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HÖLDER ESTIMATES FOR $\bar\partial$ IN ANALYTIC POLYHEDRA

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ABSTRACT. We consider Hölder estimates for $\bar\partial$ in analytic polyhedra. In the case of dimension 2, it preserves exact Hölder regularity, and it maps bounded $(0,1)$-forms into BMO with respect to volume measure.

1. Introduction and statement of results

A bounded domain $\Omega \subset \mathbb{C}^n$ is an analytic polyhedron with defining functions ϕ_j if

$$\Omega = \{z \in \mathbb{C}^n;\ |\phi_j(z)| < 1, j = 1, \dots, N\},$$

where the defining functions ϕ_j are holomorphic in some neighborhood $\widetilde\Omega$ of $\overline\Omega$. For a multiindex $I \subset \{1, \dots, N\}$ we let $\sigma_I = \{z \in \overline\Omega;\ |\phi_j(z)| = 1,\ j \in I\}$. We say that Ω is non-degenerate if $\partial\phi_{I_1} \wedge \cdots \wedge \partial\phi_{I_k} \neq 0$ on σ_I for every multiindex I. In particular, this means that no more than n of the functions ϕ_j can have modulii 1 at the same point. The polydisk D^n in $\mathbb{C}^n$ is a non-degenerate analytic polyhedron with n defining functions. It is easy to see that Ω being non-degenerate is equivalent to that Ω is locally biholomorphic to a part of the polydisk D^n. In [2], Andersson provided a weighted variant of the solution operator for $\overline\partial$ in any analytic polyhedron, and he obtained L^p estimates for $\overline\partial$. In this paper we consider Hölder estimates for $\bar\partial$ in analytic polyhedra.

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THEOREM 1.1. *Let $\Omega \subset \mathbb{C}^n$ be a non-degenerate analytic polyhedron. Let $0 < \alpha < 1$. Then there is an integral solution operator $S : C_{0,1}(\overline{\Omega}) \to C(\Omega)$ for $\overline{\partial}$ such that*

$$|Sf|_{\Lambda_{\alpha-\epsilon}(\Omega)} \leq C_{\alpha,\epsilon}|f|_{\Lambda_\alpha(\Omega)} \quad \text{for all } f \text{ with } \overline{\partial}f = 0 \text{ and } 0 < \epsilon < \alpha.$$

THEOREM 1.2. *If $n = 2$ in Theorem 1, then for $0 < \alpha < 1$ we have*
(i) $|Sf|_{\Lambda_\alpha(\Omega)} \leq C_\alpha |f|_{\Lambda_\alpha(\Omega)}$ *for all f with $\overline{\partial}f = 0$,*
(ii) $\|Sf\|_{BMO(\Omega)} \leq C|f|_{L^\infty(\Omega)}$.

In [5], Henkin and Sergeev proved uniform estimates for $\overline{\partial}$ in the strictly pseudoconvex polyhedron. Range-Siu [8] proved Hölder estimates for the $\bar{\partial}$-equation on piecewise smooth strictly pseudoconvex domains. Menini [6] proved L^p estimates for $\bar{\partial}$ on piecewise smooth strictly pseudoconvex domains.

From Theorems 1.1 and 1.2 we can see that for $n > 2$ there is a small loss $\epsilon > 0$ of regularity as in the piecewise smooth strictly pseudoconvex case ([8]). It seems that the loss cannot be removed by our method. However, if $n = 2$, we can get the optimal result.

