Dynamic Analysis and Design of Uncertain Systems Against Random Excitation Using Probabilistic Method

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In this paper, a method to obtain the sensitivity of eigenvalues and the random responses of the structure with uncertain parameters is proposed. The concept of the proposed method is that the perturbed equation of each uncertain substructure is obtained using the finite element method, and the perturbed equation of the overall structure is obtained using the mode synthesis method. By this way, the reduced order perturbed equation of the uncertain system can be obtained. And the response of the uncertain system is obtained using probability method. As a numerical example, a simple piping system is considered as an example structure. The damping and spring constants of the support are considered as the uncertain parameters. Then the variations of the eigenvalues, the correlation function and the power spectral density function of the responses are calculated. As a result, the proposed method is considered to be useful technique to analyze the sensitivities of eigenvalues and random response against random excitation in terms of the accuracy and the calculation time.

Key Words: Random Excitation, Uncertain System, Modal Analysis, Random Vibration Analysis, Design of Structure System, Sensitivity Analysis, Probabilistic Method

1. Introduction

Studies of random vibration of mechanical structures, which consist of industrial occupancies, such as the power plant, chemical plant and the bridge, etc. are very important from the view point of disaster protection against random excitation. Also, recent developments in jet and rocket propulsion have give rise to new problems in mechanical and structural vibrations. The pressure fields generated by these devices fluctuate in a random manner and contain a wide spectrum of frequencies that may result in a severe vibration in the aircraft or missile structure. In that mechanical component of the structural system, the physical parameters and characteristics of structures (mass, stiffness and damping coefficients) have deviations in the design process, manufacturing process or other reasons. Accordingly, an accurate approach is needed to analyze the vibration response and the eigenvalue of the structure system against random excitation by considering their uncertainty (Bellman, R., 1964). From the viewpoint of the dynamic response of structures against random excitation, there are many investigations (Caravanu, et al., 1973; Collins, et al., 1969; Yang, et al., 1972; Yoshino, et al., 1984). They observed the vibration of deterministic systems under a random excitation with structural systems to obtain the system responses. On the other hand, from the viewpoint

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of the dynamic response of structures with uncertain parameters, there are many investigations (Shigeru, et al., 1985 ; Bharucha, et al., 1968 ; Lin, et al., 1980).

However, there are few studies that consider both structural uncertainty and random excitation. In the analysis of complex large DOF (degree of freedom) structure system, dynamic computation of the system requires large order sets of equations of motion and it takes much calculating time. Therefore, SSM (substructure synthesis method) has been studied in the vibration analysis (Moon, et al., 1999 ; Moon, et al., 2001). They proposed an efficient analytical method of the vibration against excitation by applying the SSM and PM (perturbation method). To this end, the system is divided into some components and those are formulated according to FEM (finite element method). The perturbed equations are synthesized to the overall system and the sensitivity of eigenvalue and the random response for the overall system are analyzed.

In order to illustrate the accuracy and computation efficiency of the proposed method, a structural system with uncertainty under random excitation as a calculation example is analyzed and evaluated in accordance with the economical computation.

2. Method of Analysis

In this chapter, an analytical method to obtain the sensitivity of eigenvalues and the random responses of the structure with uncertain parameters is introduced by applying probabilistic FEM.

2.1 Modeling of complex uncertain system

In this paper, a structural system with large DOF is considered, as shown in Fig. 1. For the dynamic analysis of complex systems, the SSM can be applied. The overall system can be divided into some components. Then, the equation of motion of each components with uncertain parameter can be obtained using FEM (Nakagiri, et al., 1985).

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{f\} \qquad (1)$$

where $\{q\}$ is a relative displacement vector from an absolute coordinate, which is fixed in basement. $\{f\}$ is an random external force vector. $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrixes, which include the uncertain parameters $\alpha_i (i=1,\cdots, N_\alpha)$. They can be expanded about a design value (expected value) in terms of a series of the small parameter ε_i where the uncertain parameter is estimated as a small value. Accordingly, the dynamic characteristics of the system can be expressed as

$$\begin{aligned}
[M]&=[M^{(0)}]+\sum_{j=1}^{N_\alpha}\varepsilon_j[M^{(j)}]+\frac{1}{2}\sum_{j1=1}^{N_\alpha}\sum_{j2=1}^{N_\alpha}\varepsilon_{j1}\varepsilon_{j2}[M^{(j1j2)}]+\cdots \\
[C]&=[C^{(0)}]+\sum_{j=1}^{N_\alpha}\varepsilon_j[C^{(j)}]+\frac{1}{2}\sum_{j1=1}^{N_\alpha}\sum_{j2=1}^{N_\alpha}\varepsilon_{j1}\varepsilon_{j2}[C^{(j1j2)}]+\cdots \qquad (2)\\
[K]&=[K^{(0)}]+\sum_{j=1}^{N_\alpha}\varepsilon_j[K^{(j)}]+\frac{1}{2}\sum_{j1=1}^{N_\alpha}\sum_{j2=1}^{N_\alpha}\varepsilon_{j1}\varepsilon_{j2}[K^{(j1j2)}]+\cdots
\end{aligned}$$

where ε_i is a small variant of the uncertain parameter α_1. In this study, the expected value $\bar{\alpha}_i$ and standard deviation σ_{α_i} of the uncertain parameter α_i are regarded to be known. And it is regarded that there is no correlation among these uncertain parameters. When the system is excited. such as seismic force in the vertical direction at the supported point, the force term can be expressed as

$$\{f\}=-[M^{(0)}]\{I\}g(t)\equiv\{m\}g(t) \qquad (3)$$

where $g(t)$ is the acceleration of the seismic wave. $\{I\}$ is a unit vector which shows the direction of excitation. $\{I\}$ is used to derive the participation vector of excitation.

