y[N] & \nabla y[N]\cdot|\nabla y[N]| \end{pmatrix}$,

$X = \begin{pmatrix}\beta_1\\ \beta_2\end{pmatrix}$, $B_e = \begin{pmatrix} e[1] \\ \vdots \\ e[N]\end{pmatrix}$.

The $N\times 2$ regression matrix $A_{\nabla y}$ and N-vector B_e are composed of simulation data. This system is over determined, and so its pseudo-solution is

$$X = (A_{\nabla y}^T\cdot A_{\nabla y})^{-1}\cdot A_{\nabla y}^T\cdot B_e \quad (6)$$

The approximation error will depend on curvature of plant's phase trajectory and on ill-conditioned matrix $A_{\nabla y}^T\cdot A_{\nabla y}$.

The different modifications of quasi time-optimal control algorithm are possible in dependence of manner of amplitude reduction.

1) The control action can be decreased while phase variables are approaching to the origin

$$U_0[n] = \beta_0\cdot(|e[n]| + |\nabla y[n]| + \cdots + |\nabla^{(m-1)}y[n]|) \quad (7)$$

In particular, the simple variant $U_0[n] = \beta_0\cdot|e[n]|$ can be used.

2) The "soft" control can be obtained in neighborhood of switching surface, that is, when its sign is changing by

$$U_0[n] = \beta_0\cdot|S_a(e[n],\bar{y}[n])| \quad (8)$$

It is equivalent to approximation of relay unit by saturation nonlinear element

$$u[n] = sat(K_0\cdot S_a(e[n],\bar{y}[n])) \quad (9)$$

where $K_0 = \beta_0\cdot U_0$ is a gain factor and U_0 has constant value const which is a level of control limitation.

3) The combination of two variants above is possible too

$$U_0[n] = \begin{cases} \beta_{01}\cdot|S_a(e[n],\bar{y}[n])|, & \text{if } |e[n]| > e_0 \\ \beta_{02}\cdot|e[n]|, & \text{if } |e[n]| \le e_0 \end{cases} \quad (10)$$

The concrete modification of control algorithm may be chosen by comparison of simulation results after optimization of adjustable parameters $\beta_0, \beta_{01}, \beta_{02}$ and e_0.

3.2 Possible Structures of Fuzzy Controllers

In the general structure shown in Fig. 2 the nonlinear parts of quasi time-optimal controller can be implemented as fuzzy converters. Following four kinds of structures for quasi time-optimal fuzzy controller can be applied in practice.

1) The structure with quadratic switching surface and fuzzy approximation of saturation element (or relay unit RU).

2) The structure with fuzzy approximated switching surface (in functional converter FC) and crisp saturation element.

3) The structure with fuzzy approximation of both saturation element and switching surface.

4) The structure with full fuzzy approximation of prototype controller as a combined nonlinear functional converter.

In more detail all these structures will be considered in following simple study examples.

IV. A Design Example of Quasi Time-Optimal Fuzzy Control System

Consider the second order plant described as the following transfer function.

$$W(s)=\frac{y(s)}{u(s)}=\frac{K}{(T\cdot s+1)\cdot s} \tag{11}$$

where $K=1.0$, $T=0.05$ second.

4.1 A Design of Quasi time-optimal Discrete Controller

The optimal switching line for this plant, that is the braking phase trajectory, when $u=-U_0$, can be derived, for instance by the Sylvester formula, as follows

$$\begin{aligned} S(e,\dot{y}) = {} & e - T\cdot\dot{y} \\ & + K\cdot U_0\cdot T\cdot \ln\left(1+\frac{|\dot{y}|}{K\cdot U_0}\right)\cdot sign(\dot{y}) \end{aligned} \tag{12}$$

If we use the following approximation of derivative

$$\dot{y}[n]\approx\nabla y[n]/T_0,$$

$$\nabla y[n]=y[n]-y[n-1].$$

The switching line is described by the following equation (13).

$$S(e[n],\nabla y[n])=e[n]-F[n] \tag{13}$$

where

$$\begin{aligned} F[n] = {} & p_1\cdot\nabla y[n] \\ & + p_2\cdot\ln(1+p_3\cdot|\nabla y[n]|)\cdot sign(\nabla y[n]) \end{aligned} \tag{14}$$

and

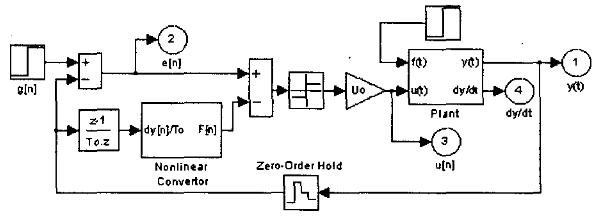
$$p_1=T/T_0,\ p_2=K\cdot U_0\cdot T,\ p_3=1/(K\cdot U_0\cdot T_0).$$

The SIMULINK model of the optimal digital control system is shown in Fig. 4.

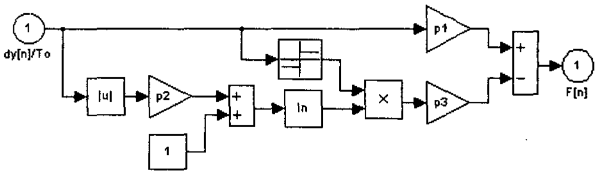
The corresponding simulation results for $U_0=10$ and $T_0=0.002$ second are shown in Fig. 5. In Fig. 5, the unit of horizontal axis is second.

In digital time-optimal control, because of relay the oscillations appear in neighborhood of the origin.

The SIMULINK model of quasi time-optimal (prototype) control system is shown in Fig. 6.



(a) Overall SIMULINK model.



(b) Block diagram of nonlinear converter in figure (a).

Fig. 4. SIMULINK model of the optimal digital control system.

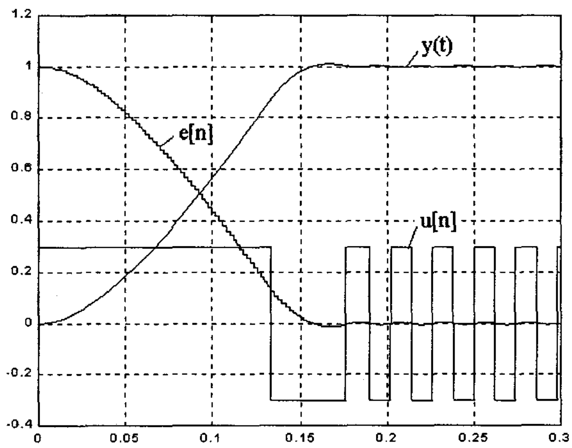
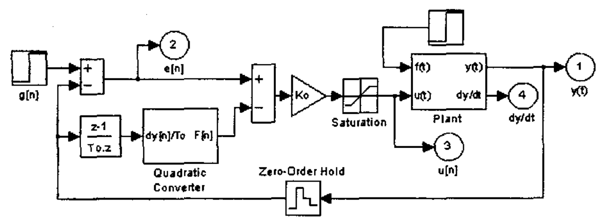
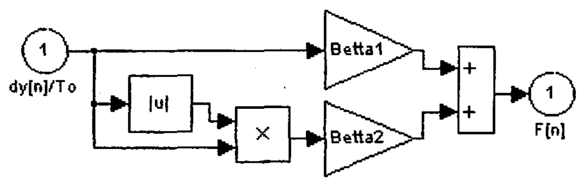


Fig. 5. Simulation results.



(a) Overall SIMULINK model.



(b) Block diagram of quadratic converter in figure (a).

Fig. 6. SIMULINK model of quasi-time-optimal control system.

