Set-Point Control of Elastic Joint Robots Using only Position Measurements

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Motivated by the dynamic output feedback passification results, point-to-point control laws for an elastic joint robot are presented when only the position measurements are available. The proposed method makes a parallel connection of the robot system and an input-dimensional linear system which obtains the effect of the desired differentiators. It is shown that the closed-loop nonlinear robot system can be rendered output strictly passive and the regulation of the system is achieved in the end. Robustness analysis is also given with regard to uncertainties on the robot parameters. Performance of the proposed control law is illustrated in the simulation studies of a manipulator with three revolute elastic joints.

Key Words : Passivity, Parallel Connection, Elastic Joint Robot, Point-to-Point Control

1. Introduction

Since passive systems theory gained renewed attention (see (Byrnes *et al.*, 1991) and references therein), passivity properties have been playing a vital role in designing various controllers for nonlinear systems and other applications (see e.g. (Lin, 1995 ; Fradkov and Hill, 1998 ; Jiang and Hill, 1998 ; Sepulchre *et al.*, 1997 ; Ortega *et al.*, 1998)). Many of the results are based on the fact that the Euler-Lagrange system e.g. a robot manipulator defines a passive operator from the control input to the generalized velocity vector (Ortega *et al.*, 1998). The passivity-based controller design results in a simple proportional-derivative (PD) law (Takegaki and Arimoto, 1981) and, similar to that used for rigid robots, the control law suffices to globally stabilize the elastic joint robots about a reference position (Tomei, 1991b).

However, the velocity measurements are not always available for the feedback law, hence, several authors have considered the set-point control problem without the velocity measurements. The need of measuring the rotor speed has been removed by using a suitable linear filter (e.g. (Berghuis and Nijmeijer, 1993 ; Kelly *et al.*, 1994)) or by observer-based controllers (e.g. (Ailon and Ortega, 1993 ; Nicosia and Tomei, 1994)). (Refer to those papers and references therein for the motivations of this practically important line of research.)

On the other hand, in the dynamic passification results (Son *et al.*, 1999 ; Son et al., 2000), it is shown that the effect of differentiators can be obtained by a parallel connection with an additional dynamic system (see also (Bar-Kana, 1987)). Motivated by the results, instead of using the velocity measurements, we consider a parallel

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connection with an additional dynamic system to solve the set-point control problem of an elastic joint robot.

We first provide a sufficient condition with which a Lyapunov candidate function is proper and positive definite and, consequently, a PD control law achieves global asymptotic stability (GAS) of the robot system. Next, the additional system is constructed such that the zero dynamics (ZD) of the augmented robot system become the closed-loop system with the PD law in (Tomei, 1991b). Our result provides another alternative way of replacing the role of velocity measurements in (Tomei, 1991b) and is essentially different from the previous results (Berghuis and Nijmeijer, 1993 ; Ailon and Ortega, 1993 ; Kelly *et al.*, 1994 ; Nicosia and Tomei, 1994).

We will consider two cases in designing the control law. In the first case, when only the rotor position is available, the GAS of the closed-loop system is guaranteed. The property of output strict passivity (OSP) can be also obtained. Moreover, robustness with respect to uncertainties on some of the robot parameters is achieved as in the PD controller (Tomei, 1991b). The performances of the control law are studied for a three link manipulator with three rigid bodies interconnected by three revolute elastic joints. In the second case, when the link position is also available, the properties in the first case are obtained by a smooth (linear) control law.

The idea of using a parallel feedforward connection has been also considered in a practical adaptive controller design i.e., simple adaptive control law (SAC) (Bar-Kana, 1987). For rigid robots, an adaptive control law is provided in (Bar-Kana and Guez, 1990; Kaufman *et al.*, 1998). Though a non-adaptive stabilizing gain K_e is provided in (Kaufman *et al.*, 1998), it contains the velocity vector and the fixed gain is not useful when the velocity measurements are not available. Main differences from the results are summarized below :

(i) An elastic robot is considered with simulation studies.

(ii) A Lyapunov function is explicitly given under the physical energy consideration (Tomei, 1991b ; Ortega *et al.*, 1998).

(iii) The additional dynamic system (V) is not a direct inverse of the transfer function of the PD controller.

(iv) Only two of the states (not whole states) are transformed into new states in normal form.

(v) The output y as well as the augmented output $\bar{y}$ is used for control laws.

(vi) Any modification to alleviate the steady state error is not required.

The paper is organized as follows. In Section 2, definitions of passivity and basic results for the PD control of elastic joint robots are presented. A new sufficient condition under which the PD law achieves GAS is also provided. