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Dynamic Analysis of a Moving Vehicle on Flexible Beam structures (I) : General Approach

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ABSTRACT

In recent years, mechanical systems such as high speed vehicles and railway trains moving on elastic beam structures have become a very important issue to consider. In this paper, a general approach, which can predict the dynamic behavior of a constrained mechanical system moving on a flexible beam structure, is proposed. Various supporting conditions for the foundation support are considered for the elastic beam structure. The elastic structure is assumed to be a non-uniform and linear Bernoulli-Euler beam with a proportional damping effect. Combined differential-algebraic equation of motion is derived using the multi-body dynamics theory and the finite element method. The proposed equations of motion can be solved numerically using the generalized coordinate partitioning method and predictor-corrector algorithm, which is an implicit multi-step integration method.

Key words : Elastic beam structure, Constrained mechanical system, Multibody dynamics, Finite element method, Bernoulli-Euler beam, Combined differential algebraic equation

1. Introduction

It is a well-known fact that a given magnitude of dynamic force gives more impact to a structure than does the same magnitude of static force. This kind of problem was first recognized during the 19th century Industrial Revolution, in the construction of bridges. Since then, related problems have been investigated by many civil engineers. Recently, mechanical engineers have become interested in the dynamic behavior of the beam because the vehicles and trains moving on the structure have become faster and lighter. Fast-moving precision machines, rocket-launching systems, overhead cranes and fast-moving escalators are other examples involving high speed movement on the structure. Timoshenko[1] has lead the way by presenting an exact solution for mass moving with constant speed on a simply supported beam. He used a mode synthesis method for the elastic beam. Many others have investigated methods to determine exact solutions with mathematical models of the Bernoulli-Euler or Timoshenko beam.[2-6]

With the advent of fast computers, the finite element method has become a more popular tool for engineering analysis. Many researchers have applied the finite element analysis technique for beams.[12-14] Some have considered the geometric nonlinear effect of the Bernoulli-Euler beam.[15-16] But most previous studies have neglected the inertia effect of the moving mass because their objective was limited to the beam structure.

The moving system has become faster and lighter nowadays, so some researchers have focused on the dynamic behavior of the moving system. To analyze the vertical direction dynamic behavior of the moving system, linear springs have been attached to the moving system.[6,16] But only the overall movement could be estimated in this case and no information was available for the components in the moving system. Meanwhile, some researchers have made an attempt to use the multibody dynamic analysis program.[17-18] Maessen[19] obtained contact force between the wheel and rail using the force-strain relationship from a finite element static analysis of the rail. This method was applied to a vehicle moving at low speeds, in which case the dynamic inertia effect can be neglected. These studies have been mostly limited to the vertical direction motion and simple supporting conditions of the rail.

This paper suggests a systematic way of analyzing a vehicle moving on the flexible beam at a high speed. The vehicle is treated as a constrained multi-rigid body system. The vertical and longitudinal motion of the system can be studied with the proposed method. Also, various supporting conditions of the beam and nonlinearities in the system can be easily applied with the proposed method.

2. Equations of Motion for the Supporting Beam

Ignoring shear strain and rotary inertia, if a cross-section of a beam is small compared to its length, then the beam can be assumed to be a Bernoulli-Euler beam. A linear non-uniform Bernoulli-Euler beam with an arbitrary cross-section along its axis is shown in Fig. 1. Equilibrium conditions of horizontal and vertical directions can be written as Eq. (1).

$$\frac{\partial}{\partial x}\left(EA(x)\frac{\partial u(x,t)}{\partial x}\right)=0$$
$$\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 v(x,t)}{\partial x^2}\right)=0 \qquad (1)$$

E is a modulus of elasticity, A(x) is a cross-section area, I(x) is the moment of inertia of the cross-section, and u(x,t) and v(x,t) represent displacements in horizontal and vertical directions, respectively. Displacements of the non-uniform elastic beam can be approximated using an assumed mode method as in Eq. (2).

$$u(x,t)=\sum_{i=1}^{N_1}\phi_i(x)u_i(t)\ ,$$
$$v(x,t)=\sum_{j=1}^{N_2}\psi_j(x)v_j(t) \qquad (2)$$

where $\phi_i(x)$ and $\psi_j(x)$ are sets of approximation functions of axial and transverse deformations, $U_i(t)$ and $V_j(t)$ are unknown time functions of nodal values, and N_1, N_2 denote the number of terms necessary in the approximations. If three coordinates are defined at each node as in Fig. 1, then $N_1=2$ and $N_2=4$.

