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S-procedure를 이용한 상태에 시변 시간지연을 가지는 이산 선형 시스템에 대한 H_∞ 제어기 설계

(H_∞ Controller Design for Discrete-time Linear Systems with Time-varying Delays in States using S-procedure)

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요 약

본 논문에서는 상태에 시변 시간지연을 가지는 이산 선형 시스템에 대한 H_∞ 제어기 설계문제를 다룬다. H_∞ 제어기가 존재할 충분조건과 설계방법에 대해 논의한다. 본 논문에서 다루는 H_∞ 제어기법은 상태궤환 제어기로서 시변 시간지연의 상한값과 S-procedure를 이용한다. 또한, 변수 치환, Schur 여수정리 등을 이용하여 충분조건을 모든 변수에 대한 선형 행렬 부등식(linear matrix inequality)으로 표현한다.

Abstract

This paper deals with the H_∞ control problems for discrete-time linear systems with time-varying delays in states. The existence condition and the design method of the H_∞ state feedback controller are given. In this paper, the H_∞ control law is assumed to be a memoryless state feedback, and the upper-bound of time-varying delay and S-procedure are used. Through some changes of variables and Schur complement, the obtained sufficient condition can be rewritten as an LMI(linear matrix inequality) form in terms of all variables.

Key Word : discrete-time linear system, time-varying delays, H_∞ control, LMI

I. Introduction

Recently the time delay is main concerns because time delays often are the causes for instability and poor performance of control systems. Since some works of H_∞ controller design methods have been developed, many H_∞ state feedback controller design algorithms of time delays systems were presented. But many related works treated the H_∞ state feedback controller design algorithms in continuous-time case only[1-4]. And most of the results are related to the systems with constant time delays and delay independent[1, 3]. Since the sizes of time delays are allowed to be arbitrary large in the delay independent case, increasing attention has been paid to time-varying delays and the delay dependent analysis[2, 4]. Therefore our objective is to find solutions at a time using LMI technique in discrete

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time-varying delays systems.

Little attention has been paid to discrete-time linear systems with time delays. Because they can be transformed into the systems with no time delays via state augmentations, the control problems for discrete-time linear systems with time delays can be solved by using the control theory of discrete-time linear systems[5]. However, in case of discrete-time linear systems with time-varying delays, the existing theory cannot be directly applied because the state augmentation approach cannot be simply applied to them due to time-varying delays. In order to solve this problems, Song et al.[6] dealt with the H_∞ control problems for discrete-time linear systems with time-varying delays in state. They showed the sufficient condition for the H_∞ control and proposed a suitable control law. However, they did not considered time-varying delays in the controlled output. And Kim et al.[7] considered the H_∞ control problem for discrete time-varying delays systems in another approach. Therefore, we deal with discrete-time linear systems with time-varying delays in states. Especially, we solve the H_∞ control problem for these systems using the upper-bound of time-varying delay and S-procedure.

In this paper, we propose the H_∞ state feedback controller design algorithm of discrete-time linear systems with time-varying delays in states. The existence condition and the design method of H_∞ state feedback controller are given. The H_∞ control law is assumed to be a memoryless state feedback, and the upper-bound of time-varying delay and S-procedure are used. Through some changes of variables and Schur complement, the obtained sufficient condition can be rewritten as an LMI form in terms of all variables. Using the LMI toolbox, the solutions can be easily obtained at a time[8].

The notations in this paper are quite standard. $\boldsymbol{R}$, $\boldsymbol{R}^n$, and $\boldsymbol{R}^{n\times m}$ denote, respectively, the set of integer numbers, the set of the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript " T " denotes the matrix transpose and the notation $X\geq Y$(respectively, $X>Y$) where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. $l_2[0,\infty]$ is the space of square summable vector sequence over $[0,\infty]$.

