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Improvement to Crack Retardation Models Using "Interactive Zone Concept"

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ABSTRACT

The load interaction effect can be best illustrated by the phenomenon of overload retardation. Some prediction methods for retardation are reviewed and the problems discussed in the present paper. The so-called under-load effect much of the retardation disappears if a very low level minimum stress follows the overload, is also of importance for a prediction model to work properly under random load spectrum. The concept of Interactive Zone (IZ) fully considering reversed plasticity during unloading was discussed. This IZ concept can be combined with existing models to derive some improved models that can naturally take account of the under-load effect.

Some simulations by IZ improved models for test under complex load sequences including multiple overloads and both over/under loads are compared with test results. It is seen that the improvement by IZ concept greatly enhanced the ability of existing models to accommodate complex load interaction effects.

Keywords: Fatigue materials , Fatigue crack growth, Fatigue life prediction, Retardation, Load interaction, Interactive zone, Variable amplitude fatigue

1. Introduction

The load interaction effect in variable amplitude fatigue test can be best illustrated by the phenomenon of overload retardation. Explanations of retardation phenomenon were partly given on concepts of crack tip plastic zone, residual stresses in the plastic zone, and the crack closure, but a fully understanding of load interaction effect still needs much research efforts.

Some prediction methods were proposed in literature, we are examining some of them in light of test results, discussing their merits and problems. Apart from the overload retardation effect, the so-called under-load effect much of the retardation disappears if a very low level minimum stress follows the overload, sometimes also called "acceleration" phenomenon, is also of importance for a prediction model to work properly. Some new concepts and models to fully account for load interaction effects will be explored here.

2. Existing Models

Wheeler model [1] is probably the most widely used retardation model. It is based purely on plastic zone size consideration as shown in Fig. 1, where r_{pi} is the current plastic zone size, λ is the residual retardation zone determined by overload plastic zone boundary.

The retarded rate is obtained by simply multiplying the original (no effect of overload retardation) rate by a retardation factor of

$$Cd=(r_{pi}/\lambda)^{w} \qquad (1)$$

i.e.

$$da/dN(ret)=(r_{pi}/\lambda)^{w} da/dN(no\ ret) \qquad (2)$$

Where w is termed as Wheeler index and supposed to be a material constant but actually is an adjustable fitting parameter.

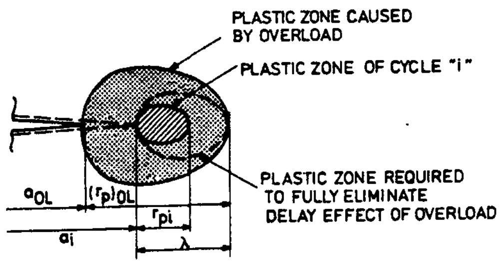

Fig. 1 Plastic zone size in Wheeler model

Wheelers retardation model is implemented in our developed FCG life prediction program and the a~N relations of Wheeler-Paris simulation for tests with different overload ratios (CCTO1, CCTO2 and CCTO9) are shown in Fig. 2.

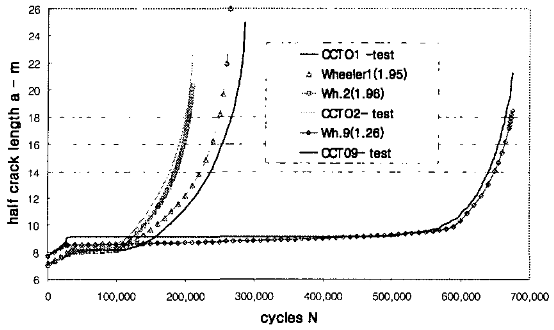

Fig. 2 Overload tests and predictions by Wheeler model

If *w* is a material constant, it should be the same for multiple test results of the same material. Table 1 lists the results from several overload tests.

It is seen that w is not a material constant even for single overload test. At the best, w may be a function of overload ratio (OLR=Pov/Pmax). So Wheeler index is not only dependent on material behavior but also on load profile. Another point of concern is that there is no means in the Wheeler model to account for the under-load effect, which will be discussed later.

Original Willenborgs model [2] combines the plastic zone size with the effective SIF concept. The residual plastic zone caused by the overload produces beneficial effect which must be overcame by the applied SIF, so the effective SIF and R ratio are changed. Crack growth rate retarded due to overload is corrected by substituting effective stress ratio R_{eff} and effective stress intensity range $\triangle K_{eff}$ into Formans equation and no addition parameter is needed.