2. Construction of the solution operator for $\overline{\partial}$

Let $\phi_j^k(\zeta, z)$ be holomorphic functions in $\widetilde{\Omega} \times \widetilde{\Omega}$ such that

$$\phi_j(\zeta) - \phi_j(z) = \sum_{k=1}^{n} \phi_j^k(\zeta, z)(\zeta_k - z_k),$$

ϕ_j^k are so-called Hefer functions to ϕ_j, and define the $(1,0)$-forms $\Phi_j = \sum_{k=1}^n \phi_j^k d\zeta_k$. Let $\beta(\zeta, z) = |\zeta - z|^2$. For any $r > 0$ we can define a kernel

(2.1)
$$K^r(\zeta, z) = \sum_{\nu=1}^{n} C_\nu \sum_{|I|=n-\nu}{}' \frac{\partial_\zeta \beta \wedge (\overline{\partial}_\zeta \partial_\zeta \beta)^{n-1}}{\beta^\nu} \prod_{j \notin I} \left(\frac{1 - |\phi_j(\zeta)|^2}{1 - \overline{\phi_j(\zeta)}\phi_j(z)} \right)^r \bigwedge_{j \in I} \frac{-r(1 - |\phi_j(\zeta)|^2)^{r-1}}{(1 - \overline{\phi_j(\zeta)}\phi_j(z))^{r+1}} \overline{\partial \phi_j} \wedge \Phi_j,$$

which induces a solution operator

$$S^r f(z) = \int_{\zeta \in \Omega} f \wedge K^r(\zeta, z), \quad z \in \Omega \tag{2.2}$$

such that $\overline{\partial}(S^r f) = f$ for a $\overline{\partial}$-closed (0,1)-form f (see [2]).

THEOREM 2.1. *Let $r \to 0$ in (2.2) and then we obtain the solution operator for $\bar{\partial}$*

(2.3)

$$Sf(z) = C\int_{\zeta\in\Omega} f(\zeta)\wedge\frac{\omega_0(\zeta,z)}{\beta^n} + \sum_{\nu=1}^{n-1} C_\nu \sum_{|I|=n-\nu}' \int_{\zeta\in\sigma_I} f(\zeta)\wedge\frac{\omega_I(\zeta,z)}{\beta^\nu\prod_{j\in I}(\phi_j(\zeta)-\phi_j(z))}, \quad z\in\Omega,$$

where ω_I are smooth $(n,\nu-1)$-forms such that $\omega_I(\zeta,z)=\mathcal{O}(|\zeta-z|)$.

PROOF. We consider a fixed term $K^r_{I,\nu}$ in (2.1). Let $1\le\nu<n$. Without loss of generality we may assume that $I=(1,\dots,n-\nu)$. Let ζ^0 be a fixed point on $\overline{\Omega}$. We may assume that $|\phi_j(\zeta^0)|=1$ for say $j\le l$ and $|\phi_j(\zeta^0)|<1$ for $l<j\le n-\nu$. Then, by the assumption on Ω, $\partial\phi_1\wedge\cdots\wedge\partial\phi_l\neq 0$ at ζ^0. Therefore, we can choose a local holomorphic coordinate system ξ at ζ^0 such that $\xi_j=\phi_j$ for $1\le j\le l$. There is a smooth $(n,n-l-1)$-form ω such that $\omega=\mathcal{O}(|\zeta-z|)$ and

$$\int_\Omega \chi\partial_\zeta\beta\wedge(\bar{\partial}_\zeta\partial_\zeta\beta)^{\nu-1}\bigwedge_1^{n-\nu}\overline{\partial\phi_j(\zeta)}\wedge\Phi_j = \int_\Omega\chi\bigwedge_1^l\overline{\partial\phi_j}\wedge\omega$$

for all cutoff functions χ with support near ζ^0. Note that

$$\lim_{r\to 0}\prod_{j\in I}\frac{1}{(1-\overline{\phi_j(\zeta)}\phi_j(z))^r}\prod_{j\notin I}\left(\frac{1-|\phi_j(\zeta)|^2}{1-\overline{\phi_j(\zeta)}\phi_j(z)}\right)^r = 1$$

and

$$\frac{-r(1-|\phi_j(\zeta)|^2)^{r-1}}{1-\overline{\phi_j(\zeta)}\phi_j(z)}\overline{\partial\phi_j} = \frac{\bar{\partial}(1-|\phi_j(\zeta)|^2)^r}{\phi_j(\zeta)-|\phi_j(\zeta)|^2\phi_j(z)}.$$