Ⅱ. H_∞ Performance Analysis

Consider the discrete-time linear system with time-varying delays in states described by the difference delay equation

$$\begin{aligned} x(k+1) &= Ax(k)+A_d x(k-d(k))+Bw(k), \\ z(k) &= Cx(k)+C_d x(k-d(k))+Dw(k), \\ x(k) &= 0, \quad \forall\ k\leq 0, \end{aligned} \tag{1}$$

where $x(k)\in \boldsymbol{R}^n$ is the state, $w(k)\in \boldsymbol{R}^l$ is the exogenous input, which belongs to $l_2[0,\infty]$, and $z(k)\in \boldsymbol{R}^p$ is the controlled output. All matrices have appropriate dimensions and we assume that all states are measurable for state feedback. the time-varying delay $d(k)\in \boldsymbol{R}$ is the positive integer term satisfying

$$0<d(k)\leq m, \quad \forall\ k\geq 0. \tag{2}$$

We discuss about Schur complement used in this paper. One of the basic ideas of LMI problem is that nonlinear (convex) inequalities are converted to LMI form using Schur complement.

Lemma 1[9] : For the symmetric matrix $L=\begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & L_{22}\end{bmatrix}$, the following are equivalent as follows:

$$\begin{aligned} &\text{i) } L<0, \\ &\text{ii) } L_{11}<0, \quad L_{22}-L_{12}^T L_{11}^{-1} L_{12}<0, \\ &\text{iii) } L_{22}<0, \quad L_{11}-L_{12}L_{22}^{-1}L_{12}^T<0. \end{aligned} \tag{3}$$

■

We will often encounter the constraint that some quadratic function be negative whenever some other quadratic functions are all negative. In this paper, we will use this S-procedure.

Lemma 2[9] : Let F_0, ..., F_p be quadratic functions of the variable $\zeta \in \boldsymbol{R}^n$:

$$F_i(\zeta) := \zeta^T T_i \zeta + 2u_i^T \zeta + v_i, \quad i = 0, \ldots, p, \qquad (4)$$

where $T_i = T_i^T$. We consider the following condition on F_0, ..., F_p:

$$F_0(\zeta) \geq 0 \ \text{ for all } \zeta \text{ such that } F_i(\zeta) \geq 0, \ i = 1, \ldots, p \qquad (5)$$

Obviously if there exist $\tau_1 \geq 0$, ..., $\tau_p \geq 0$ such that for all ζ,

$$F_0(\zeta) - \sum_{i=1}^{p} \tau_i F_i(\zeta) \geq 0, \qquad (6)$$

then (5) holds. It is a nontrivial fact that when $p = 1$, the converse holds, provided that there is some ζ_0 such that $F_1(\zeta_0) > 0$.

■

In here, we will show that the system (1) is quadratically stable with an H_∞ norm bound γ. We introduce the Lyapunov functional (7) such that the discrete time-varying delays system (1) is quadratically stable.

$$V(x, k) = x(k)^T P x(k) + \sum_{i=1}^{m} \sum_{j=k-i}^{k-1} x(j)^T R x(j), \qquad (7)$$

where P and R are positive definite matrices.

Lemma 3 : For given $m > 0$, $\gamma > 0$, and $\tau > 0$, the system (1) is quadratically stable with H_∞ norm bound γ if there exist positive definite matrices P and R such that

$$\begin{bmatrix} A^TPA + C^TC - P + mR + \tau R & A^TPA_d + C^TC_d & A^TPB + C^TD & 0 & 0 & \cdots & 0 & 0 \\ * & A_d^TPA_d + C_d^TC_d - \tau R & A_d^TPB + C_d^TD & 0 & 0 & \cdots & 0 & 0 \\ * & * & B^TPB + D^TD - \gamma^2 I & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & -R + \tau R & 0 & \cdots & 0 & 0 \\ * & * & * & * & -R + \tau R & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & * & \cdots & -R + \tau R & 0 \\ * & * & * & * & * & \cdots & * & -R + \tau R \end{bmatrix} < 0, \qquad (8)$$

holds for the time-varying delay (2).