In calculating effective SIF, the λ parameter appears in the model to compute a 'boundary SIF' Kap as

$$K_{ap} = \sigma_y \sqrt{\gamma\pi(a_{ov} + r_{ov} - a)} = \sigma_y \sqrt{\gamma\pi\lambda} \qquad (3)$$

where, σ_y is the yield stress, γ is the over-load ratio (OLR), a_{ov} is the crack length under over-load, r_{ov} is the plastic zone size under over -load.

The residual SIF, K_{res}, and effective SIF, K_{eff}, are then

$$K_{res} = K_{\max} - K_{ap} \qquad (4)$$

$$K_{eff} = K - K_{res} \qquad (5)$$

Results of applying the original Willenborg model to analyze overload tests are shown in Fig. 3. Predictions for test CCTO1 seems quite good while prediction for test CCTO2 is not so well. With the higher overload ratio of test CCTO9, both maximum and minimum effective SIF go below zero, so original Willenborg model predicts a complete crack arrest, which disagrees with experimental results.

The Generalized Willenborg Model introduced 2 new adjustable parameters by modifying effective SIF to become

$$K_{eff} = K - \Phi K_{res} \qquad (6)$$

Table 1 Calculation of Wheeler index w for several tests

Test id	overload ratio	da/dN (Paris)	da/dN(test)	Cd	(r_p/λ)	w
CCTO1	6.532/4.0	2.6075e-8	1.6085e-9	0.06122	0.2383	1.9476
CCTO2	6.592/4.0	4.1686e-8	1.9831e-9	0.04757	0.21163	1.9612
CCTO9	9.280/4.0	3.8314e-8	1.1424e-9	0.02982	0.06193	1.2628

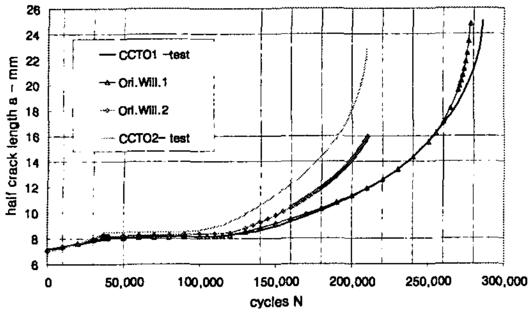

Fig. 3 Simulation of overload tests by original Willenborg Model

where

$$\Phi=\frac{1-K_{\max,th}/K_{\max}}{\gamma_c-1} \tag{7}$$

$K_{\max,th}$ is the threshold for $K_{\max}$, γ_c the "cut-off overload ratio , and they are supposed to be material properties.

With these 2 adjustable parameters, we should be able to simulate overload tests better. However, these parameters cannot be derived directly from tests. They are determined based on trial-and-error and experiences. Some of the trial simulations are shown in Fig. 4. Chang [2] also modified Willenborg model, usually called Willenborg/Chang model. Simulation for test CCTO1 by Willenborg/Chang model is also shown in Fig. 4. It is seen that the predictions by Generalized Willlenborg (G.Wil.) model are really better than by the previous models, but prediction by Willenborg/Chang (short as Ch.Wil.) model is not as good as that by Generalized Willenborg model.

The parameters for most suitable simulations (closest to test curves) are listed in Table 2. It means that the same set of $K_{\max,th}$, γ_c did not fit for different overload ratios.

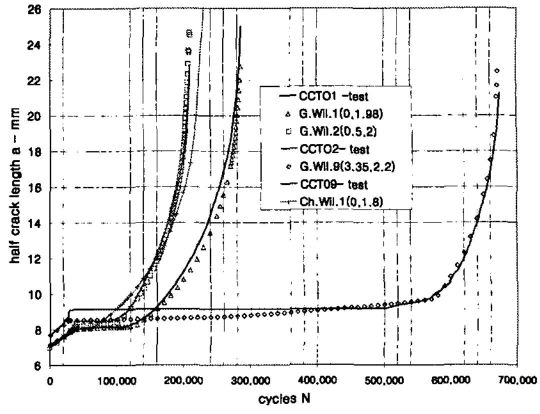

Fig. 4 Generalized (modified) Willenborg Model simulations

Table 2 Parameters for Generalized Willenborg Model

Test id	overload ratio	$K_{\max,th}$	γ_c
CCTO1	6.532/4.0	0.0	2.0
CCTO2	6.592/4.0	0.5	2.0
CCTO9	9.280/4.0	3.35	2.2