Thus we have

$$\text{(2.4)}\quad \int_\Omega\chi f\wedge K^r_{I,\nu} = \int_\Omega f\wedge\frac{\omega_I\chi}{\beta^\nu}(1+o(1))\mathcal{O}(r^{n-\nu-l})\bigwedge_1^l\frac{\bar{\partial}(1-|\xi_j|^2)^r}{\xi_j-|\xi_j|^2\phi_j(z)}$$

for χ with support near ζ^0. If $\zeta^0\in\sigma_I$ (i.e., $l=n-\nu$), then this integral tends to

$$\text{(2.5)}\qquad \int_{\sigma_I} f\wedge\frac{\omega_I\chi}{\beta^\nu\prod_{j=1}^l(\xi_j-\phi_j(z))}$$

when $r \to 0$ (see Remark 2.2). If ζ^0 is outside σ_I, then (2.4) tends to zero when $r \to 0$. The various $(n, \nu - 1)$-form ω, corresponding to points on σ_I can be pieced together to a global form ω_I defined in a neighborhood of σ_I, and thus $\int f \wedge K^r_{I,\nu}$ tends to the term $\int f \wedge K_{I,\nu}$ corresponding to I in

$$\sum_{|I|=n-\nu}' \int_{\zeta \in \sigma_I} f(\zeta) \wedge \frac{\omega_I(\zeta, z)}{\beta^\nu \prod_{j \in I}(\phi_j(\zeta) - \phi_j(z))}, \quad z \in \Omega.$$

Now we let $\nu = n$. Then there is a smooth $(n, n-1)$ form ω_0 such that $\omega_0(\zeta, z) = \mathcal{O}(|\zeta - z|)$ and

$$\int_\Omega f \wedge K^r_{0,n} = \int_\Omega f \wedge \frac{\omega_0}{\beta^n}. \qquad \square$$

REMARK 2.2. The limit process of (2.5) is justified by the following one-variable result ([1], [4])

$$\begin{aligned}
\lim_{r \to 0} \int_D F(\zeta) \frac{\bar\partial(1 - |\zeta|^2)^r}{\zeta - |\zeta|^2 z} \wedge d\zeta &= \lim_{r \to 0} \int_D \bar\partial \left(F(\zeta) \frac{(1 - |\zeta|^2)^r}{\zeta - |\zeta|^2 z} \right) \wedge d\zeta \\
&\quad - \lim_{r \to 0} \int_D (1 - |\zeta|^2)^r \, \bar\partial \left(\frac{F(\zeta)}{\zeta - |\zeta|^2 z} \right) \wedge d\zeta \\
&= \lim_{r \to 0} \left[- \int_T F(\zeta) \frac{(1 - |\zeta|^2)^r}{\zeta - |\zeta|^2 z} d\zeta \right] \\
&\quad - \int_D \bar\partial \left(\frac{F(\zeta)}{\zeta - |\zeta|^2 z} \right) \wedge d\zeta \\
&= \int_T \frac{F(\zeta)}{\zeta - z} d\zeta.
\end{aligned}$$

3. Proofs of the main results

Theorem 1.1 is a consequence of the following proposition.

PROPOSITION 3.1. *There exists a constant $C_{\alpha,\epsilon}$ such that*

$$|d_z Sf(z)| \le C_{\alpha,\epsilon} |f|_{\Lambda_\alpha} \operatorname{dist}(z, \partial\Omega)^{\alpha - \epsilon - 1} \quad \textit{for } z \in \Omega \textit{ and } 0 < \epsilon < \alpha.$$