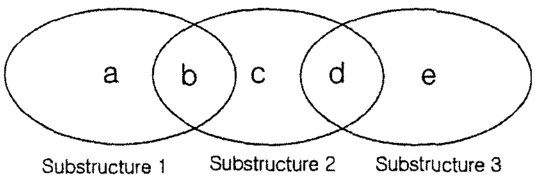
2.2 Equations of motion for uncertain substructure

Each substructure's equation is formulated sequentially by using the PM to the first order according to the SSM procedures, which is divided into three substructures, as shown in Fig. 1.

Here, a, c and e are the internal region. b and d are the assembling region. The equation of motion for substructure 1 can be expressed as

$$[M_1]\{\ddot{q}_1\}+[C_1]\{\dot{q}_1\}+[K_1]\{q_1\}=\{f_1\} \qquad (4)$$

where $[M_1]$, $[C_1]$ and $[K_1]$ are the mass, damping and stiffness matrix of substructure 1,



a : Internal region of substructure 1
c : Internal region of substructure 2
e : Internal region of substructure 3
b, d : Assembling region

Fig. 1 Substructuring of the system

respectively. They can be perturbed in accordance with uncertain parameter $\alpha_i (i=1, \cdots, N_a)$ as a small value

$$[M_1]=\begin{bmatrix} M^1_{aa} & M^1_{ab} \\ M^1_{ba} & M^1_{bb} \end{bmatrix}=[M_1^{(0)}]+\sum_{j=1}^{N_a}\varepsilon_j[M_1^{(j)}]$$

$$[C_1]=\alpha\begin{bmatrix} K^1_{aa} & K^1_{ab} \\ K^1_{ba} & K^1_{bb} \end{bmatrix}=[C_1^{(0)}]+\sum_{j=1}^{N_a}\varepsilon_j[C_1^{(j)}] \tag{5}$$

$$[K_1]=\begin{bmatrix} K^1_{aa} & K^1_{ab} \\ K^1_{ba} & K^1_{bb} \end{bmatrix}=[K_1^{(0)}]+\sum_{j=1}^{N_a}\varepsilon_j[K_1^{(j)}]$$

$$\{q_1\}=\begin{Bmatrix} q_a^1 \\ q_b^1 \end{Bmatrix}, \qquad \{f_1\}=\begin{Bmatrix} F_a^1 \\ F_b^1 \end{Bmatrix}$$

where $\{F_a^1\}$, $\{F_b^1\}$ are the external force of the region **a**, **b**, respectively. The damping term is considered to be proportional to (damping coefficient α) the stiffness term, which is used for structural dynamics in general. The substructures are synthesized using the modes $[\Phi^1]$, which are obtained by constraining the assembling region. The displacement $\{q_a^1\}$ of internal region a can be expressed as

$$\{q_a^1\}=\{q_1^{1N}\}+\{q_a^{1c}\} \tag{6}$$

where $\{q_a^{1N}\}=[\Phi^1]\{\xi^1\}$, $\{q_a^{1c}\}=[T_b^1]\{q_b^1\}$. $\{\xi^1\}$ is a modal participation vector of substructure 1. $\{q_a^{1c}\}$ is a static elastic deformation caused by $\{q_b^1\}$. The sensitivities of eigenvalue are defined as

$$\lambda_1=\lambda_1^{(0)}+\sum_{j=1}^{N_a}\varepsilon_j\lambda_1^{(j)} \tag{7}$$

$[T_b^1]$ is obtained by applying Guyan's method to the equation of substructure 1. And $[T_b^1]=[T_b^{1(0)}]+\sum_{j=1}^{N_a}\varepsilon_j[T_b^{1(0)}]$, $[T_b^1]=-[K_{aa}^1]^{-1}[K_{ab}^1]$. To apply the SSM, the coordinate transformation is carried out

$$\begin{Bmatrix} q_a^1 \\ q_b^1 \end{Bmatrix}\begin{bmatrix} \Phi^1 & T_b^1 \\ 0 & E_{n_b} \end{bmatrix}\begin{Bmatrix} \xi^1 \\ q_b^1 \end{Bmatrix}=[T_p^1]\begin{Bmatrix} \xi^1 \\ q_b^1 \end{Bmatrix} \tag{8}$$

where $[E_{n_b}]$ is unit matrix. By using this transformation equation, the equation of substructure *1 can be expressed*

$$[\overline{M_1}]\{\ddot{\zeta}_1\}+[\overline{C_1}]\{\dot{\zeta}_1\}+[\overline{K_1}]\{\zeta_1\}=\{\overline{f_1}\}$$

$$[\overline{M_1}]=[T_p^1]^T[M_1][T_p^1]=\begin{bmatrix} E_{na} & \overline{M^1_{ab}} \\ \overline{M^1_{ba}} & \overline{M^1_{bb}} \end{bmatrix}$$

$$[\overline{C_1}]=[T_p^1]^T[C_1][T_p^1]=\alpha\begin{bmatrix} \Lambda_{na} & 0 \\ 0 & \overline{K^1_{bb}} \end{bmatrix} \tag{9}$$

$$[\overline{K_1}]=[T_p^1]^T[K_1][T_p^1]=\alpha\begin{bmatrix} \Lambda^1 & 0 \\ 0 & \overline{K^1_{bb}} \end{bmatrix}$$

$$\{\zeta_1\}=\begin{Bmatrix} \xi^1 \\ q_b^1 \end{Bmatrix}, \qquad \{\overline{f_1}\}=[T_p^1]^T\{f_1\}=\begin{Bmatrix} \overline{F_a^1} \\ \overline{F_b^1} \end{Bmatrix}$$

$[\Lambda^1]$ is composed of eigenvalues. The equation of substructure 2 is obtained by similar procedure with substructure 1 applying the FEM and modal analysis. By using the modal matrix and the transformation equation, the equation of substructure 2 can be obtained as

$$[\overline{M_2}]\{\ddot{\zeta}_2\}+[\overline{C_2}]\{\dot{\zeta}_2\}+[\overline{K_2}]\{\zeta_2\}=\{\overline{F_2}\}$$

$$[\overline{M_2}]=[T_p^2]^T[M_2][T_p^2]=\begin{bmatrix} \overline{M^2_{bb}} & \overline{M^2_{bc}} & \overline{M^2_{bd}} \\ \overline{M^2_{cb}} & \overline{E^2_{nc}} & \overline{M^2_{cd}} \\ \overline{M^2_{db}} & \overline{M^2_{dc}} & \overline{M^2_{dd}} \end{bmatrix}$$

$$[\overline{C_2}]=[T_p^2]^T[C_2][T_p^2]=\alpha\begin{bmatrix} \overline{K^2_{bb}} & 0 & \overline{K^2_{bd}} \\ 0 & \Lambda^2 & 0 \\ \overline{K^2_{db}} & 0 & \overline{K^2_{dd}} \end{bmatrix} \tag{10}$$

$$[\overline{K_2}]=[T_p^2]^T[K_2][T_p^2]=\begin{bmatrix} \overline{K^2_{bb}} & 0 & \overline{K^2_{bd}} \\ 0 & \Lambda^2 & 0 \\ \overline{K^2_{db}} & 0 & \overline{K^2_{dd}} \end{bmatrix}$$

$$\{\zeta_2\}=\begin{Bmatrix} q_b^2 \\ \xi^2 \\ q_d^2 \end{Bmatrix}, \{\overline{f_2}\}=[T_p^2]^T\{f_2\}=\begin{Bmatrix} \overline{F_b^2} \\ \overline{F_c^2} \\ \overline{F_d^2} \end{Bmatrix}$$

where $[M_2]$, $[C_2]$, $[K_2]$ are the mass, damping and stiffness matrix of substructure 2, respectively. $\{q_b^2\}$, $\{q_d^2\}$ are the displacement of assembling and the internal region of substructure 2. $\{F_b^2\}$, $\{F_d^2\}$, $\{F_c^2\}$ are the internal forces. The displacement $\{q_c^2\}$ of internal region c is obtained as

$$\{q_c^2\}=[\Phi^2]\{\xi^2\}+[T_b^2 \;\; T_d^2]\begin{Bmatrix} q_b^2 \\ q_d^2 \end{Bmatrix} \tag{11}$$

where $[\Phi^2]$ is modal matrix, which is obtained from the eigenvalue problem of $([K_{cc}^2]-\lambda^2[M_{cc}^2])\{\Phi_r^2\}=\{0\}$ $(r=1, 2, \cdots, n_c)$. Static modes $[T_b^2\ T_d^2]$ are obtained by applying Guyan's method to the equation of substructure 2, such as $[T_b^2\ T_d^2]=-[K_{cc}^2]^{-1}[K_{cb}^2\ K_{cd}^2]$.

The coordinate transformation is carried out as

$$\begin{Bmatrix} q_b^2 \\ q_c^2 \\ q_d^2 \end{Bmatrix}=\begin{bmatrix} E_{n_b} & 0 & 0 \\ T_b^2 & \Phi^2 & T_d^2 \\ 0 & 0 & E_{n_d} \end{bmatrix}\begin{Bmatrix} q_b^2 \\ \xi^2 \\ q_d^2 \end{Bmatrix}=[T_p^2]\begin{Bmatrix} q_b^2 \\ \xi^2 \\ q_d^2 \end{Bmatrix} \quad (12)$$

Then, $\{q_c^2\}$ is transformed into modal coordinates. By the similar way, the equation of substructure 3 obtained in modal coordinates as

$$[\overline{M_3}]\{\ddot{\zeta}_3\}+[\overline{C_3}]\{\dot{\zeta}_3\}+[\overline{K_3}]\{\zeta_3\}=\{\overline{f_3}\} \quad (13)$$

Therefore, all of the equations for substructures are obtained in modal coordinates.

2.3 Mode synthesis of uncertain structure and sensitivity analysis

The overall structure is modeled three components and those equations of motion were obtained and transformed into modal coordinate. Then, those equations can be synthesized together. When assembling region is rigid, according to the condition for the compatibility of assembling, following conditions have to be satisfied.

$$\begin{aligned}&\{q_b\}=\{q_b^1\}=\{q_b^2\},\ \{q_d\}=\{q_d^2\}=\{q_d^3\}\\&\{F_b^1\}+\{F_b^2\}=\{0\},\ \{F_d^2\}+\{F_d^3\}=\{0\}\end{aligned} \quad (14)$$

By superposing each modal equation of substructure, and applying above conditions, the perturbed equation for the overall structure can be obtained

$$\tilde{M}\ddot{\tilde{q}}+\tilde{C}\dot{\tilde{q}}+\tilde{K}\tilde{q}=\tilde{F}$$

$$\tilde{M}=\begin{bmatrix} E_{n_a} & \overline{M_{ab}^1} & 0 & 0 & 0 \\ \overline{M_{ba}^1} & \overline{M_{bb}} & \overline{M_{bc}^2} & \overline{M_{cd}^2} & 0 \\ 0 & \overline{M_{cb}^2} & E_{nc} & \overline{M_{cd}^2} & 0 \\ 0 & \overline{M_{ab}^2} & \overline{M_{dc}^2} & \overline{M_{dd}} & \overline{M_{de}^3} \\ 0 & 0 & 0 & \overline{M_{ed}^3} & E_{ne} \end{bmatrix}$$

$$\overline{M_{bb}}=\overline{M_{bb}^1}+\overline{M_{bb}^2}, \qquad \overline{M_{dd}}=\overline{M_{dd}^2}+\overline{M_{dd}^3}$$

$$\tilde{C}=\alpha\begin{bmatrix} E_{n_a} & \overline{K_{ab}^1} & 0 & 0 & 0 \\ \overline{K_{ba}^1} & \overline{K_{bb}} & \overline{K_{bc}^2} & \overline{K_{cd}^2} & 0 \\ 0 & \overline{K_{cb}^2} & E_{nc} & \overline{K_{cd}^2} & 0 \\ 0 & \overline{K_{ab}^2} & \overline{K_{dc}^2} & \overline{K_{dd}} & \overline{K_{de}^3} \\ 0 & 0 & 0 & \overline{K_{ed}^3} & E_{ne} \end{bmatrix}$$

$$\tilde{K}=\begin{bmatrix} E_{n_a} & \overline{K_{ab}^1} & 0 & 0 & 0 \\ \overline{K_{ba}^1} & \overline{K_{bb}} & \overline{K_{bc}^2} & \overline{K_{cd}^2} & 0 \\ 0 & \overline{K_{cb}^2} & E_{nc} & \overline{K_{cd}^2} & 0 \\ 0 & \overline{K_{ab}^2} & \overline{K_{dc}^2} & \overline{K_{dd}} & \overline{K_{de}^3} \\ 0 & 0 & 0 & \overline{K_{ed}^3} & E_{ne} \end{bmatrix}$$

$$\overline{K_{bb}}=\overline{K_{bb}^1}+\overline{K_{bb}^2}$$

$$\overline{K_{dd}}=\overline{K_{dd}^2}+\overline{K_{dd}^3}$$

$$\tilde{q}=\begin{Bmatrix} \xi^1 \\ q_b \\ \xi^2 \\ q_d \\ \xi^3 \end{Bmatrix},\ \tilde{F}=\begin{bmatrix} \overline{F_a^1} \\ [T_b^1]^T F_a^1+[T_b^2]^T F_c^2 \\ \overline{F_c^2} \\ [T_b^2]^T F_a^2+[T_b^3]^T F_e^3 \\ \overline{F_e^3} \end{bmatrix} \quad (15)$$

By this way, the reduced order of perturbed equation can be obtained using only low modes without the analysis of the original overall structures.

Then, the sensitivity of eigenvalue and the random response for the overall system can be obtained, which is subjected to change into original coordinates. When the truncated perturbation equation of structure are obtained, it can be perturbed in accordance with uncertain parameter α_1. Accordingly, general sensitivity analysis of eigenvalue can be applied as

$$\lambda_i=\lambda_i^{(0)}+\sum_{j=1}^{Na}\varepsilon_j\lambda_i^{(j)} \quad (16)$$

2.6 Random response analysis using probabilistic method

The response $\{q(t)\}$ of Eq. (1) can be expressed approximately as

$$\{q\}=\{q^{(0)}(t)\}+\sum_{j=1}^{Na}\varepsilon_j\{q^{(j)}(t)\} \quad (17)$$

By substituting this equation into Eq. (1), the equation can be arranged again as

$$\begin{aligned}&[M^{(0)}]\{\ddot{q}^{(0)}(t)\}+[C^{(0)}]\{\dot{q}^{(0)}(t)\}+[K^{(0)}]\{q^{(0)}(t)\}\\&=\{f(t)\}\end{aligned} \quad (18)$$

$$[M^{(0)}]\{\ddot{q}^{(j)}(t)\}+[C^{(0)}]\{\dot{q}^{(j)}(t)\}+[K^{(0)}]\{q^{(j)}(t)\}$$
$$=-[M^{(j)}]\{\ddot{q}^{(0)}(t)\}-[C^{(j)}]\{\dot{q}^{(0)}(t)\}-[K^{(j)}]\{q^{(0)}(t)\} \quad (19)$$
$$=\{f^{(j)}(t)\} \quad (j=1, \cdots, N_a)$$

From Eq. (18) impulse response function matrix $[H^{(0)}(t)]$ can be obtained. Accordingly, using impulse response function, the expected value $\{q^{(0)}(t)\}$ of the response and the first order sensitivity $\{q^{(j)}(t)\}$ can be obtained as

$$\{q^{(0)}(t)\}=\int_0^1 [H^{(0)}(t-\tau)]\{f(\tau)\}d\tau$$
$$\{q^{(j)}(t)\}=\int_0^1 [H^{(0)}(t-\tau)]\{f^{(j)}(\tau)\}d\tau \quad (20)$$

There is a relation between the impulse response function matrix $[\tilde{H}^{(0)}(t)]$, which is obtained from truncated perturbation equation of overall system, and $[H^{(0)}(t)]$ as

$$[H^{(0)}(t)]=[T_p^{(0)}][\tilde{H}^{(0)}(t)][T_p^{(0)}]^T \quad (21)$$

Thereby, the expected value of the response and the first order sensitivity can be rewritten as

$$\{q^{(0)}(t)\}=[T_p^{(0)}]\int_0^t \{\tilde{h}_{(0)}(t-\tau)\}g(\tau)\,d\tau$$
$$\{q^{(j)}(t)\}=[T_p^{(0)}]\int_0^t [\tilde{H}^{(0)}(t-\tau)][T_p^{(0)}]^T\{f^{(j)}(\tau)\}d\tau \quad (22)$$

where $\{\tilde{h}^{(0)}(t)\}=-[\tilde{H}^{(0)}(t)][T_p^{(0)}]^T[M^{(0)}]\{I\}$. Hereby, correlation function of the response $[R_x(t_1, t_2)]$ obtained as

$$[R_x(t_1, t_2)]=\overline{\{q(t_1)\}\{q(t_2)\}^T}$$
$$=\{q^{(0)}(t_1)\}\{q^{(0)}(t_2)\}^T$$
$$+\sum_{j=1}^{Na}\sigma_{\alpha j}^2\{q^{(j)}(t_1)\}\{q^{(j)}(t_2)\}^T \quad (23)$$

Here, right side term of the Eq. (28) is defined as

$$\{q^{(0)}(t_1)\}\{q^{(0)}(t_2)\}^T$$
$$=[T_p^{(0)}]\int_0^{t_1}\int_0^{t_2}\{\tilde{h}_{(0)}(t_1-\tau_1)\}\{\tilde{h}_{(0)}(t_1-\tau_1)\}^T R_g(\tau_1, \tau_2)\,d\tau_1 d\tau_2[T_p^{(0)}]^T$$
$$\{q^{(j)}(t_1)\}\{q^{(j)}(t_2)\}^T$$
$$=[T_p^{(0)}]\int_0^{t_1}\int_0^{t_2}[\tilde{H}_{(0)}(t_1-\tau_1)[T_p^{(0)}]^T\{f^{(j)}(\tau_1)\}\{f^{(j)}(\tau_2)\}^T[T_p^{(0)}][\tilde{H}^{(0)}(t_2-\tau_2)]^T d\tau_1 d\tau_2[T_p^{(0)}]^T \quad (25)$$

where $R_g(t_1, t_2)$ is an autocorrelation function of $g(t)$. PSD (Power spectrum density) of the response can be obtained by transforming the $[R_x(t_1, t_2)]$ into frequency domain with the PSD function of $g(t)$.

3. Numerical Examples

3.1 Model for analysis

A structure system, which is found easily in mechanical structure as shown in Fig. 2, is considered to verify the proposed method.

The model of the system is constrained at both

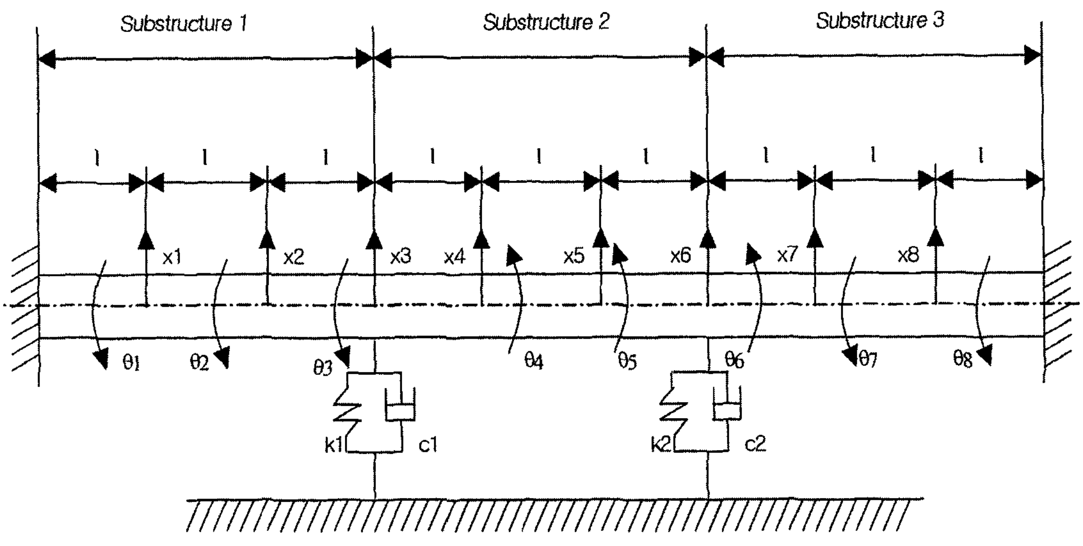


Fig. 2 Model of the system
(Uncertain parameters : c_1, c_2 ; k_1, k_2)

ends. Support of the system is modeled as bearing with spring and damping. To apply the SSM, the pipe system is divided into 3 components. Those components are assembled together in rigid as sembling region. FEM model discretizes the distributed mass and stiffness properties of each substructure. Then, using the local dynamic characteristics obtained for each substructure element, the overall system dynamic characteristics are obtained. By considering external force and the constraints of the system, the equations of motion of the overall system can be derived. Total degrees of freedom of overall system is 16. In this analysis, the damping of the system is assumed as the proportional damping of stiffness as $[C_p]=\alpha[K_p]$, $[K_p]$ is stiffness matrix of the pipe system. The coefficient $\alpha(=0.0005)$ is decided with the first mode of the system to become 0.05. The damping coefficient $c_i(i=1, 2)$, and spring coefficient $k_i(i=1, 2)$ of the supports are subjected to have uncertainty. The properties of the system are tabulated in Table 1.

Table 1 Properties of the system

Length of element	7.50 [m]
Material Density	7.7×10^3 [kg/m^3]
Young's Modulus	206.0×10^9 [N/m^2]
Outer Diameter	1.0 [m]
Inner Diameter	0.9 [m]
Spring Coefficient k_1, k_2 (k_{s0})	1.0×10^7 [N/m]
Damping Coefficient c_1, c_2 (c_{s0})	5.4×10^4 [N·sec/m]
Spring Coefficient of Beam k_b	1.3×10^6 [N/m]
Damping Coefficient of Beam c_b	6.7×10^2 [N·sec/m]

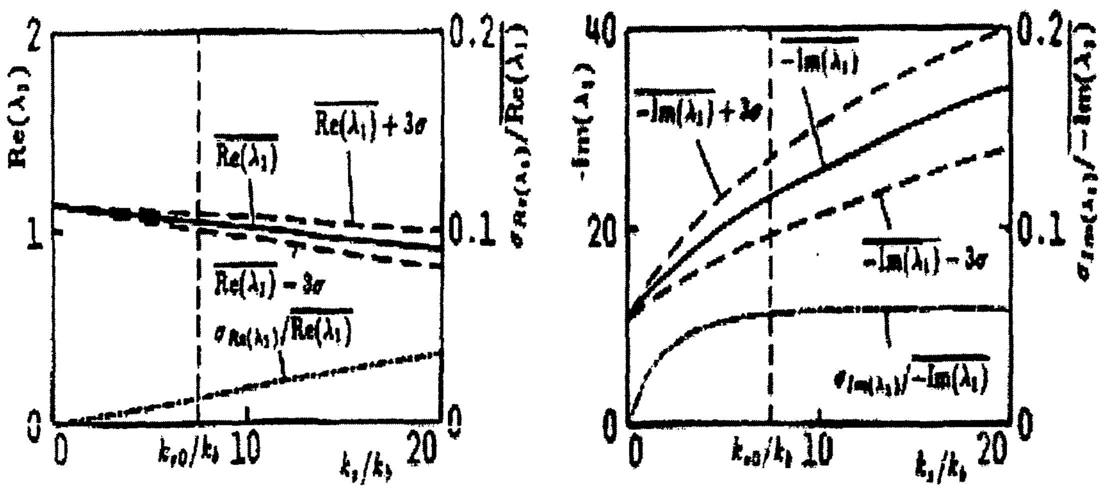
Table 2 Eigenvalue of the deterministic system (ζ is damping ratio)