HeQingZhis model [3] utilized the residual compress stress concept in plastic zone and also took the stress relaxation into account. It is a model which bridges Willenborg model and Elbers closure concept [4]. The modified effective SIF range is similar to Elbers formula as:

$$\triangle K_{eff}=U\triangle K_{app} \tag{8}$$

where

$$U=1+(1-\eta)/(1-R)-(1-\eta)K_{ap}/\triangle K \tag{9}$$

K_{ap} is the same as in Willenborg model, η is the stress relaxation coefficient and expressed as

$$\eta=1-(1-\triangle K_{th}/\triangle K)/(\gamma_c-1) \tag{10}$$

R is the stress ratio.

The two adjustable parameters are $\triangle K_{th}$ and γ_c now. Simulations for overload tests by HeQZ model are shown in Fig. 5.

The parameters for most suitable simulations (closest to test curves) are list in Table 3. It is seen that 2 sets of different parameter combinations can give almost the same good predictions. It also shows that the same set

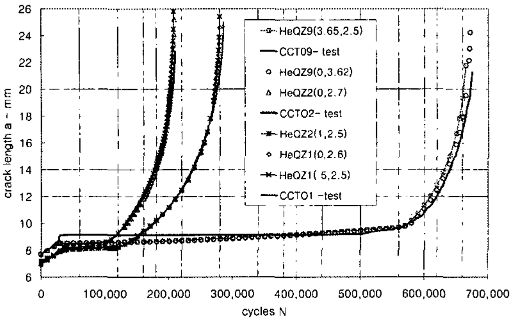

Fig. 5 Simulations of HeQZ model compared with test curves

of parameters did not suit for all cases of different overload ratios.

All the above models are basically retardation models only considering overload influence. There is no · means in these models to account for the under-load effect.

Table 3 Parameters for HeQZ Model

Test id	overload ratio	$\triangle K_{th}$	γ_c	$\triangle K_{th}$	γ_c
CCTO1	6.532/4.0	0.5	2.5	0	2.6
CCTO2	6.592/4.0	1.0	2.5	0	2.7
CCTO9	9.280/4.0	3.65	2.5	0	3.62

3. The "Interactive (influence) Zone"

In the above models, the all- important parameter λ can be termed as overload influence zone since it outlines the boundary of the overload effect. There is only one dominant λ in operation. If another overload is bigger than the previous one, then the new λ for the bigger overload will become dominant.

Taking the unloading reversed plastic zone into account, we think the reversed plasticity will release part of the residual stress built up during overload and the influence region of overload will be shrunk by some extent. We propose the Interactive Zone (IZ) Z based on a simple assumption that the reversed plasticity will reduce the residual plastic region as:

$$Z = \lambda - r_{\triangle p} \tag{11}$$

Where $r_{\triangle p}$ represents the dominant unloading reversed plastic zone after an overload.

There are 2 issues that must be cleared. Firstly, the reversed plasticity will be controlled by cyclic yield stress range ($\triangle\sigma_y$) instead of the monotonic (normal) yield stress (σ_y). The simplest assumption will be $\triangle\sigma_y = 2\sigma_y$ if adopting a kinetic hardening law. Other cyclic strain hardening laws can be used to fine-tune the model. Secondly, the dominant unloading stress range ($\sigma_{ov} - \sigma_{\min}$) will be counted from the dominant overload to the lowest minimum stress after that overload. If the minimum stress of a new load cycle is higher than the previous one, the dominant ($\sigma_{ov} - \sigma_{\min}$) will remain unchanged. If the minimum stress of a new cycle is lower than the present dominant one, the dominant ($\sigma_{ov} - \sigma_{\min}$) will change but still be counted from the same overload to the present lowest stress. It is obvious that the retardation effect will vanish when ($\sigma_{ov} - \sigma_{\min}$) $= 2\sigma_{ov}$ assuming kinetic hardening law.

This concept of Interactive Zone (IZ) can be combined with Wheeler or Willenborg or other models to derive some improved models (improvement to existing models) which can naturally take account of the under-load effect.