PROOF. The first term in the right hand part of (2.3) is given by integrating f against the Bochner-Martinelli kernel over Ω. It is well-known that the Hölder norm of the term is dominated by the L^∞ norm of f, so that it is enough to consider the case $1 \le \nu \le n-1$ and $|I| = n-\nu$. We may assume that $I = (1,\dots,n-\nu)$. Let

$$(3.1)\qquad S_{I,\nu}f(z) = \int_{\zeta\in\sigma_I} f(\zeta)\wedge \frac{\omega_I(\zeta,z)}{\beta^\nu \prod_{j=1}^{n-\nu}(\phi_j(\zeta)-\phi_j(z))}.$$

By differentiating under the integral sign in (3.1), one obtains

$$d_z S_{I,\nu}f(z) = J_0(z) + \sum_{\mu=1}^{n-\nu} J_1^\mu(z) + J_2(z),$$

where

$$J_0(z) = \int_{\sigma_I} f(\zeta)\wedge \frac{A_0(\zeta,z)}{\beta^\nu \prod_{j=1}^{n-\nu}(\phi_j(\zeta)-\phi_j(z))},$$
$$J_1^\mu(z) = \int_{\sigma_I} f(\zeta)\wedge \frac{A_1(\zeta,z)}{\beta^\nu(\phi_\mu(\zeta)-\phi_\mu(z))^2 \prod_{j\ne\mu}(\phi_j(\zeta)-\phi_j(z))},$$
$$J_2(z) = \int_{\sigma_I} f(\zeta)\wedge \frac{A_2(\zeta,z)}{\beta^{\nu+1} \prod_{j=1}^{n-\nu}(\phi_j(\zeta)-\phi_j(z))}.$$

The expressions $A_j(\zeta,z)$ are double forms which satisfy $|A_j(\zeta,z)| \lesssim |\zeta-z|^j, j=0,1,2$.

We will prove that for each pair of points $z^0 \in \overline{\Omega}$ and $\zeta^0 \in \sigma_I$, we can find neighborhoods U^{z^0} and U^{ζ^0} such that if χ is a smooth cutoff function with support in U^{ζ^0}, then the estimate

$$|J_1^\mu(z)| \lesssim |f|_{\Lambda_\alpha} \operatorname{dist}(z,\partial\Omega)^{\alpha-\epsilon-1} \quad \text{for} \quad \alpha>\epsilon>0$$

holds uniformly for all $z\in U^{z^0}\cap\Omega$.

As usual, we assume that $|\phi_j(z^0)| = 1$ for $1\le j\le l$ and $|\phi_j(z^0)|<1$ for $l<j\le n-\nu$. Near z^0, $w_1=\phi_1,\dots,w_l=\phi_l$ are part of a local coordinate system $w_1,\dots,w_n$ and moreover $\xi_1=\phi_1,\dots,\xi_n=\phi_n$ are local coordinates near ζ^0. In these coordinates, the integral to estimate is

$$J_1^\mu(z) = \int_{\xi\in T^{n-\nu}\times D^\nu} \frac{f(\xi)\chi(\xi)A_1(\xi,w)}{|\xi-w|^{2\nu}(\xi_\mu-w_\mu)^2\prod_{j\ne\mu}(\xi_j-w_j)}.$$

We will consider the only term $J_1^1(z)$. We can write $J_1^1(z) = J_{1,1}^1(z) + J_{1,2}^1(z)$, where

$$(3.2)\qquad J_{1,1}^1(z) = \int_{\xi\in T^{n-\nu}\times D^\nu} \frac{(f(\xi)-f(w_1))\chi(\xi)A_1(\xi,w)}{|\xi-w|^{2\nu}(\xi_1-w_1)^2\prod_{j=2}^{l}(\xi_j-w_j)}$$

and

$$(3.3)\qquad J_{1,2}^1(z) = \int_{\xi\in T^{n-\nu}\times D^\nu} \frac{f(w_1)\chi(\xi)A_1(\xi,w)}{|\xi-w|^{2\nu}(\xi_1-w_1)^2\prod_{j=2}^{l}(\xi_j-w_j)},$$

where $f(w_1) = f(w_1,\xi_2,\dots,\xi_n)$.