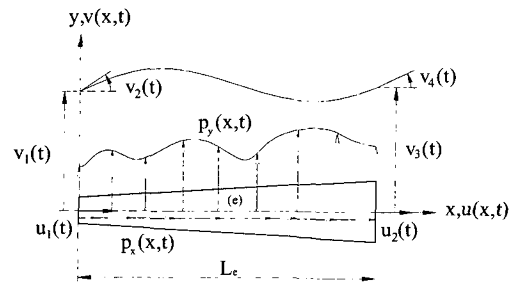

Fig. 1 Finite element of linear non-uniform Bernoulli-Euler beam

Also, shape functions should satisfy the following boundary conditions.

$$\phi_1(0)=1,\phi_1(L_e)=0,\phi_2(0)=0,\phi_2(L_e)=1$$
$$\psi_1(0)=1,\psi_1'(0)=\psi_1(L_e)=\psi_1'(L_e)=0$$
$$\psi_2'(0)=1,\psi_2(0)=\psi_2(L_e)=\psi_2'(L_e)=0$$
$$\psi_3(L_e)=1,\psi_3(0)=\psi_3'(0)=\psi_3'(L_e)=0$$
$$\psi_4'(L_e)=1,\psi_4(0)=\psi_4'(0)=\psi_4(L_e)=0 \qquad (3)$$

Shape functions which satisfy Eq. (1) with boundary conditions of Eq. (3) can be determined as Eq. (4) and (5).

$$\phi_1(x)=1-\frac{x}{L_e}$$
$$\phi_2(x)=\frac{x}{L_e} \qquad (4)$$

$$\psi_1(x)=1-3\left(\frac{x}{L_e}\right)^2+2\left(\frac{x}{L_e}\right)^3$$
$$\psi_2(x)=x-2L_e\left(\frac{x}{L_e}\right)^2+L_e\left(\frac{x}{L_e}\right)^3$$
$$\psi_3(x)=3\left(\frac{x}{L_e}\right)^2-2\left(\frac{x}{L_e}\right)^3$$
$$\psi_4(x)=-L_e\left(\frac{x}{L_e}\right)^2+L_e\left(\frac{x}{L_e}\right)^3 \qquad (5)$$

The strain energy and the kinetic energy of the non-uniform and linear Bernoulli-Euler beam are defined by the following equations.

$$V=\frac{1}{2}\int_0^{L_e}EA(x)(u'(x,t))^2\,dx+\frac{1}{2}\int_0^{L_e}EI(x)(v''(x,t))^2\,dx \qquad (6)$$

$$T = \frac{1}{2}\int_0^{L_e} \rho A(x)(\dot{u}(x,t))^2 dx + \frac{1}{2}\int_0^{L_e} \rho A(x)(\dot{v}(x,t))^2 dx \tag{7}$$

Mass and stiffness coefficients for horizontal and vertical nodal coordinates can be obtained by substituting Eqs. (2), (4) and (5) into Eqs, (6) and (7).

$$^1m_{ij} = \int_0^{L_e} \rho A(x)\phi_i\phi_j dx,\ ^1k_{ij} = \int_0^{L_e} EA(x)\phi_i'\phi_j' dx \quad (i,j=1,\cdots,N_1) \tag{8}$$

$$^2m_{ij} = \int_0^{L_e} \rho A(x)\psi_i\psi_j dx,\ ^2k_{ij} = \int_0^{L_e} EI(x)\psi_i''\psi_j'' dx \quad (i,j=1,\cdots,N_2) \tag{9}$$

where superscripts 1 and 2 denote x and y directions, respectively. If external forces exist, then horizontal and vertical forces can be transformed into the nodal coordinate system using the virtual work theory, as in Eq. (10).

$$\begin{aligned}\delta W &= \int_0^{L_e} p_x(x,t)\delta u(x,t)dx + \int_0^{L_e} p_y(x,t)\delta v(x,t)dx \\ &= \sum_{i=1}^{N_1} {}^1P_i(t)\delta u_i + \sum_{j=1}^{N_2} {}^2P_j(t)\delta v_j\end{aligned} \tag{10}$$

where

$$^1P_i(t) = \int_0^{L_e} p_x(x,t)\phi_i dx,\ ^2P_j(t) = \int_0^{L_e} p_y(x,t)\psi_j dx.$$

The equations of motion for an element in matrix form can be obtained by substituting Eqs. (8), (9) and (10) into the Euler-Lagrange equation.

$$[M]_e\{\ddot{v}\}_e + [K]_e\{v\}_e = \{P\}_e \tag{11}$$

where

$$[M]_e = \begin{bmatrix} ^1m_{11} & 0 & 0 & ^1m_{12} & 0 & 0 \\ 0 & ^2m_{11} & ^2m_{12} & 0 & ^2m_{13} & ^2m_{14} \\ 0 & ^2m_{21} & ^2m_{22} & 0 & ^2m_{23} & ^2m_{24} \\ ^1m_{21} & 0 & 0 & ^1m_{22} & 0 & 0 \\ 0 & ^2m_{31} & ^2m_{32} & 0 & ^2m_{33} & ^2m_{34} \\ 0 & ^2m_{41} & ^2m_{42} & 0 & ^2m_{43} & ^2m_{44} \end{bmatrix}_e,$$

$$[K]_e = \begin{bmatrix} ^1k_{11} & 0 & 0 & ^1k_{12} & 0 & 0 \\ 0 & ^2k_{11} & ^2k_{12} & 0 & ^2k_{13} & ^2k_{14} \\ 0 & ^2k_{21} & ^2k_{22} & 0 & ^2k_{23} & ^2k_{24} \\ ^1k_{21} & 0 & 0 & ^1k_{22} & 0 & 0 \\ 0 & ^2k_{31} & ^2k_{32} & 0 & ^2k_{33} & ^2k_{34} \\ 0 & ^2k_{41} & ^2k_{42} & 0 & ^2k_{43} & ^2k_{44} \end{bmatrix}_e,$$

$$\{v\}_e^T = [u_1\ \ v_1\ \ v_2\ \ u_2\ \ v_3\ \ v_4]_e,$$

$$\{P\}_e^T = [^1P_1\ \ ^2P_1\ \ ^2P_2\ \ ^1P_2\ \ ^2P_3\ \ ^2P_4]_e$$

The equations of motion for the system can be obtained by assembling the element equations.

$$[M]\{\ddot{v}\} + [K]\{v\} = \{P\} \tag{12}$$

Contact force due to impact between the moving system and the beam can be written as the nodal force vector of Eq. (13)

$$\{P\} = p_x(t)\{T(x)\} + p_y(t)\{S(x)\} \tag{13}$$

where {T(x)} and {S(x)} are vector functions for nodal degrees of the freedom of the element i where impact forces $P_x(t)$ and $P_y(t)$ are applied.

$$\{T(x)\}^T = [0, 0, 0, \cdots, \{T(x)\}_i^T, 0, 0, \cdots, 0] \tag{14}$$

$$\{S(x)\}^T = [0, 0, 0, \cdots, \{S(x)\}_i^T, 0, 0, \cdots, 0] \tag{15}$$

where

$$\{T(x)\}_i^T = [\phi_1(x), 0, 0, \phi_2(x), 0, 0]_i$$

$$\{S(x)\}_i^T = [0, \psi_1(x), \psi_2(x), 0, \psi_3(x), \psi_4(x)]_i$$

Boundary conditions are applied to the node depending on the supporting condition between the beam and the foundation.