Proof : Firstly, we define a Lyapunov functional as (7). Taking the difference of the Lyapunov functional (7) yields

$$\begin{aligned} \Delta V(x, k) &= V(x, k+1) - V(x, k) \\ &= x(k+1)^T P x(k+1) + \\ &\quad \sum_{i=1}^{m} \sum_{j=k+1-i}^{k} x(j)^T R x(j) - x(k)^T P x(k) \\ &\quad - \sum_{i=1}^{m} \sum_{j=k-i}^{k-1} x(j)^T R x(j). \end{aligned} \qquad (9)$$

When assuming zero input, we have

$$\Delta V(x, k) = \begin{bmatrix} x(k) \\ x(k-d(k)) \\ x(k-1) \\ x(k-2) \\ \vdots \\ x(k-m+1) \\ x(k-m) \end{bmatrix}^T \begin{bmatrix} A^TPA - P + mR & A^TPA_d & 0 & 0 & \cdots & 0 & 0 \\ * & A_d^TPA_d & 0 & 0 & \cdots & 0 & 0 \\ * & * & -R & 0 & \cdots & 0 & 0 \\ * & * & * & -R & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & \cdots & -R & 0 \\ * & * & * & * & \cdots & * & -R \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-d(k)) \\ x(k-1) \\ x(k-2) \\ \vdots \\ x(k-m+1) \\ x(k-m) \end{bmatrix} < 0, \qquad (10)$$

where (10) is not an LMI form in terms of all variables. In order to make (10) to LMI, we use the upper-bound of time-varying delay (2), i.e. the fact that the time-varying delay $d(t)$ is less than or equal to the upper-bound of time-varying delay, m. Therefore, in addition to Lemma 2, (10) is converted as follows:

$$\Delta V(x,\ k)$$

$$= \begin{bmatrix} x(k) \\ x(k-d(k)) \\ x(k-1) \\ x(k-2) \\ \vdots \\ x(k-m+1) \\ x(k-m) \end{bmatrix}^T \begin{bmatrix} A^TPA-P+mR+\tau R & A^TPA_d & 0 & 0 & \cdots & 0 & 0 \\ * & A_d^TPA_d-\tau R & 0 & 0 & \cdots & 0 & 0 \\ * & * & -R+\tau R & 0 & \cdots & 0 & 0 \\ * & * & * & -R+\tau R & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & \cdots & -R+\tau R & 0 \\ * & * & * & * & \cdots & * & -R+\tau R \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-d(k)) \\ x(k-1) \\ x(k-2) \\ \vdots \\ x(k-m+1) \\ x(k-m) \end{bmatrix} < 0, \tag{11}$$

where ensures the quadratic stability of the closed loop system (1). In the next place, assume the zero initial condition and introduce

$$J = \sum_{k=0}^{\infty} [z(k)^T z(k) - \gamma^2 w(k)^T w(k)]. \tag{12}$$

Noting

$$J \le \sum_{k=0}^{\infty} [z(k)^T z(k) - \gamma^2 w(k)^T w(k) + \Delta V(x,\ k)], \tag{13}$$

and further substituting (11) into (13) and let

$$\begin{aligned} \xi(k) &= [x(k)^T\ x(k-d(k))^T\ w(k)^T\ x(k-1)^T \\ &\quad x(k-2)^T \cdots x(k-m+1)^T\ x(k-m)^T]^T, \end{aligned} \tag{14}$$

then

$$J \le \sum_{k=0}^{\infty} \xi(k)^T Z \xi(k), \tag{15}$$

where Z is defined

$$Z = \begin{bmatrix} A^TPA+C^TC-P+mR+\tau R & A^TPA_d+C^TC_d & A^TPB+C^TD & 0 & 0 & \cdots & 0 & 0 \\ * & A_d^TPA_d+C_d^TC_d-\tau R & A_d^TPB+C_d^TD & 0 & 0 & \cdots & 0 & 0 \\ * & * & B^TPB+D^TD-\gamma^2 I & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & -R+\tau R & 0 & \cdots & 0 & 0 \\ * & * & * & * & -R+\tau R & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & * & \cdots & -R+\tau R & 0 \\ * & * & * & * & * & \cdots & * & -R+\tau R \end{bmatrix}, \tag{16}$$

where $*$ means the symmetric term. Therefore, when $Z<0$, $k\ge 0$, the system (1) is quadratically stable with an H_∞ norm bound γ.

■

In Lemma 3, the sufficient condition for the H_∞ control problem of discrete-time linear systems with time-varying delays in states is presented. From this results, we can solve the problem designing the linear memoryless state feedback control law such that the resulting closed loop system is quadratically stable for the time-varying delay, and the H_∞ norm of the closed loop system is bounded by given value γ. Therefore, we will design the H_∞ state feedback controller using some changes of variables, Schur complement, and S-procedure.

Ⅲ. H_∞ State Feedback Control

Consider the discrete-time linear system with time-varying delays in states described by the difference delay equation

$$\begin{aligned} x(k+1) &= Ax(k) + Ax(k-d(k)) + B_w w(k) + B_u u(k), \\ z(k) &= Cx(k) + C_d x(k-d(k)) + D_w w(k) + D_u u(k), \\ x(k) &= 0, \quad \forall\ k \le 0, \end{aligned} \tag{17}$$

where $x(k) \in \boldsymbol{R}^n$ is the state, $u(k) \in \boldsymbol{R}^m$ is the control input, $w(k) \in \boldsymbol{R}^l$ is the exogenous input, which belongs to $l_2[0,\ \infty]$, and $z(k) \in \boldsymbol{R}^p$ is the controlled output. All matrices have appropriate dimensions and we assume that all states are measurable for state feedback. the time-varying delay $d(k) \in \boldsymbol{R}$ is the positive integer term satisfying

(2).

We introduce the linear memoryless state feedback *control law*

$$u(k) = Kx(k) \tag{18}$$

such that the resulting closed loop system is quadratically stable for time-varying delay, and the H_∞ norm of the closed loop system is bounded by given value γ.

When we apply the control (18) to the discrete time-varying delays system (17), the closed loop system from $w(k)$ to $z(k)$ is given by

$$\begin{aligned} x(k+1) &= A_K x(k) + A_d x(k-d(k)) + B_w w(k), \\ z(k) &= C_K x(k) + C_d x(k-d(k)) + D_w w(k), \end{aligned} \tag{19}$$

where

$$\begin{aligned} A_K &= A + B_u K, \\ C_K &= C + D_u K. \end{aligned} \tag{20}$$

Therefore, we discuss *the sufficient condition* such that the closed loop system is quadratically stable for time-varying delay, and the H_∞ norm of the closed loop system is bounded by given value γ.

Theorem 1 : Consider the discrete time-varying delay system (19). For given $m>0$, $\gamma>0$, and $\tau>0$, if there exist positive definite matrices Q, S, and matrix M such that

$$\begin{bmatrix} -Q & Q & Q & 0 & 0 & QA^T+M^TB_u^T & QC^T+M^TD_u^T & 0 & 0 \\ * & -m^{-1}S & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\tau^{-1}S & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\tau^{-1}S & 0 & \tau^{-1}SA_d^T & \tau^{-1}SC_d^T & 0 & 0 \\ * & * & * & * & -\gamma^2 I & B_w^T & D_w^T & 0 & 0 \\ * & * & * & * & * & -Q & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -\Phi_m & \Phi_m \\ * & * & * & * & * & * & * & * & -\tau^{-1}\Phi_m \end{bmatrix} < 0, \tag{21}$$

holds for the time-varying delay (2), then the closed loop system (19) is quadratically stable with an H_∞ norm bound γ. In here, some variables are defined as follows:

$$\begin{aligned} Q &= P^{-1}, \\ S &= R^{-1}, \\ M &= KP^{-1}, \\ \Phi_m &= diag\{S,\ S,\ \cdots,\ S,\ S\}, \end{aligned} \tag{22}$$

where the dimension of Φ_m is m(the upper bound of time-varying delay) of the dimension of S.

