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# 반복 학습 제어기의 properness 제한에 관한 연구

# A Study on the Properness Constraint on Iterative Learning Controllers

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## 요 약

본 논문은 초기 조건 문제의 관점에서 반복 학습 제어기가 proper 해야 할 필요성에 대하여 연구한다. 반복 학습 제어기가 proper하지 않으면, 모든 반복에 있어서 시스템의 초기 상태와 요구되는 시스템의 상태가 완전히 일치하지 않는다면 학습 입력의 크기가 무한대로 증가하는 경우가 생겨 실제 구현이 불가능해진다. 따라서 이론적으로 학습 제어의 수렴이 보장되더라도 proper하지 않은 학습 제어기는 실제 시스템에는 적용할 수 없음을 보인다. 또한 반복 학습 제어 시스템의 초기 조건의 불일치가 시스템의 수렴 특성에 미치는 영향에 대하여 분석한다.

## ABSTRACT

This note investigates the necessity of properness constraint on iterative learning controllers from the viewpoint of the initial condition problem. It is shown that unless the iterative learning controller is proper, the learning control input may grow unboundedly and thus not be feasible in practice, though the convergence of tracking error is theoretically guaranteed. In addition, this note analyzes the effects of initial condition misalignment in the iterative learning control system on the control input and convergence property.

**Key Words** : iterative learning control, convergence, initial condition problem, controller properness.

## 1. Introduction

Iterative learning control is a type of intelligent control scheme for improving tracking accuracy of systems performing repeated tasks. In this control scheme, a single tracking task is repeated over and over, always starting from the same initial condition, and the control action at each trial is created utilizing control results obtained from previous trials. Even with an imperfect plant model, a well-designed iterative learning control system can provide improved tracking performance after fully learning the given task.

Although iterative learning control was originally proposed in the time-domain, it has also been explored in the frequency-domain in parallel [2]-[7]. The frequency-domain design provides more design freedom and physical insights not available in time-domain designs, such as tracking bandwidth, or convergence rate varying with frequency.

Designing an iterative learning controller in the frequency domain can be thought of as choosing appropriate filters operating on previously acquired information so that the iterative process may converge. In some literature, improper filters are employed to assure the convergence of the tracking error. Though any reasonable implementation of an improper filter is noncausal, a noncausal filter can be implemented without difficulty in iterative learning control systems because we have an entire history of errors to filter from the previous trial.

This note shows that the learning control input created by improper filters is not feasible in practice unless a strict condition on the initial state of the plant is perfectly satisfied. Unless the condition is met, the learning control input or tracking error may not converge in practice as opposed to the theory. As a result, it is shown that properness constraint on the iterative learning controller is inevitable for practical purposes. In addition, this note analyzes the effects of initial condition misalignment on the convergence property.

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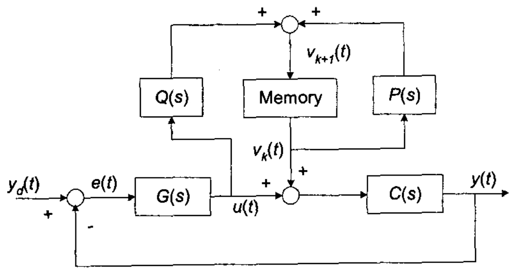


그림 1. 반복 학습 제어 시스템
Fig. 1. An iterative learning control system

## 2. System Statement

Consider the iterative learning control system shown in Fig.1. In this figure, $y_d(t)$ is the desired output trajectory, $y(t)$ is the system output, $e(t)$ is the tracking error, $u(t)$ is the feedback control input, and $v_k(t)$ is the iterative learning control input. $C(s)$ and $G(s)$ are proper transfer functions of the feedback controller and the controlled plant, respectively. The feedback controller $C(s)$ is designed independently of the iterative learning controller. The learning filters $P(s)$ and $Q(s)$ are added to the existing feedback control system with a view to improving the tracking performance.

The input update law of the iterative learning controller is given in the frequency-domain as

$$V_{k+1}(s)=P(s)V_k(s)+Q(s)U_k(s) \tag{1}$$

where $V_k(s)$ and $U_k(s)$ denote the Laplace transforms of the learning control input and feedback control input at the $k$th iteration. $P(s)$ and $Q(s)$ are proper and stable transfer functions, which characterize the iterative learning process. The input update law (1) can be modified to time-domain learning algorithms such as the so-called $P$-type learning law

$$v_{k+1}(t)=v_k(t)+Ke_k(t)$$

depending upon the choice of $P(s)$ and $Q(s)$. $P(s)$ and $Q(s)$ are designed such that the learning control input $v_k(t)$ converges as $k\to\infty$ with the input update law (1).