These boundary conditions must be considered when assembling finite elements. If displacement of a node is constrained, then the system equation of motion, Eq. (12), can be partitioned into constrained degrees of freedom and active degrees of freedom as in Eq. (16).

$$\begin{bmatrix} M_{aa} & M_{ac} \\ M_{ca} & M_{cc} \end{bmatrix}\begin{Bmatrix} \ddot{v}_a \\ \ddot{v}_c \end{Bmatrix} + \begin{bmatrix} K_{aa} & K_{ac} \\ K_{ca} & K_{cc} \end{bmatrix}\begin{Bmatrix} v_a \\ v_c \end{Bmatrix} = \begin{Bmatrix} P_a \\ P_c \end{Bmatrix} \tag{16}$$

Since constrained displacements and accelerations are zeroes, Eq. (16) can be rewritten as Eq. (17) and Eq. (18).

$$[M_{aa}]\{\ddot{v}_a\} + [K_{aa}]\{v_a\} = \{P_a\} \tag{17}$$

$$\{P_c\} = [M_{ca}]\{\ddot{v}_a\} + [K_{ca}]\{v_a\} \tag{18}$$

Reaction forces on the constrained nodes can be obtained from the equations of motion of active degrees of freedom. If forces are applied as boundary conditions, the forces can be transformed to nodal forces and applied to the right side of the equations of motion.

In many cases, the influence of damping upon the response of a system is important. But the damping matrix is difficult to define because the structural damping effect is caused by complicated material properties. For simplicity, the damping matrix can be expressed as a linear combination of the mass and stiffness matrices known as the proportional damping.

$$[C_{aa}] = a[M_{aa}] + b[K_{aa}] \tag{19}$$

Two constants a, b can be defined as [11]

$$a = 2\omega_1\omega_2 \frac{\zeta_1\omega_2 - \zeta_2\omega_1}{\omega_2^2 - \omega_1^2}, \qquad b = 2\frac{\zeta_2\omega_2 - \zeta_1\omega_1}{\omega_2^2 - \omega_1^2} \tag{20}$$

where ω_1, ω_2 are two distinct natural frequencies and ζ_1, ζ_2 are corresponding modal damping ratios. The equations of motion with structural damping can be written as follows

$$[M_{aa}]\{\ddot{v}_a\} + [C_{aa}]\{\dot{v}_a\} + [K_{aa}]\{v_a\} = \{P_a\} \tag{21}$$

3. Various Supporting Conditions

The foundation which supports an elastic structure can be modeled in many ways. In this study, an elastic structure is considered to be as a uniform beam, but a non-uniform beam could be considered also.

3.1 Kinematic conditions for a foundation

Simply supported, damped or cantilever types are used for the kinematic boundary conditions for an elastic structure. These boundary conditions are used to obtain system equations of motion as in Eq. (17). Also, system equations of motion of a more complicated structure, as in multi-span beams, can be obtained using kinematic boundary conditions.

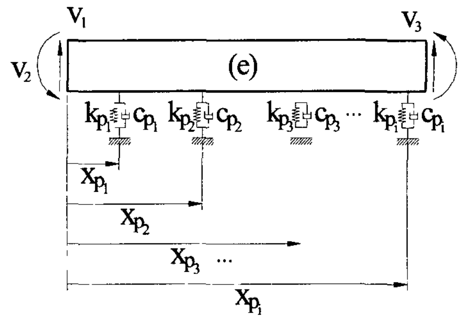

Fig. 2 Beam supported by vertical and discrete foundation

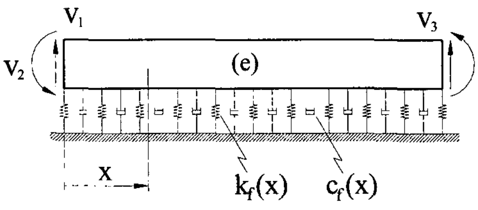

Fig. 3 Beam supported by vertical and continuous foundation

3.2 Vertical and discrete foundation

The vertical and discrete foundation is shown in Fig. 2. The beam is supported by springs and dampers at distance x_{pi} from the left end. The supporting force f at p_i can be calculated using the displacement and velocity at that point.

$$f_{pi} = k_{pi} v(x_{pi}, t) + c_{pi}\dot{v}(x_{pi}, t), \quad i = 1, 2, \cdots, NS \tag{22}$$

where k_{pi} and c_{pi} denote spring and damping constants at the point p_i and NS is the number of supporting foundations. The supporting forces can be transformed to the nodal coordinate system of a beam as follows.

$$\{F_{VD}\}_e = \int_0^{L_e} \sum_{i=1}^{NS} f_{pi}\, {}^D\delta(x - x_{pi}) \{S(x_{pi})\}_e \, dx = \sum_{i=1}^{NS} f_{pi} \{S(x_{pi})\}_e \tag{23}$$

where ${}^D\delta(x)$ denotes the Dirac delta function. If vertical and discrete foundations are continued over the entire beam, then external forces due to this support can be obtained by assembling the finite elements. So vertical and discrete foundations can be considered by adding force terms to the right side of the system equations of motion.

$$[M_{aa}]\{\ddot{v}_a\}+[C_{aa}]\{\dot{v}_a\}+[K_{aa}]\{v_a\}=\{P_a\}+\{F_{VD}\} \tag{24}$$

3.3 Vertical and Continuous foundation.

The vertical and continuous foundation is shown in Fig. 3. The vertical displacement and velocity at the distance x from the left end can be found using the nodal degrees of freedom and the shape function as follows.

$$v(x,t)=\{S(x)\}_e^T\{v\}_e$$
$$\dot{v}(x,t)=\{S(x)\}_e^T\{\dot{v}\}_e \tag{25}$$

The strain energy due to stiffness in the vertical and continuous foundation is

$$U_S=\frac{1}{2}\int_0^{L_e} v^T k_f(x) v\,dx = \frac{1}{2}\{v\}_e^T[K_{VC}]_e\{v\}_e \tag{26}$$

where $k_f(x)$ represents the vertical spring constant. The stiffness matrix caused by the vertical and continuous support can be calculated as Eq. (27).

$$[K_{VC}]_e=\int_0^{L_e}\{S(x)\}_e k_f(x)\{S(x)\}_e^T dx \tag{27}$$

This term should be added to the equations of motion of the beam.

The dissipation energy due to the vertical damping effect can be calculated as Eq. (28).

$$U_C=\frac{1}{2}\int_0^{L_e}\dot{v}^T c_f(x)\dot{v}\,dx = \frac{1}{2}\{\dot{v}\}_e^T[C_{VC}]_e\{\dot{v}\}_e \tag{28}$$

where $c_f(x)$ is the vertical damping coefficient. So the damping matrix which will be added to the beam can be calculated as Eq. (29).

$$[C_{VC}]_e=\int_0^{L_e}\{S(x)\}_e c_f(x)\{S(x)\}_e^T dx \tag{29}$$

The effect of the vertical and continuous support can be ascertained by adding damping and stiffness matrices to the system equations of motion as in Eq. (30).

$$[M_{aa}]\{\ddot{v}_a\}+([C_{aa}]+[C_{VC}])\{\dot{v}_a\}+([K_{aa}]+[K_{VC}])\{v_a\}=\{P_a\} \tag{30}$$

3.4 Longitudinal and discrete foundation

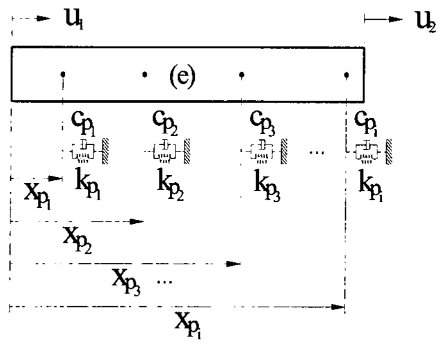

Fig. 4 Beam supported by longitudinal and discrete foundation

The longitudinal and discrete foundation is shown in Fig. 4. The axial force of the support whose distance is x_{pi} from the left end can be determined using the axial displacement and velocity of the supporting point in the beam.

$$g_{pi}=k_{pi}u(x_{pi},t)+c_{pi}\dot{u}(x_{pi},t) \quad i=1,2,\cdots,NA \tag{31}$$

where NA represents the number of supporting points in the beam. The axial force can be transformed to the nodal coordinate system of the beam as in Eq. (32).

$$\{F_{LD}\}_e=\int_0^{L_e}\sum_{i=1}^{NA} g_{pi}{}^D\delta(x-x_{pi})\{T(x_{pi})\}_e dx = \sum_{i=1}^{NA} g_{pi}\{T(x_{pi})\}_e \tag{32}$$

The external force vector due to the longitudinal and discrete foundations can be obtained by assembling the finite elements of the system. Thus the effect of the longitudinal and discrete foundation can be considered by adding the force vector to the right side of the system equations of motion.

$$[M_{aa}]\{\ddot{v}_a\}+[C_{aa}]\{\dot{v}_a\}+[K_{aa}]\{v_a\}=\{P_a\}+\{F_{LD}\} \tag{33}$$

3.5 Longitudinal and continuous foundation

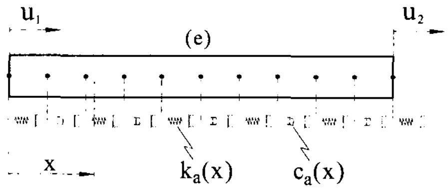

Fig. 5 Beam supported by longitudinal and continuous foundation

The Longitudinal and continuous foundation is shown in Fig. 5. The displacement and velocity at the distance x from the left end can be found using the nodal degrees of freedom and the corresponding shape vector.

$$u(x,t)=\{T(x)\}_e^T\{v\}_e$$

$$\dot{u}(x,t)=\{T(x)\}_e^T\{\dot{v}\}_e \qquad (34)$$

The potential energy of the longitudinal and continuous foundation is

$$U_S=\frac{1}{2}\int_0^{L_e}u^T k_a(x)u\,dx =\frac{1}{2}\{v\}_e^T[K_{LC}]_e\{v\}_e \qquad (35)$$

where $k_a(x)$ denotes the longitudinal spring constant. The stiffness matrix due to the longitudinal and continuous foundation can be calculated as in Eq. (36).

$$[K_{LC}]_e=\int_0^{L_e}\{T(x)\}_e k_a(x)\{T(x)\}_e^T dx \qquad (36)$$

The dissipation energy due to the axial direction damping effect is

$$U_C=\frac{1}{2}\int_0^{L_e}\dot{u}^T c_a(x)\dot{u}\,dx =\frac{1}{2}\{\dot{v}\}_e^T[C_{LC}]_e\{\dot{v}\}_e \qquad (37)$$

where $c_a(x)$ represents the axial direction damping coefficient. The damping matrix acting on the beam can be written as

$$[C_{LC}]_e=\int_0^{L_e}\{T(x)\}_e c_a(x)\{T(x)\}_e^T dx \qquad (38)$$

The effect of the longitudinal and continuous foundation can be achieved by adding the damping and stiffness matrices to the system equations of motion.

$$[M_{aa}]\{\ddot{v}_a\}+([C_{aa}]+[C_{LC}])\{\dot{v}_a\}+([K_{aa}]+[K_{LC}])\{v_a\}=\{P_a\} \qquad (39)$$

3.6 Rotational and discrete foundation

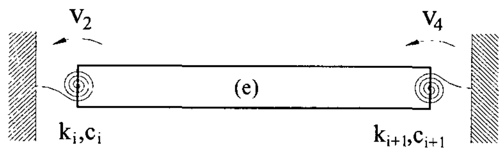

Fig. 6 Beam supported by rotational and discrete foundation

The nodal force vector due to rotational springs and dampers which are connected at both ends of the beam as shown in Fig. 6 can be written as Eq. (40).

$$\{F_{NR}\}_e=[K_{NR}]_e\{v\}_e+[C_{NR}]_e\{\dot{v}\}_e \qquad (40)$$

where $[K_{NR}]_e=diag[0,0,k_i,0,0,k_{i+1}]_e$ and $[C_{NR}]_e=diag[0,0,c_i,0,0,c_{i+1}]_e$. If the same rotational and discrete foundation is applied to the entire system, the nodal force vector can be obtained by assembling the element equations. Thus, the effect of the rotational and discrete foundation can be obtained by adding the corresponding force vector to the right side of the system equations of motion.

$$[M_{aa}]\{\ddot{v}_a\}+[C_{aa}]\{\dot{v}_a\}+[K_{aa}]\{v_a\}=\{P_a\}+\{F_{NR}\} \qquad (41)$$

4. Equations of motion of the moving system

The mechanical system moving on the elastic structure is composed of many parts. The motion of these parts is constrained by the geometrical constraints or dynamic force relationship. Every component of the moving system is considered to be a rigid body.