Proof : Firstly, we define a Lyapunov functional as (7). Taking the difference of the Lyapunov functional (7) yields (9).

When assuming zero input, we have

$$\Delta V(x,\ k)$$

$$= \begin{bmatrix} x(k) \\ x(k-d(k)) \\ x(k-1) \\ x(k-2) \\ \vdots \\ x(k-m+1) \\ x(k-m) \end{bmatrix}^T 0, \begin{bmatrix} A_K^TPA_K-P+mR+\tau R & A_K^TPA_d 0 & 0 & 0 & \cdots & 0 & 0 \\ * & A_d^TPA_d-\tau R & 0 & 0 & \cdots & 0 & 0 \\ * & * & -R+\tau R & 0 & \cdots & 0 & 0 \\ * & * & * & -R+\tau R & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & \cdots & -R+\tau R & 0 \\ * & * & * & * & \cdots & * & -R+\tau R \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-d(k)) \\ x(k-1) \\ x(k-2) \\ \vdots \\ x(k-m+1) \\ x(k-m) \end{bmatrix} < 0, \tag{23}$$

which ensures the quadratic stability of the closed loop system (19). In the next place, assume the zero initial condition and introduce

$$J = \sum_{k=0}^{\infty} [z(k)^T z(k) - \gamma^2 w(k)^T w(k)]. \tag{24}$$

Noting

$$J \le \sum_{k=0}^{\infty} [z(k)^T z(k) - \gamma^2 w(k)^T w(k) + \Delta V(x,\ k)], \tag{25}$$

and further substituting (23) into (52) and let

$$\xi(k)=[x(k)^T\ x(k-d(k))^T\ w(k)^T\ x(k-1)^T\ x(k-2)^T \cdots x(k-m+1)^T\ x(k-m)^T]^T, \quad (26)$$

then

$$J \le \sum_{k=0}^{\infty} \xi(k)^T Z \xi(k), \quad (27)$$

where Z is defined

$$Z=\begin{bmatrix} A_K^TPA_K+C_K^TC_K-P+mR+\tau R & A_K^TPA_d+C_K^TC_d & A_K^TPB_w+C_K^TD_w & 0 & 0 & \cdots & 0 & 0 \\ * & A_d^TPA_d+C_d^TC_d-\tau R & A_d^TPB_w+C_d^TD_w & 0 & 0 & \cdots & 0 & 0 \\ * & * & B_w^TPB_w+D_w^TD_w-\gamma^2 I & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & -R+\tau R & 0 & \cdots & 0 & 0 \\ * & * & * & * & -R+\tau R & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & * & \cdots & -R+\tau R & 0 \\ * & * & * & * & * & \cdots & * & -R+\tau R \end{bmatrix}. \quad (28)$$

where $*$ means the symmetric term. Therefore, when $Z<0$, $k\ge 0$, the system (19) is quadratically stable with an H_∞ norm bound γ.

Using the lemma 1, $Z<0$ in (28) is transformed as follows:

$$\begin{bmatrix} -P+mR+\tau R & 0 & 0 & A_K^T & C_K^T & 0 & 0 & \cdots & 0 & 0 \\ * & -\tau R & 0 & A_d^T & C_d^T & 0 & 0 & \cdots & 0 & 0 \\ * & * & -\gamma^2 I & B_w^T & D_w^T & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & -P^{-1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & -R+\tau R & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & * & -R+\tau R & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & * & * & * & \cdots & -R+\tau R & 0 \\ * & * & * & * & * & * & * & \cdots & * & -R+\tau R \end{bmatrix} < 0. \quad (29)$$