Now we recall the following lemma in the unit disk in $\mathbb{C}^1$ (see 1.4.10 in [9]).

LEMMA 3.2. *For c real, define*

$$I_c(z) = \int_T \frac{d\zeta}{|\zeta - z|^{1+c}}.$$

If $c>0$, then $I_c(z)\approx(1-|z|^2)^{-c}$ and $I_0(z)\approx \log\frac{1}{1-|z|^2}$.

Using Fubini's theorem and Lemma 3.2 in (3.2), it follows that

$$\begin{aligned}
|J_{1,1}^1(z)| &\lesssim |f|_{\Lambda_\alpha}\int_{\xi\in T^{n-\nu}\times D^\nu}\frac{1}{|\xi-w|^{2\nu-1}|\xi_1-w_1|^{2-\alpha}\prod_{j=2}^{l}|\xi_j-w_j|}\\
&\lesssim |f|_{\Lambda_\alpha}\frac{1}{(1-|w_1|^2)^{1-\alpha}}\prod_{j=2}^{l}\log\frac{1}{1-|w_j|^2}\\
&\lesssim |f|_{\Lambda_\alpha}\left[\min_{1\le j\le l}(1-|w_j|^2)\right]^{\alpha-\epsilon-1}\\
&\lesssim |f|_{\Lambda_\alpha}\operatorname{dist}(z,\sigma_I)^{\alpha-\epsilon-1}\\
&\lesssim |f|_{\Lambda_\alpha}\operatorname{dist}(z,\partial\Omega)^{\alpha-\epsilon-1}\quad\text{for}\quad 0<\epsilon<\alpha
\end{aligned}$$

uniformly for all $z\in U^{z^0}\cap\Omega$.

In order to estimate (3.3), we use integration by parts. Since χ has a compact support in U^{ζ^0}, using integration by parts, we have

$$\begin{aligned}
J_{1,2}^1(z) &= \int_{\xi\in T^{n-\nu}\times D^\nu}\frac{f(w_1)\chi(\xi)A_1(\xi,w)}{|\xi-w|^{2\nu}(\xi_1-w_1)^2\prod_{j=2}^{l}(\xi_j-w_j)}\\
&= -\int_{\xi\in T^{n-\nu}\times D^\nu}\frac{f(w_1)}{\prod_{j=2}^{l}(\xi_j-w_j)}\frac{\partial}{\partial\xi_1}\left(\frac{1}{\xi_1-w_1}\right)\frac{\chi(\xi)A_1(\xi,w)}{|\xi-w|^{2\nu}}\\
&= \int_{\xi\in T^{n-\nu}\times D^\nu}\frac{f(w_1)}{\prod_{j=1}^{l}(\xi_j-w_j)}\frac{\partial}{\partial\xi_1}\left[\frac{\chi(\xi)A_1(\xi,w)}{|\xi-w|^{2\nu}}\right].
\end{aligned}$$

Thus it follows that

$$|J_{1,2}^1(z)| \lesssim |f|_{L^\infty} \int_{\xi\in T^{n-\nu}\times D^\nu} \frac{1}{|\xi - w|^{2\nu}\prod_{j=1}^{l}|\xi_j - w_j|}.$$

For $\xi \in T^{n-\nu}\times D^\nu$, it follows that

$$|\xi - w|^{2\nu}\prod_{j=1}^{l}|\xi_j - w_j| \geq \prod_{1}^{l}|\xi_j - w_j|^{1+(2s\nu)/l}\prod_{n-\nu+1}^{n}|\xi_j - w_j|^{2-2s}$$

for $0 < s < \frac{1}{2}$. By Fubini's theorem and Lemma 3.2, we have

$$\int_{\xi\in T^{n-\nu}\times D^\nu} \frac{1}{\prod_1^l|\xi_j - w_j|^{1+(2s\nu)/l}\prod_{n-\nu+1}^{n}|\xi_j - w_j|^{2-2s}}$$
$$\lesssim \prod_1^l \frac{1}{(1-|w_j|^2)^{(2s\nu)/l}}.$$