Let us denote the zero-input response of the plant as $y^0(t)=L^{-1}[Y^0(s)]$ and assume that the initial state of the plant is identical for every iteration, so $Y_k^0(s)=Y^0(s), \forall k$. In other words, $y^0(t)$ is invariant with respect to the iteration. In [6], it is proved that if

$$|P(j\omega)-Q(j\omega)T(j\omega)|<1, \quad \forall\omega, \tag{2}$$

then as $k\to\infty$, the learning control input $v_k(t)$ converges in the $L_2$-norm sense to $v_*(t)$ defined by

$$\begin{aligned} v_*(t) &= L^{-1}[V_*(s)] \\ &= L^{-1}\left[\frac{Q(s)C(s)}{1-P(s)+Q(s)T(s)}\,\frac{Y_d(s)-Y^0(s)}{1+G(s)C(s)}\right] \end{aligned} \tag{3}$$

where

$$T(s)=\frac{G(s)C(s)}{1+G(s)C(s)}.$$

Obviously, if we set $P(s)=1$ and find a proper $Q(s)$ so that the convergence condition (2) is satisfied, then perfect tracking, i.e., $y(t)\equiv y_d(t)$, is achieved. However, this choice is feasible only in the case where $T(s)$ is a stable and minimum-phase system of relative degree 0 [4]. Putting it differently, if $T(s)$ is strictly proper or of nonminimum phase, it is impossible to find a proper filter $Q(s)$ guaranteeing the convergence of the iterative process. This is too restrictive because most plants in real situations are strictly proper; hence $T(s)$ is strictly proper as well.

The introduction of the filter $P(s)$ gives extra design freedom and makes the convergence condition less restrictive at the cost of tracking accuracy in the high-frequency band. Now that any reference trajectory to be tracked virtually remains in the low frequency region, accurate tracking is normally needed up to the bandwidth of importance. Therefore, despite the theoretically inevitable error, accurate tracking of the desired trajectory can be preserved within the bandwidth if $P(s)$ is chosen in such a way that it is close to 1 in low frequency range where tracking is important and that it rolls off in high frequency range to guarantee system convergence [2], [3], [6], [7].

## 3. Main Results

The elimination of the properness constraint on $Q(s)$ relaxes the convergence condition and enables us to obtain a perfect tracking property even for nonminimum-phase systems or strictly proper systems. The adoption of an improper filter $Q(s)$, however, causes a critical problem when the condition that the initial plant state should remain invariant for every trial is violated. Suppose that $Q(s)$ is not proper. Defining

$$H(s)=\frac{Q(s)C(s)}{1-P(s)+Q(s)T(s)}\,\frac{1}{1+G(s)C(s)}, \tag{4}$$

we see that $H(s)$ should be improper. Let us define

$$y^{(j)}(t)=\frac{d^j}{dt^j}y(t).$$

Then, from the initial-value theorem, we obtain

$$\begin{aligned} v_*(0) &= \lim_{s\to\infty} sV_*(s) \\ &= \lim_{s\to\infty} sH(s)\,(Y_d(s)-Y^0(s)) \\ &= \lim_{s\to\infty} sH(s)\,(s^{-1}(y_d(0)-y^0(0)) \\ &\quad + s^{-2}(\dot{y}_d(0)-\dot{y}^0(0)) + \cdots \\ &\quad + s^{-l}(y_d^{(l-1)}(0)-y^{0^{(l-1)}}(0)) \\ &\quad + s^{-(l+1)}L[\,y_d^{(l)}(t)-y^{0^{(l)}}(t)]) \end{aligned}$$

where $l$ denotes the relative degree of $H^{-1}(s)$ and $L$ denotes the Laplace operator. Thus, if $y_d^{(j)}(0) \neq y^{0^{(j)}}(0)$ for some $0 \leq j \leq l-1$, then $v_*(0)$ grows unboundedly, which means $v_*(t)$ has an impulse at $t=0$. Consequently, if the initial values of zero-input response resulting from the initial plant state and its derivatives up to $l-1$th order do not completely equal the desired ones, an impulse should appear at $t=0$ in the converged learning control input.

It is virtually impossible in practice to make the desired and actual initial values identical because no physical system is free of noise or disturbance that perturbs the plant state. Moreover, the quantization that always exists in digital computations may cause the problem, though desired and actual initial values are identical. If the initial state is misaligned, without regard to the amount of the misalignment, the overall control system does not work as the theory predicts, and in some cases it may be even destabilized. In a similar context, The importance of initial state matching in a D-type learning law without a feedback controller was discussed in I1].

The reason an impulse appears in the converged control input can be easily understood by intuition. If the initial plant state deviates from the desired one, there will be a nonzero initial tracking error, which cannot be suppressed by a bounded control input. Therefore, for achieving perfect tracking despite an initial state mismatch, a control with infinite magnitude should be applied. Apparently, the above problem can be simply solved by restricting $Q(s)$ to be proper. When $Q(s)$ is proper, $H(s)$ is also proper and $v_*(t)$ always has a finite initial value

$$v_*(0) = \lim_{s\to\infty} H(s)(y_d(0) - y^0(0)).$$

Hence, in a learning control scheme that adopts proper filters, $v_*(0)=0$ if either $T(s)$ or $Q(s)$ is strictly proper or $y_d(0) = y^0(0)$.

In consequence, the above discussion shows that filters in practical iterative learning controllers need to be proper to generate feasible learning control input. Despite the theory allowing the use of improper learning controllers, real iterative learning control systems adopting improper filters may not yield tracking accuracy as predicted by the theory because the control input created by the theory includes an impulse.

Another subject to consider is the assumption of invariant initial state of the plant. In derivation of (2) and (3), it was assumed that the initial state of the plant is invariant with respect to iteration. Actually, every iterative learning control scheme needs some kind of initial condition requirement, depending upon the type of input update law. In practice, however, it is not always possible to make the initial state of the plant completely identical at each trial, and the measurement of plant output utilized by the learning controller can be contaminated by sensor noise. Hence, it is necessary to analyze how misalignment in the initial state affects the convergence property and system performance.

Suppose $\delta Y_k^0(s) = Y_k^0(s) - Y^0(s)$ and $\|\delta Y_k^0\|_2 < \delta_{\max}$ for $k=1,2,\cdots$. After some manipulations, we get

$$\begin{aligned} E_k - E_* &= (P-QT)(E_{k-1}-E_*) \\ &\quad + \frac{P}{1+GC}\delta Y_{k-1}^0 - \frac{1}{1+GC}\delta Y_k^0 \\ &= (P-QT)^k(E_0 - E_*) \\ &\quad + \sum_{i=1}^{k-1}(P-QT)^i\left(\frac{P}{1+GC}\delta Y_{k-i}^0 - \frac{1}{1+GC}\delta Y_{k-i+1}^0\right) \end{aligned}$$

And

$$\begin{aligned} \|E_k - E_*\|_2 &\leq \|P-QT\|_\infty^k \|E_0 - E_*\|_2 \\ &\quad + \sum_{i=1}^{k-1}\|P-QT\|_\infty^i \left(\|\frac{P}{1+GC}\delta Y_{k-i}^0\|_2 \right. \\ &\qquad \left. + \|\frac{1}{1+GC}\delta Y_{k-i+1}^0\|_2\right). \end{aligned}$$

It can be assumed that $\|P(s)\|_\infty \leq 1$ because $P(s)$ is supposed to be a low-pass filter whose pass-band gain is 1. Thus we get

$$\begin{aligned} \|E_k - E_*\|_2 &\leq \|P-QT\|_\infty^k \|E_0 - E_*\|_2 \\ &\quad + 2\sum_{i=1}^{k-1}\|P-QT\|_\infty^i \|\frac{1}{1+GC}\|_\infty \delta_{\max}. \end{aligned}$$

Now it follows that

$$\lim_{k\to\infty}\|e_k(t) - e_*(t)\|_2 \leq \frac{2\sigma\delta_{\max}}{1-\gamma}$$

where

$$\gamma = \|P-QT\|_\infty \text{ and } \sigma = \|\frac{1}{1+GC}\|_\infty.$$

As seen above, the converged error remains within a ball of radius $2\sigma\delta_{\max}/(1-\gamma)$ centered at $e_*(t)$ in the $L_2$-norm sense. The bound of the ball can be made arbitrarily small by reducing the variation of the initial state. Furthermore, it also depends on the sensitivity function of the feedback control system and $\gamma$. The smaller the $H_\infty$-norm of the sensitivity function and $\gamma$ are, the smaller the bound is. It is interesting that the decrease of sensitivity leads to more robust learning control system against the misalignment of the initial state. From the standpoint of performance and disturbance rejection in the feedback control system, it is also desirable to reduce sensitivity.

## 4. Conclusions

Even though theory allows the use of an improper learning controller, it may cause a critical problem when applied to plants in real environments. Thus, it is imperative to impose the properness constraint on the iterative learning controller, not only for system robustness to high-frequency noise but for feasible control input as well. Additionally, we analyzed the effect of the variable plant initial state on convergence property. Though we only focused on SISO systems, it is straightforward to extend the obtained results to

MIMO systems.

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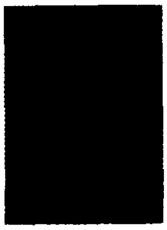
## 저 자 소 개

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