Using some changes of variables and nonsingular matrices, the proof is completed. the inequality (29) is equivalent to

$$\begin{bmatrix} -P^{-1} & P^{-1} & P^{-1} & 0 & 0 & P^{-1}A^T+P^{-1}K^TB_u^T & P^{-1}C^T+P^{-1}K^TD_u^T & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & -m^{-1}R^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & -\tau^{-1}R^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & -\tau^{-1}R^{-1} & 0 & \tau^{-1}R^{-1}A_d^T & \tau^{-1}R^{-1}C_d^T & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & * & -\gamma^2 I & B_w^T & D_w^T & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & -P^{-1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & * & * & -R^{-1} & 0 & \cdots & 0 & 0 & R^{-1} & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & * & * & * & -R^{-1} & \cdots & 0 & 0 & 0 & R^{-1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & * & * & * & * & * & \cdots & -R^{-1} & 0 & 0 & 0 & \cdots & R^{-1} & 0 \\ * & * & * & * & * & * & * & * & * & \cdots & * & -R^{-1} & 0 & 0 & \cdots & 0 & R^{-1} \\ * & * & * & * & * & * & * & * & * & \cdots & * & * & -\tau^{-1}R^{-1} & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \cdots & * & * & * & -\tau^{-1}R^{-1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & * & * & * & * & * & \cdots & * & * & * & * & \cdots & -\tau^{-1}R^{-1} & 0 \\ * & * & * & * & * & * & * & * & * & \cdots & * & * & * & * & \cdots & * & -\tau^{-1}R^{-1} \end{bmatrix} < 0. \quad (30)$$

Using some changes of variables, (30) is changed to (21).

■

(21) is an LMI form in terms of Q, S, and M.

Therefore, the H_∞ state feedback controller gain K can be calculated from the $M=KP^{-1}$ after finding the LMI solution, Q, wS, and M from the (21) and (22). Using the LMI Toolbox, the solutions can be easily obtained at a time because (21) is an LMI form in terms of all variables[8].

Ⅳ. Example

Consider the discrete time-varying delays system

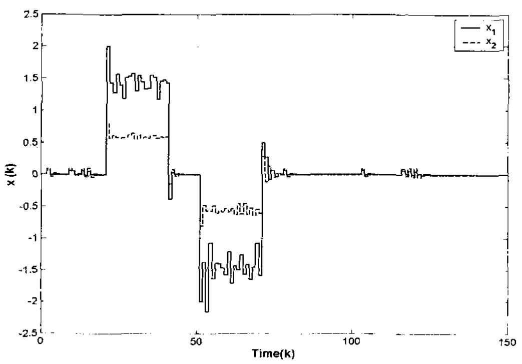
$$\begin{aligned} x(k+1) &= \begin{bmatrix} -0.4 & 1 \\ 0 & 0.9 \end{bmatrix} x(k) + \begin{bmatrix} -0.04 & 0.1 \\ 0 & 0.09 \end{bmatrix} \\ &\quad x(k-d(k)) + \begin{bmatrix} 1 \\ 0.4 \end{bmatrix} w(k) + \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix} u(k), \\ z(k) &= [1 \ \ 6]\, x(k) + [0.1 \ \ 0.6]\, x(k-d(k)) \\ &\quad + 2w(k) + u(k), \end{aligned} \tag{31}$$

We take $m=5$, $\gamma=5$, and $\tau=0.99$. Using the LMI toolbox, all solutions are obtained at the same time as follows[8]:

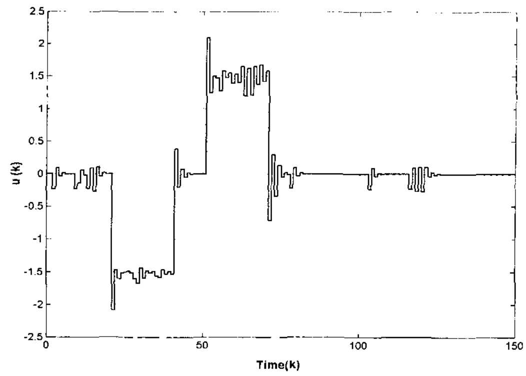
$$\begin{aligned} P &= \begin{bmatrix} 2.3004 & 5.8808 \\ 5.8808 & 37.5332 \end{bmatrix}, \\ R &= \begin{bmatrix} 0.0580 & 0.08988 \\ 0.0898 & 2.5734 \end{bmatrix}, \\ M &= [0.2807 \ \ -0.1125]. \end{aligned} \tag{32}$$