The equations of motion of the moving system are differential algebraic equations as in Eq. (42) and constraint equations can be written as Eq. (43).

$$\begin{bmatrix} N & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} g \\ \gamma \end{Bmatrix} \tag{42}$$

$$\{\Phi(q,t)\} = \{0\} \tag{43}$$

where [N] is the mass matrix of the rigid body system, {q} is the generalized coordinate vector, {λ} is the Lagrange multiplier vector, {g} is the generalized force vector and [Φ] is the independent kinematic constraint equations for the system. $[\Phi_q]$ is the Jacobian matrix and {γ} is the right hand side of the second derivative of equation (43) as in Eq. (44).

$$\{\gamma\} = -[([\Phi_q]\{\dot{q}\})_q]\{\dot{q}\} - 2[\Phi_{qt}]\{\dot{q}\} - \{\Phi_{tt}\} \tag{44}$$

The generalized force vector includes external forces applied to the rigid body system and forces due to a system like the linear spring-damper-actuator or the rotational spring actuator. Also, nonlinear Hertzian contact force can be used to consider low speed impact between the rigid body and the supporting structure. The forces generated by the impact of two bodies which have arbitary curves is shown in Figures 7 and 8.

The normal force due to contact between two bodies is [22]

$$\{f_{nj}^c\} = \left(k\alpha^{1.5} + c\dot{\alpha}\right)\frac{\{d_{ij}\}}{|\{d_{ij}\}|} = -\{f_{ni}^c\} \tag{45}$$

where $|\{d_{ij}\}|$ is the magnitude of the vector $\{d_{ij}\}$ and k and c are the Hertzian contact stiffness and damping coefficients, respectively, and α is the normal direction penetration distance of the two contact points P_i, P_j. The distance can be calculated as

$$\alpha = (R_i + R_j) - \{d_{ij}\}^T\{d_{ij}\} \tag{46}$$

where $\{d_{ij}\}=\{r_j\}+\{s_j^c\}-\{r_i\}-\{s_i^c\}$ and R_i, R_j are the radii of the curvatures of the instantaneous point of contact. Two bodies are in the impact state when $\alpha > 0$. Two bodies are apart when $\alpha < 0$. If the curvature of one body is a straight line, the radius of curvature becomes infinite. In this case, the penetration distance can be calculated as

$$\alpha = R_j - \{d_{ij}\}^T\{d_{ij}\} \tag{47}$$

The Hertzian contact stiffness is defined using the geometric and physical properties of the contact body as [22],

$$k = \frac{4}{3\pi(h_i + h_j)}\left[\frac{R_i R_j}{R_i + R_j}\right]^{0.5} \tag{48}$$

where

$$h_l = \frac{1-\nu_l^2}{\pi E_l}, \quad l = i, j$$

E_l : Young's Modulus, ν_l : Poisson's ratio.

The tangential force is the result of the friction between two impacting or contacting bodies. The magnitude of the tangential force is a function of the vertical force and the friction coefficient.

$$\{f_{tj}^c\} = \mu_0 \tanh\left(\frac{\dot{\beta}}{\dot{\beta}_\varepsilon}\right)\{f_{ni}^c\}\left[A\left(\frac{\pi}{2}\right)\right]\frac{\{d_{ij}\}}{|\{d_{ij}\}|} = -\{f_{ti}^c\} \tag{49}$$

where μ_0 is the friction coefficient, $\dot{\beta}$ is the tangential velocity, $\dot{\beta}_\varepsilon$ is the transition speed and A is the rotational transformation matrix. Using this impact or contact force between two bodies, the generalized force vector can be calculated.

$$\{g_i^c\} = \begin{Bmatrix} f_{(x)i}^c \\ f_{(y)i}^c \\ -(y_i^P - y_i)f_{(x)i}^c + (x_i^P - x_i)f_{(y)i}^c \end{Bmatrix} = -\{g_j^c\} \tag{50}$$