The output of the quadratic converter in figure (a) of Fig. 6 is written by the following equation (15).

$$F[n]=\beta_1\cdot \dot{y}[n]+\beta_2\cdot \dot{y}[n]\cdot|\dot{y}[n]|\cdot sign(\dot{y}[n]) \quad (15)$$

where $\dot{y}[n]=\nabla y[n]/T_0$. The two parameters $\beta_1=0.01$ and $\beta_2=0.001$ of quadratic converter in the prototype controller were firstly defined by the regression method and then optimized so that to minimize oscillations in the sliding mode.

The parameter $K_0=500.0$ of the saturation element, which approximates the relay unit, was adjusted so that to eliminate the sliding mode in neighborhood of the origin.

The results of simulation of the prototype control system is shown in Fig. 7. In Fig. 7, the unit of horizontal axis is second.

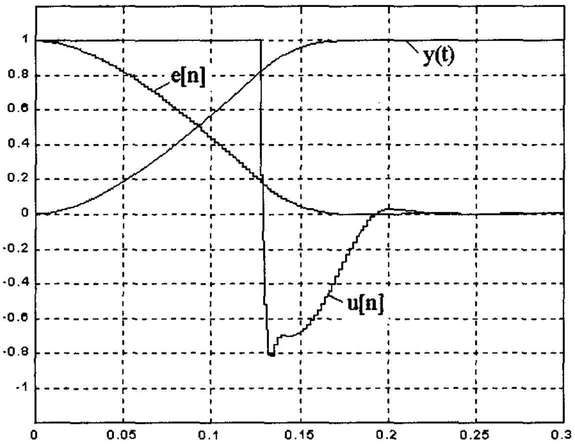


Fig. 7. Simulation results of the prototype control system.

4.2 Fuzzy Approximation of Quasi Time-Optimal Controller

The design of all fuzzy converters was carried out by using the Mamdani and the Sugeno fuzzy inference systems of MATLAB Fuzzy toolbox [3-5, 9].

The necessary input-output data were obtained by simulation with use of the SIMULINK model shown in Fig. 8.

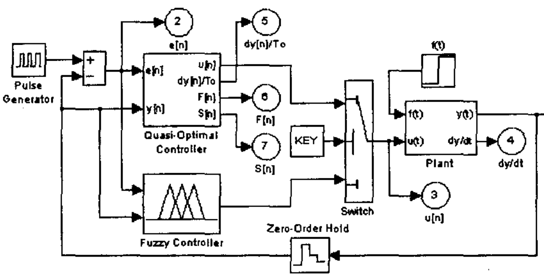


Fig. 8. SIMULINK model.

For learning the Sugeno-type fuzzy converters by ANFIS technology [6-8] the double step response illustrated in Fig. 9 was used. In Fig. 9, the unit of horizontal axis is second.

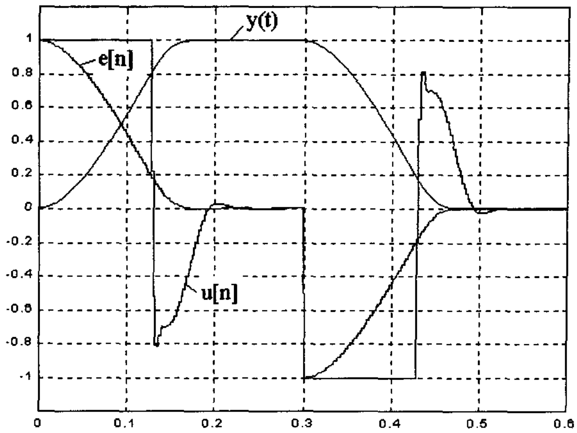


Fig. 9. Control $u(n)$ and step response $y(t)$.

4.2.1 First Variant of Fuzzy Controller

Fig. 10 shows the model of fuzzy controller with the crisp quadratic switching line and the Mamdani-type single-input fuzzy converter approximating the saturation element.