Therefore, the final H_∞ state feedback controller gain is obtained as

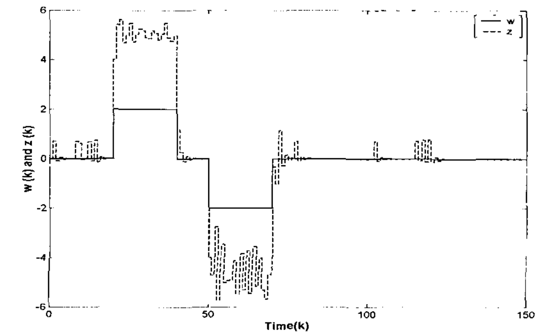
The simulation results are shown in Fig. 1. The trajectories of states converge to zero as time goes to infinity in (a) of Fig. 1. From this result, the obtained controller stabilizes the discrete-time linear



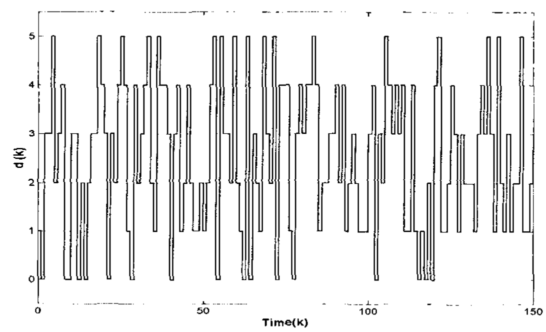
(a) $x_1(k)$ and $x_2(k)$



(b) $u(k)$



(c) $w(k)$ and $z(k)$



(d) $d(k)$

그림 1. 상태, 제어 입력, 외부 입력, 제어될 출력, 시변 시간지연의 궤적

Fig. 1. The trajectories of states, control input, exogenous input, controlled output, and time-varying delay.

$$K = [-0.0159 \ \ -2.5720]. \tag{33}$$

system against the time-varying delay and the exogenous input. And the trajectory of the control input $u(k)$ is shown in (b) of Fig. 1. Also the H_∞

norm bound of the closed loop system can be calculated by the induced norm property between $w(k)$ and $z(k)$. Therefore, we investigate that the value of γ is less than given value($=5$) in (c) of Fig. 1. Here, the initial values of states are zero and the time-varying delay $d(k)$ is shown in (d) of Fig. 1. And the value of $w(k)$ is defined by

$$w(k)=\begin{cases}2, & \text{if } 20\le k\le 40,\\ -2, & \text{if } 50\le k\le 70,\\ 0, & \text{otherwise}.\end{cases} \tag{34}$$

V. Conclusion

In this paper, we presented the sufficient condition for the H_∞ control problem of discrete-time linear systems with time-varying delays in states and proposed the design method of the H_∞ state feedback controller of discrete-time linear systems with time-varying delays in states. The discrete time-varying delays system problems were solved on the basis of LMI technique. Therefore the H_∞ state feedback controller was obtained and the obtained controller guaranteed the quadratic stability and the H_∞ norm bound γ of the closed loop system. Whenever the parameter uncertainties exist in the discrete time-varying delays systems, the approach proposed in this paper can be easily applied to solve them.

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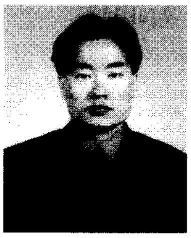

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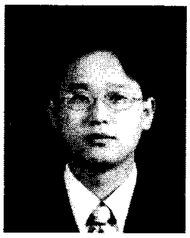

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