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A Coupled Moisture and Heat Flow Analysis Model in Unsaturated Soil

불포화토에서의 복합적 습기와 열흐름의 분석모델

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요 지

포장내의 흙들의 함수비는 강우, 온도 등의 기후적 요인에 따라 계절적으로 변화한다. 그것은 강우로인한 동수경사가 습기의 흐름을 유발하고, 온도변화로인한 온도경사가 직 · 간접적으로 열흐름뿐만아니라 습기의 흐름을 유발하기 때문이다. 포장내의 흙들은 보통 불포화상태로 존재한다. 그리고 열의 흐름과 습기의 흐름은 상호간에 복잡한 작용을 하는 복합적인 과정들로 인식되어 왔다. 이 논문은 불포화토내의 복합적인 열의 흐름과 습기의 흐름에 대해 유한요소법을 이용한 하나의 일차원적 분석모델을 제시한다. 이 모델은 온도와 함수비 변화뿐만아니라 시간에 따른 동상을 예측하기 위하여 사용될 수 있다. 온도 및 함수비의 변화 그리고 동상의 예측은 포장의 설계 및 유지관리를 위해 의미 있는 일이 될 것이다. 이 모델은 다른 모델의 결과들과 비교를 통해서 검증된다.

Abstract

Water content of soils within pavement varies seasonally depending on climatic factors such as rainfall, temperature and so on, since a hydraulic gradient due to rainfall causes moisture flow, and a thermal gradient due to temperature change induces not only heat flow but also moisture flow directly and indirectly. Soils within pavement are usually in an unsaturated state, and heat flow and moisture flow have been recognized as coupled processes with complex interactions between them. This paper presents a one-dimensional analysis model by the finite element method for the coupled heat flow and moisture flow in unsaturated soils. The model can be used to predict not only the change of temperature and water content, but also frost heave with time. It will be a meaningful work for the design and maintenance of pavement to predict the change of the temperature and water content and frost heave. The model is tested through comparisons with the results by other models.

Keywords : Finite element method, Heat flow, Moisture flow, Soil-water characteristic curve, Unsaturated soil

1. Introduction

There have been many attempts to model the moisture and temperature change within unsaturated soils. Some models were developed to predict the change of moisture or the change of temperature independently. Some models were developed to predict the change of moisture and temperature by coupled moisture and heat flow analysis. But though the models have the same purpose, they show some differences in the applications of numerical equations and soil properties. This may cause different results for the prediction of moisture and temperature within pavements.

To solve the governing equations of the coupled

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moisture and heat flow, two equations which give the relationships between soil parameters and soil suction are required. One is the relationship between water content and suction for the soil. The other is the relationship between permeability and suction. Many equations for these relationships have been presented by many researchers. In the model presented here two groups of equations are used for the equations to show the relationship between water content and suction and the relationship between permeability and suction. One group consists of the equations by Gardner (1958) and Guymon et al. (1993). The other group consists of the equations by Fredlund and Xing (1994) and Fredlund et al. (1994).

2. Literature Review

The attempts to analyze the coupled heat and moisture flow using mathematical methods have been studied by many researchers such as Harlan (1973), Guymon and Luthin (1974), Jame and Norum (1980), Guymon et al. (1981, 1993), and Newman and Wilson (1997). These theories are basically similar to one another. They are based on some forms of the equations given by Harlan (1973).

There are differences in the applications of the numerical equations and soil properties. For example, Harlan (1973), Jame and Norum (1980), and Newman and Wilson (1997) neglected the convective (or advective) term of the heat equation but Guymon and Luthin (1974), and Guymon et al. (1981, 1993) considered the term. Harlan (1973) used the finite difference method to model the one-dimensional coupled heat and moisture flow. He assumed that the soil-water characteristic curve and the suction versus hydraulic conductivity relationship would be same in the unfrozen and the frozen soils. Jame and Norum (1980) used the finite difference methods like Harlan (1973) but they noted that there were some differences between the numerical results based on Harlan (1973) and the results obtained from their measurements. They thought the difference was caused by the ice accumulation which disrupted the flow paths and hence reduced the flow rate. To account for the reduced flow in the frozen soil, Jame and Norum (1980) introduced an impedance factor.