mode	λ	ω(rad/s)	ζ
1	$-1.04+j\,23.2$	23.2	0.045
2	$-1.61+j\,38.3$	38.3	0.042
3	$-1.00+j\,59.2$	59.2	0.017
4	$-3.28+j\,99.8$	99.9	0.033
5	$-6.99+j\,149.0$	149.0	0.047

First, the eigenvalue is obtained by regarding the system as deterministic system. Modal damping ratio is obtained as shown in Table 2 using $\zeta_i=-Re(\lambda_i)/\omega_i$, $\omega_i=\sqrt{Re(\lambda_i)^2+Im(\lambda_i)^2}$.

3.2 Sensitivity analysis of uncertain system

For the analysis, spring coefficient k_{s0}, and damping coefficient c_{s0} of support are changed as an uncertain parameter. The effect of those variance coefficients on the eigenvalue of overall



(a) Real part of eigenvalue $Re(\lambda_1)$ (b) Imaginary part of eigenvalue $Im(\lambda_1)$

Fig. 3 Variation of first natural frequency of the system (k_s is changed)
($\sigma_{k1}\sqrt{\overline{k_1}}=\sigma_{k2}\sqrt{\overline{k_2}}=0.3$, $\sigma_{c2}\sqrt{\overline{c_2}}=\sigma_{k2}\sqrt{\overline{k_2}}=0.0$, $\overline{k_1}=\overline{k_2}=k_s$, $\overline{c_1}=\overline{c_2}=c_s$)

system is reviewed. Base of spring coefficient, damping coefficient of the support are regarded as equivalent spring coefficient k_p, damping coefficient c_p, which are obtained from the first eigenvalue of the system. Then, the variation of the first eigenvalue is reviewed when the spring coefficient, damping coefficient of support are changed. k_s, k_b are the stiffness of support and beam, which come from the modeling of pipe system. When k_b is fixed, k_s is varied to observe the effect of changes in eigenvalue of overall system, which means the effect of uncertainty. The same things can be applied to the damping term. Figure 3 shows variation of the eigenvalue, when the spring coefficient k_s of support is changed by fixing the damping coefficient c_{s0}. Figure 4 shows variation of the eigenvalue, when the damping coefficient c_s of support is changed by fixing the spring coefficient k_{s0}. Figure 4 (a) and (b) show real part and imaginary part of eigenvalue variation, respectively. It can be concluded from the *Fig. 3, Fig. 4 that the coefficients of* eigenvalue variations are relatively *small.*

To show the accuracy and effectiveness of proposed analysis, five different analysis type cases are considered

Case 1; *deterministic system,*

Case 2; indeterministic, Hierarchy method,

Case 3; indeterministic, perturbation method,

Case 4; indeterministic, proposed method, adopting 1~4 modes,

Case 5; indeterministic, proposed method, adopting 1~2 modes.

Sensitivity analysis of eigenvalue is carried out when the spring coefficient, damping coefficient of support at substucture 1 are uncertain as $(\sigma_{c1}\sqrt{c_1}=\sigma_{k1}\sqrt{k_1}=0.3,\ \sigma_{c2}\sqrt{c_2}=\sigma_{k2}\sqrt{k_2}=0.0)$.

Table 3 shows the result of variation of low eigenvalues in the case of 3~5. The result of *Monte Carlo simulation*(1000 *iteration*) *is also*

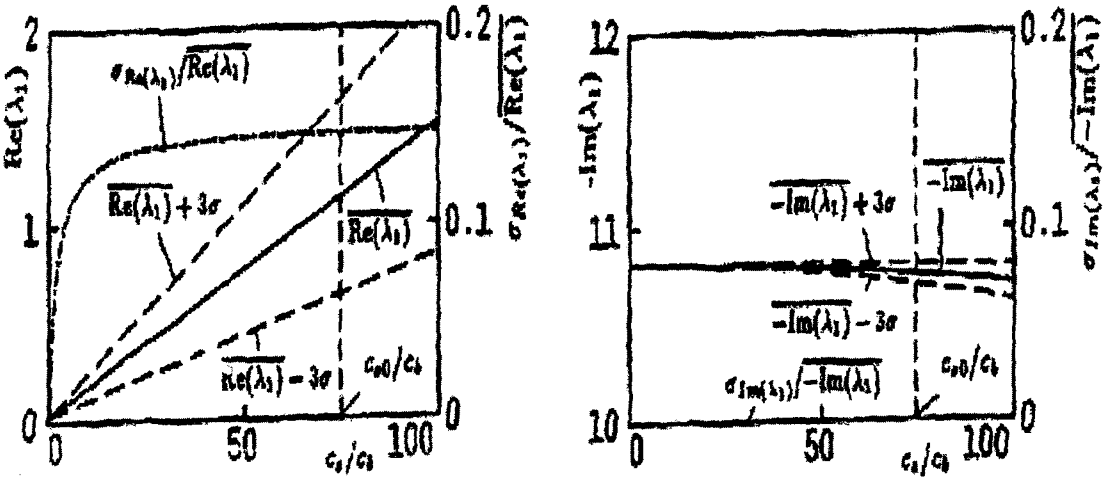
Table 3 Coefficient of eigenvalue variation