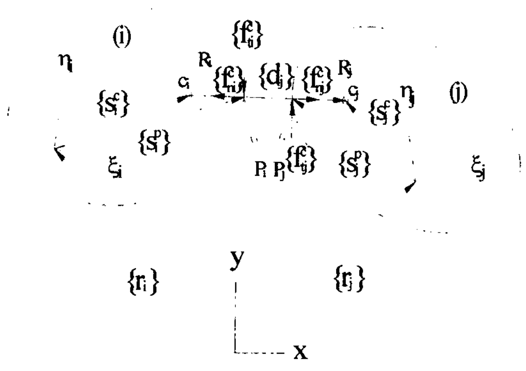

Fig. 7 Arc-arc contact

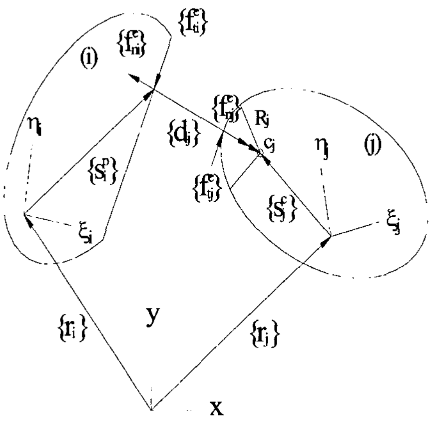

Fig. 8 Arc-line contact

These forces can be added to the force vector in the system equations of motion to represent the Hertzian impact or contact.

5. Combined System Equations of Motion

If a constrained multibody system is moving on the flexible beam which has various types of foundation, the equations of motion of the flexible beam and the multibody system can be written as,

$$[M_{aa}]\{\ddot{v}_a\}+[C]^*\{\dot{v}_a\}+[K]^*\{v_a\}=\{P(q,\dot{q},v_a,\dot{v}_a,t)\}^* \qquad (51)$$

$$\begin{bmatrix} N & \Phi_q^T \\ \Phi_q & 0 \end{bmatrix}\begin{Bmatrix} \ddot{q} \\ \lambda \end{Bmatrix}=\begin{Bmatrix} g(q,\dot{q},v_a,\dot{v}_a,t) \\ \gamma(q,\dot{q},t) \end{Bmatrix} \qquad (52)$$

Where
[C]*=[Caa]+[CVC]+[CLC]
[K]*=[Kaa]+[KVC]+[KLC]
{P}*={Pa}+{FVD}+{FLD}+{FNR}

The generalized force vector {g} and the nodal force vector {P}* are both functions of generalized coordinates and nodal coordinates. Thus Eqs. (51) and (52) can be combined as one system equation.

$$\begin{bmatrix} M_{aa} & 0 & 0 \\ 0 & N & \Phi_q^T \\ 0 & \Phi_q & 0 \end{bmatrix}\begin{Bmatrix} \ddot{v}_a \\ \ddot{q} \\ \lambda \end{Bmatrix}=\begin{Bmatrix} Q \\ g \\ \gamma \end{Bmatrix} \qquad (53)$$

where

$$\{Q\}=\{P\}^*-[C]^*\{\dot{v}_a\}-[K]^*\{v_a\}$$

The generalized coordinates of Eq. (53) must satisfy the constraint equations of Eq. (43).

To solve combined differential algebraic equations, the generalized coordinate partitioning method [23] which is regarded as the most stable solution method for DAE, can be used. The generalized coordinates are partitioned into independent and dependent coordinates. The independent generalized coordinates and nodal coordinates are integrated using the multistep integration method and the dependent generalized coordinates can be obtained by solving the constraint equation of Eq. (43).

6. Conclusions

A method analyzing a mechanical system which moves on the non-uniform linear Bernoulli-Euler beam is presented in this paper. Hamilton's principle is applied to derive vertical and horizontal equations of motion for the beam. Various supporting conditions are considered for the foundation of the beam. The effect of the various supporting conditions are considered in the equations of motion for the beam.

The equations of motion for the beam and the system differential algebraic equations of the moving mechanical system are combined. The combined system equations of motion will enable us to analyze the mechanical system which moves on the elastic beam. The vertical and horizontal motion can be analyzed with this system. Also, reaction forces between the moving system and the elastic beam can be calculated. And finally, reaction forces between interconnected parts in the moving system can be obtained, which could be used to estimate the life cycle of the parts.

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