We choose s sufficiently small so that $0 < 2s\nu < 1-\alpha+\epsilon$. Then we have

$$\begin{aligned}|J_{1,2}^1(z)| &\leq C_{\alpha,\epsilon}|f|_{L^\infty}\left[\min(1-|\phi_j(z)|^2)\right]^{\alpha-\epsilon-1}\\ &\leq C_{\alpha,\epsilon}|f|_{L^\infty}\operatorname{dist}(z,\partial\Omega)^{\alpha-\epsilon-1} \quad \text{for} \quad 0<\epsilon<\alpha.\end{aligned}$$

Hence we have

$$|J_1^\mu(z)| \lesssim |f|_{\Lambda_\alpha}\operatorname{dist}(z,\partial\Omega)^{\alpha-\epsilon-1} \quad \text{for} \quad 0<\epsilon<\alpha.$$

In cases of $J_0(z)$ and $J_2(z)$ we can see that $|J_2(z)| \lesssim |J_0(z)|$. The estimate $|J_0(z)| \lesssim |f|_{\Lambda_\alpha}\operatorname{dist}(z,\partial\Omega)^{\alpha-\epsilon-1}$ for $\epsilon > 0$ can be obtained similarly and we omit the details. Thus we obtain

$$|d_z S_{I,\nu} f(z)| \lesssim |f|_{\Lambda_\alpha}\operatorname{dist}(z,\partial\Omega)^{\alpha-\epsilon-1}$$

for $0<\epsilon<\alpha$ and all I and ν. Hence the proposition is proved. □

In the proof of Proposition 3.1, if $n = 2$, then it follows that

$$|d_z Sf(z)| \lesssim |f|_{\Lambda_\alpha}\operatorname{dist}(z,\partial\Omega)^{\alpha-1} \quad \text{for} \quad 0<\alpha<1 \tag{3.4}$$

and

$$|d_z Sf(z)| \lesssim |f|_{L^\infty}\operatorname{dist}(z,\partial\Omega)^{-1}. \tag{3.5}$$

No integration by parts is needed for the proof of (3.5). By classical arguments, Theorem 1.2 is a consequence of (3.4) and (3.5) ([7]).

4. An example

We consider the sharpness of the estimates in Theorem 1.2.

Let D^2 be the unit polydisk in $\mathbb{C}^2$. Let $0 < \alpha < 1$. Let $f = \overline{\partial} v$, where $v(z_1, z_2) = (1 - z_1)^\alpha \overline{z}_2$. Then $f = (1 - z_1)^\alpha d\overline{z}_2$ is a (0,1)-form in D^2, $\overline{\partial} f = 0$ is obvious, and $f \in \Lambda_\alpha(D^2)$.

Suppose $u \in \Lambda_\beta(D^2)$ satisfies $\overline{\partial} u = f$ in D^2. For $0 < d < 1/2$, we consider the integral

$$I(d) = \int_{|z_2|=1/2} [u(1-d, z_2) - u(1-2d, z_2)]\, dz_2. \tag{4.1}$$

Since $u \in \Lambda_\beta(D^2)$, it follows that

$$|I(d)| \lesssim d^\beta. \tag{4.2}$$

On the other hand, $\overline{\partial}(u - v) = 0$, so $u = v + h$, with $h \in \mathcal{O}(D^2)$. By Cauchy integral theorem we can replace u by v in the integral (4.1). Therefore we have

$$\begin{aligned} I(d) &= [d^\alpha - (2d)^\alpha] \int_{|z_2|=1/2} \overline{z}_2\, dz_2 \\ &= [d^\alpha - (2d)^\alpha]\, \frac{\pi i}{2}. \end{aligned} \tag{4.3}$$

If $\beta > \alpha$, (4.2) and (4.3) lead to a contradiction as $d \to 0$.

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