The one-dimensional coupled heat and moisture flow modeling using the finite element method was developed by Guymon and Luthin (1974), Guymon et al. (1981, 1993) and Newman and Wilson (1997). Guymon et al. (1993) used Gardner's (1958) equation to establish the soil-water characteristic curve and the suction versus hydraulic conductivity relationship. They also thought ice might be partly blocking soil pores and reduce hydraulic conductivity, and thus used an E parameter which is similar to the impedance factor proposed by Jame and Norum (1980). Newman and Wilson (1997) used the soil-water characteristic curve equation presented by Fredlund and Xing (1994) and the suction versus hydraulic conductivity relationship presented by Fredlund et al. (1994). But they did not use an impedance factor to consider the decrease of hydraulic conductivity due to ice. Newman and Wilson (1997) thought that the decrease of hydraulic conductivity is caused by using an inaccurate suction versus hydraulic conductivity relationship. They showed that there was good agreement between the results by modeling and the experimental results, though they did not consider an impedance factor.

3. Moisture and Heat Flow Model

Two systems of equations are required to describe mathematically the coupled heat and moisture flow. One is the moisture flow equation which is an application of the principle of mass conservation and the other is the heat flow equation which is an application of the principle of energy conservation.

The moisture flow equation in an unsaturated soil is

$$\frac{\partial}{\partial y}\left(k(\psi)\left(\frac{\partial \psi}{\partial y}+1\right)\right)=c(\psi)\frac{\partial \psi}{\partial t}+\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} \tag{1}$$

where y is the depth, $k(\Psi)$ is the hydraulic conductivity, Ψ is the suction, $c(\Psi)$ is specific moisture capacity, ρ_i is the density of ice, ρ_w is the density of liquid water, and θ_i is the volumetric ice content. The specific moisture capacity is defined as

$$c(\psi) = \frac{d\theta}{d\psi} \tag{2}$$

where θ is the volumetric water content. To solve equation (1) the specific moisture capacity and the hydraulic conductivity must be known. The specific moisture capacity can be determined from the soil-water characteristic curve and the hydraulic conductivity can be determined from the suction versus hydraulic conductivity relationship. In the model, two groups of equations were used. One is soil-water characteristic curve by Gardner (1958) and suction versus hydraulic conductivity relationship by Guymon et al. (1993) which are relatively simple and thus have been used by many researchers. The other is soil-water characteristic curve by Fredlund and Xing (1994) and suction versus hydraulic conductivity relationship by Fredlund et al. (1994). The equations by Fredlund and Xing (1994) and Fredlund et al. (1994) are more complicated than the equations by Gardner (1958) and Guymon et al. (1993) but show relatively good fitting results with experimental data. These equations are shown in appendix.

The heat flow equation is

$$\frac{\partial}{\partial y}\left(K_T \frac{\partial T}{\partial y}\right) - vC_w \frac{\partial T}{\partial y} = C_m \frac{\partial T}{\partial t} - L\frac{\rho_i}{\rho_w}\frac{\partial \theta_i}{\partial t} \tag{3}$$

where K_T is thermal conductivity, T is temperature, v is the velocity of the water flow, C_w is the heat capacity of water, C_m is the volumetric heat capacity, L is the latent heat of fusion of water which is the thermal energy required for a change of phase, ρ_i and ρ_w are the densities of the ice and water, and θ_i is the volumetric ice content. The thermal parameters in equation (3) can be computed from DeVries (1966) relationship:

$$C_m = C_w\theta_u + C_i\theta_i + C_s(1-\theta_0) \tag{4}$$

$$K_T = K_w\theta_u + K_i\theta_i + K_s(1-\theta_0) \tag{5}$$

where θ_u is unsaturated volumetric water content, θ_0 is porosity of soil, C_w, C_i and C_s are volumetric heat capacities of water, ice and soil, and K_w, K_i and K_s are thermal conductivities of water, ice and soil. The velocity of the water flow is calculated from Darcy's law:

$$v = -k\frac{\partial h}{\partial y} \tag{6}$$

In equations (1) and (3), the last terms are a sink or source due to the change of ice phase. These terms are approximated by an isothermal phase change process presented by Hromadka et al. (1981). Then, dropping off the last terms of equations (1) and (3) become

$$\frac{\partial}{\partial y}\left(k(\psi)\left(\frac{\partial \psi}{\partial y}+1\right)\right) = c(\psi)\frac{\partial \psi}{\partial t} \tag{7}$$

$$\frac{\partial}{\partial y}\left(K_T \frac{\partial T}{\partial y}\right) - vC_w \frac{\partial T}{\partial y} = C_m \frac{\partial T}{\partial t} \tag{8}$$

The following equations are the numerical equations for equations (7) and (8) which were derived by the finite element method:

$$\begin{aligned}([C(\psi)] + \omega\Delta t[K(\psi)])\{\psi\}_{t+\Delta t} &= ([C(\psi)] - (1-\omega)\Delta t[K(\psi)])\{\psi\}_t \\ &\quad + \Delta t((1-\omega)\{F\}_t + \omega\{F\}_{t+\Delta t})\end{aligned} \tag{9}$$

$$\begin{aligned}([C] + \omega\Delta t[D])\{T\}_{t+\Delta t} &= ([C] - (1-\omega)\Delta t[D])\{T\}_t \\ &\quad + \Delta t((1-\omega)\{F\} + \omega\{F\}_{t+\Delta t})\end{aligned} \tag{10}$$

In equation (9), $[C(\Psi)]$ is called the capacitance matrix and $[K(\Psi)]$ is called the conductance matrix. ω is the value depending on the numerical method and $\{F\}$ is the specified rate of the flow. In equation (10), $[C]$ is the capacitance matrix and $[D]$ is the conduction-advection matrix.

In the model, first, water content change at each node is calculated using the equation (9) and temperature change at each node is calculated using the equation (10). And then water content calculated by moisture flow equation and temperature calculated by heat flow equation are corrected by the isothermal phase change process.

4. Model Verification

Two examples are presented to verify the model. The analysis results compared for the model verification show a particular part of the model such as water content change or/and temperature change. If the validation of the model is established through the comparison with other analysis results individually, the model may be used for more complex problems with confidence.

Example 1)

Philip (1957) solved infiltration problems in Yolo light clay using a quasi-analytical method. The soil-water characteristic curve and permeability function used in Philips solution were expressed by Haverkamp et al. (1977) as the following equations:

$$\theta = \frac{739(0.495-0.124)}{739+(\ln|h|)^4} + 0.124 \qquad for \quad h < -1\ cm$$

$$\theta = 0.495 \qquad for \quad h \geq -1\ cm \tag{11}$$

$$K = K_s \frac{124.6}{124.6+|h|^{1.77}} \tag{12}$$

where $K_s = 0.04428 \quad cm/hr$

A problem was solved with the following boundary and initial conditions:

t = 0	$0 \geq z \geq -50$cm	$\theta = 0.2376$(or $\Psi = -600$cm)
t>0	$z = 0$	$\theta = 0.495$(or $\Psi = 0$)
t>0	$z = -50$cm	$\theta = 0.2376$(or $\Psi = -600$cm)

This problem is a one-dimensional problem. The problem domain was divided into 40 elements which are the top 30 equal sized elements with 1 cm length and the bottom 10 equal sized elements with 2 cm length. The time step size is 0.2 hour. Figure 1 shows the results by the model and Philips solution.

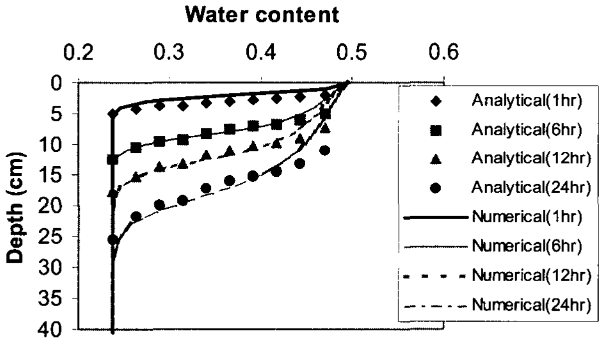


Fig.1. Comparison between the results by the model (numerical) and Philips solution (analytical)

Example 2)

Seasonal variations of moisture and temperature within pavements were monitored by the Seasonal Monitoring Program (SMP) which is included as a part of the Strategic Highway Research Program (SHRP). Figure 2 shows a pavement section of U.S. Route 23 in Delaware County, Ohio where instrumentation sensors were included. The pavement section consists of 28 cm portland cement concrete (PCC), 15.2 cm dense graded aggregate base

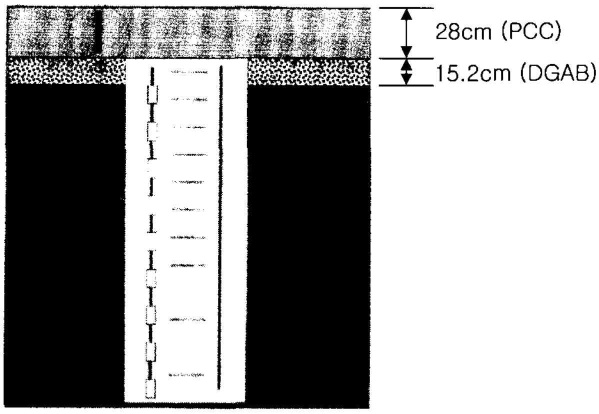
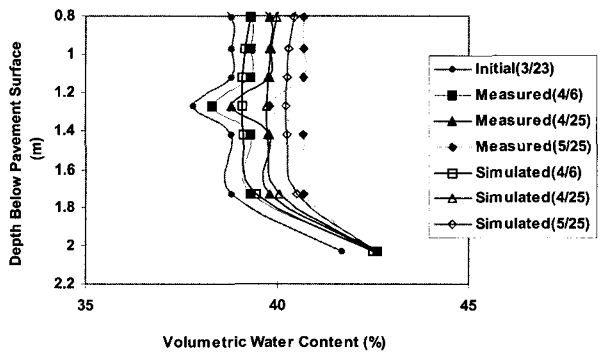
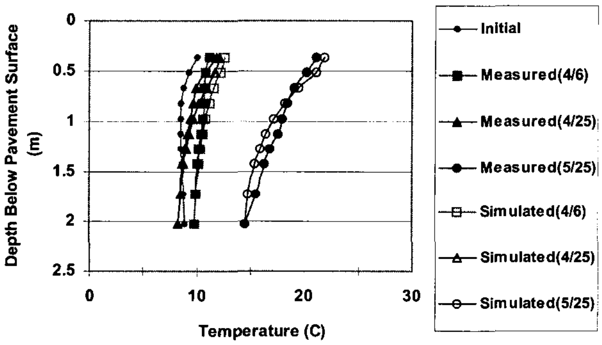


Fig. 2. A pavement section of U.S. Route 23 in Delaware County, Ohio



(a) Water content change analysis in the subgrade soil



(b) Temperature change analysis

Fig. 3. Comparison between the results by measurement and new model

(DGAB) and subgrade soil.

The results by monitoring and by the model presented here for the change of volumetric water content and temperature are shown in Figure 3. The results show relatively good agreement as shown in the figures. In the middle part of Figure 3 (a) the measured water contents are less than the simulated water contents. It is considered that there is some inhomogeneous subgrade soil, though it was assumed that the subgrade soil is homogeneous for the numerical analysis.

5. Summary and Conclusions

An analysis model for the coupled heat and moisture flow was presented. In moisture flow, two equations for the relationships between soil parameters are used. They are soil-water characteristic curve and suction versus permeability relationship. In the model, two groups of equations were used. One is soil-water characteristic curve by Gardner (1958) and suction versus permeability relationship by Guymon et al. (1993). The other is soil-water characteristic curve by Fredlund and Xing (1994) and suction versus permeability relationship by Fredlund et al. (1994). In the model one group of them can be selected.

The model presented here was developed to analyze frost heave as well as water content and temperature change. In the present work, the verification for the analysis of water content and temperature variation within pavement has been tested. The results by the model show good agreement with the results by others.

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Appendix

Soil-water characteristic curve by Gardner (1958):

$$\theta_u = \frac{\theta_s}{A_w|\psi|^a + 1}$$

where θ_u is the unsaturated water content, θ_s is the volumetric water content at saturation, A_w and a are best fit parameters, and Ψ is the suction.

Soil-water characteristic curve by Fredlund and Xing (1994):

$$\theta_u = C(\psi)\frac{\theta_s}{\left\{\ln\left[e + \left(\frac{\psi}{a}\right)^n\right]\right\}^m}$$

where $C(\Psi)$ is a correcting factor, e is exponent value, and a, n and m are best fit parameters.

Suction versus permeability relationship by Guymon et al. (1993):

$$k_h(\psi) = \frac{k_s}{A_k|\psi|^b + 1}$$

where k_h is the unsaturated hydraulic conductivity, k_s is the saturated hydraulic conductivity, A_k and b are best fit parameters.

Suction versus permeability relationship by Fredlund et al. (1994): Fredlund et al. presented an equation for the relative coefficient of permeability to predict the permeability of unsaturated soil using the soil-water characteristic curve equation by Fredlund and Xing (1994).

$$k_r = \frac{\int_{\ln(\psi)}^{b} \frac{\theta(e^y) - \theta(\psi)}{e^y} \theta'(e^y) dy}{\int_{\ln(\psi_{aev})}^{b} \frac{\theta(e^y) - \theta_s}{e^y} \theta'(e^y) dy}$$

where k_r is the relative coefficient of permeability, b = ln(1000000), θ' is the derivative of equation by Fredlund and Xing (1994) with respect to Ψ and Ψ_{aev} is the air entry value. k_r is the ratio of the unsaturated permeability to the saturated permeability in a soil.

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