(a) Variation of 1st eigenvalue

Method	$\sigma Re(\lambda)$	$\sigma Re(\lambda)\sqrt{Re(\lambda)}$	$\sigma I_m(\lambda)$	$\sigma I_{m(\lambda)}\sqrt{I_m(\lambda)}$
Monte Carlo	0.1580	0.1506	1.380	0.0599
Case 3	0.1521	0.1458	1.304	0.0561
Case 4	0.1521	0.1458	1.304	0.0561
Case 5	0.1520	0.1457	1.302	0.0561

(b) Variation of 2nd eigenvalue

Method	$\sigma Re(\lambda)$	$\sigma Re(\lambda)\sqrt{Re(\lambda)}$	$\sigma I_m(\lambda)$	$\sigma I_{m(\lambda)}\sqrt{I_m(\lambda)}$
Monte Carlo	0.2082	0.1295	1.073	0.0280
Case 3	0.2084	0.1293	1.084	0.0283
Case 4	0.2084	0.1293	1.084	0.0283
Case 5	0.2082	0.1292	1.083	0.0283



(a) Real *part of eigenvalue* $Re(\lambda_1)$ (b) Imaginary part of eigenvalue $Im(\lambda_1)$

Fig. 4 Variation of first natural frequency of the system (c_s is changed)

$(\sigma_{k1}\sqrt{k_1}=\sigma_{k2}\sqrt{k_2}=0.0,\ \sigma_{c2}\sqrt{c_2}=\sigma_{k2}\sqrt{k_2}=0.3,\ \overline{k_1}=\overline{k_2}=k_s,\ \overline{c_1}=\overline{c_2}=c_s)$

shown. From Table 3, there is good agreement between the results of Case 4 and Case 3. The result of Case 5, which is reduced order, also simulated well with accuracy. Those results are in good agreement with the result of Monte-Carlo method, which shows variation of eigenvalue in good calculation accuracy by the first perturbation order.

3.3 Analysis of random response

The mean square of the response is obtained against the earthquake excitation to the pipe system. The seismic acceleration $g(t)$ by earthquake can be modeled as stationary narrow band random process and stationary wide band random process. Then, those autocorrelation $R_g(t_1, t_2)$ and PSD (power spectrum density) function $\Phi_g(\omega)$ are given readily. In the case of damping coefficient of support is indeterministic, ($\sigma_{c1}\sqrt{c_1}=\sigma_{c2}\sqrt{c_2}\equiv\sigma_c\sqrt{c}$, $\sigma_{k1}\sqrt{k_1}=\sigma_{k2}\sqrt{k_2}=0.0$) are considered. Stationary narrow band random process can be expressed as ($\tau=|t_1-t_2|$)

$$R_g(t_1, t_2)=S\exp(\beta\tau)\cos(\omega_t\tau),$$
$$\Phi_g(\omega)=\frac{2S\beta(\omega_t^2+\beta^2+\omega^2)}{(\omega_t^2+\beta^2+\omega^2)^2-4\omega_t^2\omega^2} \tag{26}$$

where β is coefficient of dominant frequency. ω_t is a dominant frequency. Stationary wide band random process can be expressed as

$$R_g(t_1, t_2)=2P\sin(\omega_n\tau)/\tau, \ (\tau=|t_1-t_2|),$$
$$\Phi_g(\omega)=\begin{cases}2P\pi & (|\omega|<\omega_n)\\ P\pi & (|\omega|=\omega_n)\\ 0 & (|\omega|>\omega_n)\end{cases} \tag{27}$$

And mean square of both process is introduced as to become same value as $S=2P\omega_n$. Then the root mean square value $\sqrt{x_3^2}$ of response at nodal pint 3 against stationary narrow band random process is shown in Fig. 5. Monte Carlo simulation (1000 iteration) is also shown together, which is regarded as exact solution. Hierarchy method of Case 2 is the result, which is calculated to the perturbation 2nd order. Thus, this is closer to the exact solution and efficient analyzing method compared with the result of Case 3～Case 5, which are calculated to the perturbation first order. The results of Case 3～Case 5 are almost

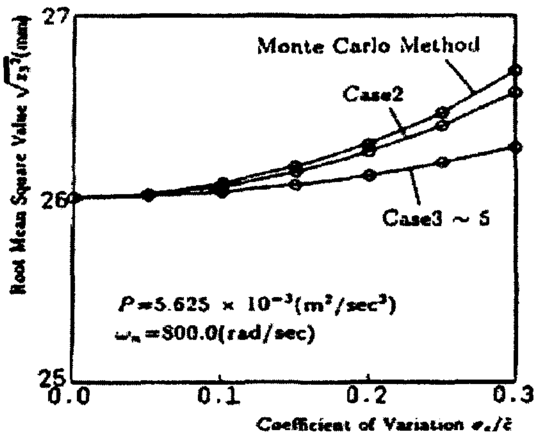


Fig. 5 Mean square of response (narrow band)

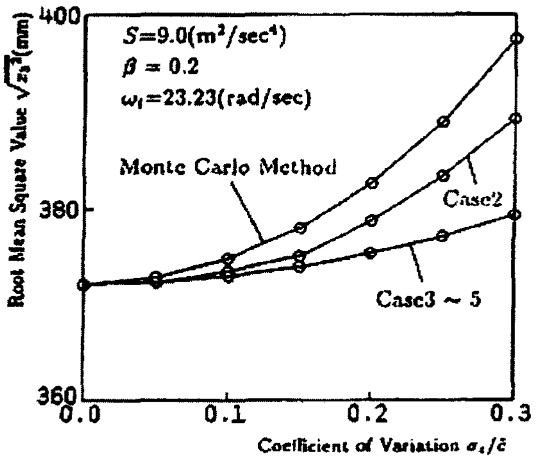


Fig. 6 Mean square of response (wide band)

same. This is estimated that dominant frequency ω_t of excitation is close to the first natural frequency of the system. Thus, accurate result is obtained in spite of neglecting higher modes.

The root mean square value $\sqrt{x_3^2}$ of response at nodal point 3 against stationary wide band random process is shown in Fig. 6. Also, Hierarchy method of Case 2 is close to the exact solution. The results of Case 3-Case5 are almost same. This is estimated that frequency range of excitation include all of the natural frequency of the system. Nevertheless, the modal damping ratio of higher modes are bigger than the modal damping ratio of lower modes. Thus, there is no effect of higher modes excitation in the response.

PSD $\Phi_{x3}(\omega)$ of response against stationary

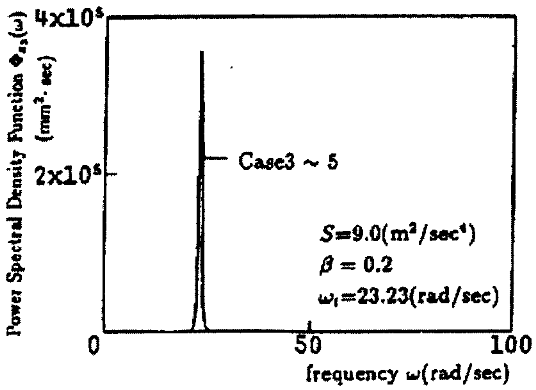


Fig. 7 PSD function of narrow band

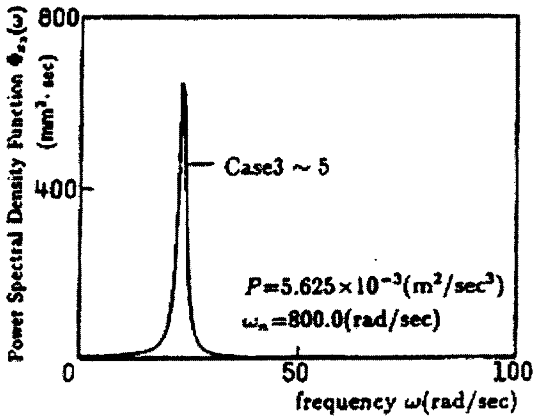


Fig. 8 PSD function of wide band

narrow band random process and stationary wide band random process are shown in Fig. 7 and Fig. 8, respectively. There is a high value around the first natural frequency because the modal damping ratio of higher modes are bigger than the modal damping ratio of lower modes.

To show the effectiveness of the proposed method, the calculation time is compared. When analyzing the correlation function of response, the calculation time is compared. Case 3 is regarded as a normal of calculation time. Case 4 takes same time with the Case 3. Case 5 takes about 0.15 times of Case 3. This comes from the effect of reduced order of degrees of freedom.

4. Conclusion

In this paper, a method to obtain the sensitivity of eigenvalues and the random responses of the structure with uncertain parameters using a probabilistic method is proposed. According to the proposed method, the reduced order perturbed equation for the overall system is obtained without analysis of the original overall structures even though the overall structure is large system. As a numerical example, a simple structure system is considered, which has a uncertain parameter. Then the variations of the eigenvalue and the random responses are calculated. The accurate results are obtained comparing with the other general method in spite of reducing the DOF. As a result, the proposed method is proved to be an useful technique to analyze the sensitivity of eigenvalues and random response against random excitation in terms of the accuracy and the calculation time.

Acknowledgment

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References

Bellman, R., 1964, *Perturbation Technique in Mathematics, Physics and Engineering*, Holt, Rinehart and Winston.

Bharucha-Reid, 1968, *Probabilistic Methods in Applied Mathematics*, Vol. 1, p. 48, Academic press.

Byungyoung Moon, Jin-Wook Kim and Bo-suk Yang, 1999, "Non-Linear Vibration Analysis of Mechanical Structure System using Substructure Synthesis Method," *KSME International Journal*, Vol. 13, No. 9, pp. 620~629.

Byungyoung Moon, Beomsoo Kang, 2001, "Non-Linear Vibration Analysis of Rotor System using Substructure Synthesis Method (Analysis with Consideration of Nonlinear Sensitivity)," *JSME International Journal Series C*, Vol. 44, No. 1, pp. 12~19.

Caravanu, P. and Thomson, W. T., 1973, "Frequency Response of a Dynamic System with Statistical Damping," *AIAA Journal*, Vol. 11, No. 2, p. 170.

Collins, J. D. and Thomson, W. T., 1969, "The

Eigenvalue Problem for Structural System with Statistical Properties," *AIAA Journal*, Vol. 7, No. 4, p. 642.

Lin, C. W. and Loceff, F., 1980, "A new Approach to Compute System Response with Multiple Support Response Spectra Input," *Nuclear Engineering and Design*, Vol. 60, pp. 347~352.

Nakagiri, S. and Hisada, T. A., 1985, "Introduction of Finite Element method by Statistical Method," *Haihukann Press*, p. 163.

Shigeru Nakagiri, Toshiaki Hisada, Kazuhiko Toshimitsu, 1985, "Time History Response Analysis of Uncertain Damping System by Probalilistic FEM," *Trans. Jpn. Soc. Mech. Eng.*, (in Japanese) Series C, Vol. 51, No. 472, p. 3172.

Yang, J. N. and Shinozuka, M., 1972, "On the First Excursion Probability in Stationary Narrow Band Random Vibration," *Trans. ASME Jouranl of Applied Mechanics*, Vol. 39, No. 3, p. 733.

Yoshino Inoue, Takeshi Hujikawa, Etsujiro Imanishi, Toru Abe, 1984, "Sensitivity Analysis and Modal Parameter Estimation after Design change," *Trans. JSME Journal* (in Japanese) Series C, Vol. 50, No. 452, p. 597.