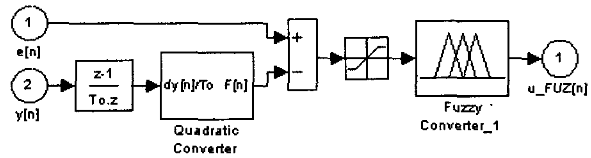


Fig. 10. First variant model of fuzzy controller.

The number of triangular input and output membership functions is 7 and 7, respectively and the number of rules is 7 too. As a defuzzification method the center of gravity method was chosen. The obtained input-output characteristics of the fuzzy converter is shown in Fig. 11.

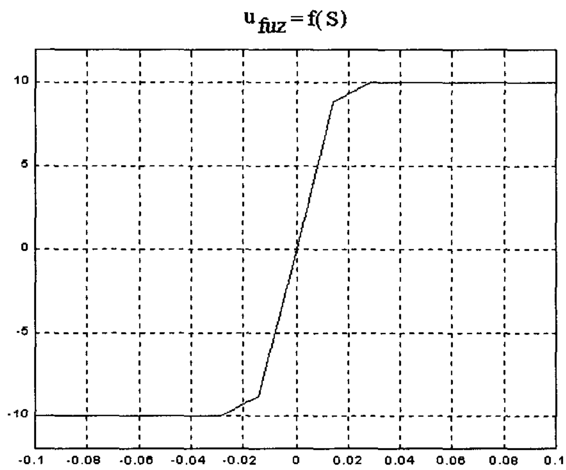


Fig. 11. Input-output characteristics of fuzzy converter.

The step response of the control system with first variant of fuzzy controller is shown in Fig. 12. In Fig. 12, the unit of horizontal axis is second.

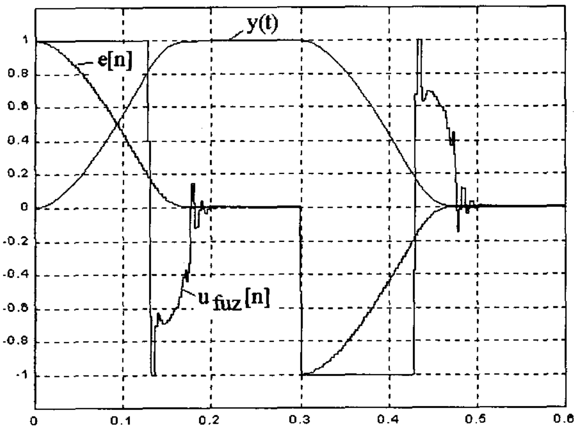


Fig. 12. Step response of control system with first variant of fuzzy controller.

4.2.2 Second Variant of Fuzzy Controller

Fig. 13 illustrates the model of fuzzy controller with the crisp saturation element and the Sugeno-type single-input fuzzy converter approximating the switching line.

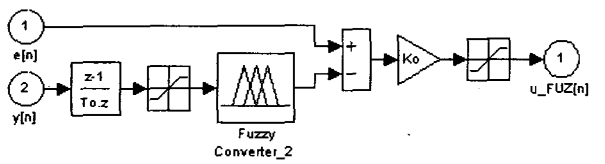


Fig. 13. Second variant model of fuzzy controller.

In the first-order Sugeno fuzzy model the number of the Gaussian-type input membership functions is 4, the number of parameters in consequence is 4, and the number of rules is 4 too. The switching line characteristics of the approximated fuzzy converter is shown in Fig. 14.