4. Improved Model

Here firstly we shall check whether the IZ concept has be correctly implemented in the program by simulating some overload tests.

In a combined IZ+Wheeler model, we use Z to replace λ in Wheeler model. Of course, this time the index w is different from the simple Wheeler model. Then,

$$da/dN(ret) = (r_{pi}/Z)^{w} da/dN(no\ ret) \tag{12}$$

In IZ+Willenborg model, Z is also used to replace the λ in calculating Kap

$$K_{ap} = \sigma_y\sqrt{\gamma\pi(a_{ov} + r_{ov} - a - r_{\triangle p})} = \sigma_y\sqrt{\gamma\pi Z} \tag{13}$$

Since generalized Willenborg model, Willenborg/ Chang and HeQZ models all used Kap in their corresponding formula, IZ concept is readily extendible to all these models in a very similar manner (we shall omit the details here). We have proposed a new model (Chen-Lee model [5]) using simple empirical yet very powerful expression of

$$U = (K_{\max} / K_{ap})^m \tag{14}$$

This new model can also be easily combined with the IZ concept. Here some examples of simulation by IZ improved models are given in Fig. 6.

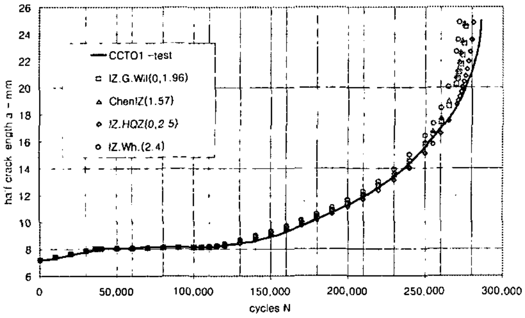

Fig. 6 Simulations of CCTO1 by IZ improved models

From the above Fig.6 we may expect that IZ concept improved models can be used with slightly adjusted parameters wherever those un-improved (original, simple) models are suitable as indicated in Fig. 2.

5. Under-load (acceleration) effects

The interaction effect includes both overload retardation and under-load acceleration phenomena. Apart from the overload retardation effect, the so-called under-load effect, sometimes also called acceleration phenomenon much of the retardation disappears if a very low level minimum stress follows the overload, is also of importance for a prediction model to work properly. The real strong point of IZ concept improved models is in considering under-load effects.

We have tested some CCT specimens under specially designed load profile to investigate the load interaction effects. One test case will be described here and used to examine the suitability of the above models for FCG life prediction under complex load sequences. The CCT specimen was mostly subjected to CA fatigue cycling with some over load and over/under loads applied at various points. The CA cyclic parameters are as following, maximum stress = 80Mpa, minimum stress = 40Mpa, fatigued at frequency of 7Hz. 3 overloads of 130 Mpa stress were applied after 50,000 CA cycles with a 500 CA cycles in between. Other overloads of 130Mpa followed immediately by under-load of 20Mpa were applied at points after 190,000 cycles and 250,000 cycles (after completely recovered from the previous load interaction influence).

In Fig. 7 shown below, the joint line between the dark square points is the test curve. Also shown in the figure are some predictions made by the various models discussed above. The original Willenborg model has no adjustable parameter so it can not predict even correct tendency not to mention the complete a~N curve for this load sequence. The Wheeler model predicted correct behavior for first group 3 overloads, but failed for the other two over/under load combinations. Generalized Willenborg model and HeQZ model have more adjustable parameters so they gave better predictions than Wheeler model. Some simulations by IZ improved models are compared in the Fig. 7. It is seen from the figure that the improvement by IZ concept greatly enhanced the ability of existing models to accommodate complex load interaction effects.

6. Conclusions

We have examined some existing retardation models for FCG life prediction in light of test results. It is found that though those models worked well for single overload case they could not correctly simulate overload/ under-load test behavior. A concept of Interactive Zone (IZ) considering reversed plasticity during unloading was discussed. This concept of Interactive Zone (IZ) can be combined with existing models to derive some improved models which can naturally take account of the under-load effect.

The IZ concept has be proven correctly implemented in the program by simulating some overload tests. Some simulations by IZ improved models for test under complex load sequences including multiple overloads and both over/under loads are compared in the Fig. 7. It is seen that the improvement by IZ concept greatly enhanced the ability of existing models to accommodate complex load interaction effects.

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