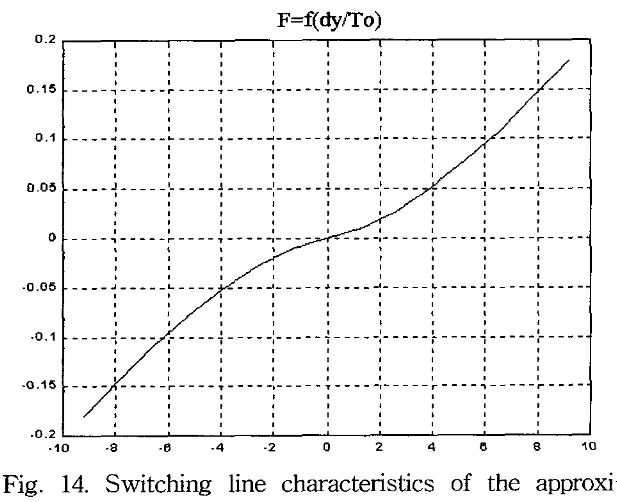


Fig. 14. Switching line characteristics of the approximated fuzzy converter.

The step response of the control system with second variant of fuzzy controller is shown in Fig. 15. In Fig. 15, the unit of horizontal axis is second.

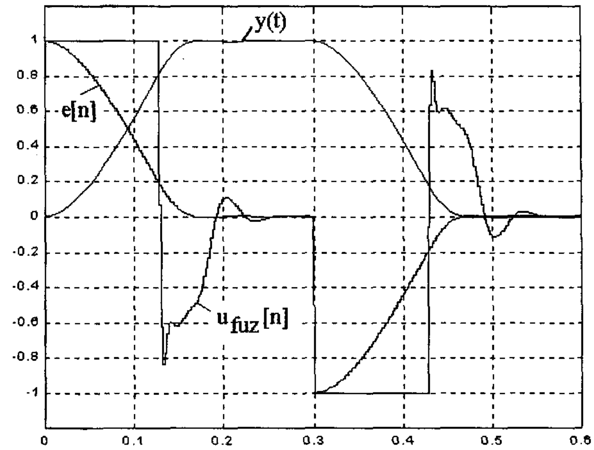


Fig. 15. Step response of control system with second variant of fuzzy controller.

4.2.3 Third Variant of Fuzzy Controller

The third model of fuzzy controller with two fuzzy converters is shown in Fig. 16.

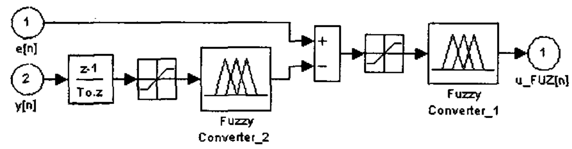


Fig. 16. Third variant model of fuzzy controller.

The first Mamdani-type fuzzy converter approximates the saturation element, and the second Sugeno-type fuzzy converter approximates the switching line.

The step response of the control system with third variant of fuzzy controller is shown in Fig. 17. In Fig. 17, the unit of horizontal axis is second.

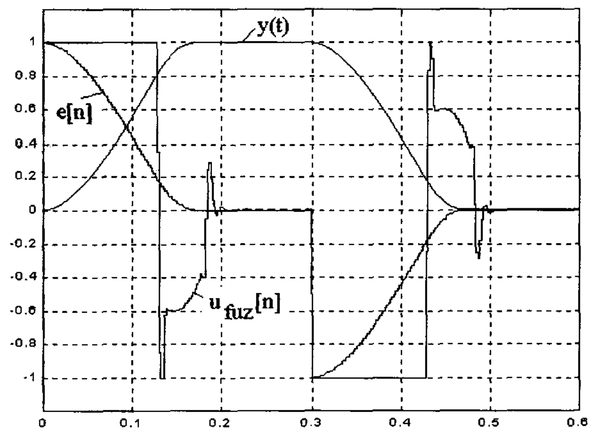


Fig. 17. Step response of control system with third variant of fuzzy controller.

4.2.4 Fourth Variant of Fuzzy Controller

Fig. 18 illustrates the fourth model of fuzzy controller with the Sugeno-type two-input fuzzy converter, which approximates the nonlinear parts of prototype controller on the whole.

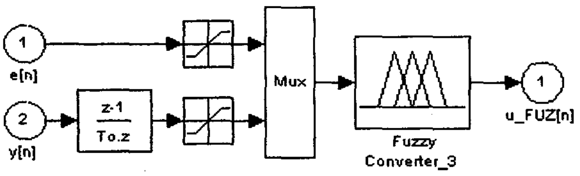


Fig. 18. Fourth variant model of fuzzy controller.

In the zero-order Sugeno fuzzy model the numbers of the Gaussian-type input membership functions are 4 and the number of parameters in consequence is 16, and also the number of rules is 16, too. The corresponding input-output characteristics of fuzzy converter is shown in Fig. 19.

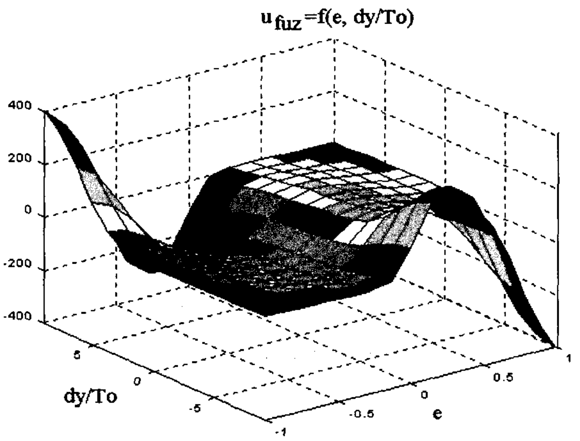


Fig. 19. Input-output characteristics of the fourth variant of fuzzy controller.

The step response of the control system with fourth variant of fuzzy controller is shown in Fig. 20. In Fig. 20, the unit of horizontal axis is second.

4.3 Robustness Analysis of Fuzzy Control Systems

The order of actual plant may be higher order than two and the parametric uncertainties in plant model may take place too. So the robustness analysis is necessary. The simulation results show that the robustness of fuzzy control systems is close to one of prototype control system.

V. Concluding Remarks

The time-optimal digital control synthesized by theoretical methods is a nonlinear time-variant function, which can not be directly implemented as a closed-loop algorithm. In practice, the synthesis of quasi time-optimal discrete control is more preferable.

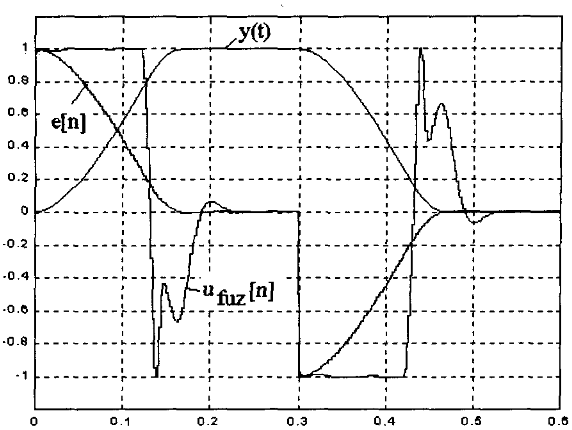


Fig. 20. Step response of control system with fourth variant of fuzzy controller.

The main idea of methodology suggested for quasi time-optimal fuzzy controller design consists in an approximation of some nonlinear parts of prototype discrete controller by fuzzy converters. The proposed approach was illustrated by studying the examples with the linear plant of 2nd order, but this methodology is universal and can be applied for computer-aided design of fuzzy control systems with linear and nonlinear plants of high order. In digital time-optimal control, because of relay the oscillations appear in neighborhood of the origin. But in the proposed quasi time-optimal fuzzy control, since the amplitude of relay output is varied with depending on error and derivatives of output, there are no oscillations in neighborhood of origin. From the simulation results, we know that the step responses of four kinds of fuzzy approximations for quasi time-optimal fuzzy controller are very similar. This means that the proposed fuzzy controller is robust.

In case of complex multi-input fuzzy converters its design has to be carried out by the Sugeno fuzzy inference systems using ANFIS technology.

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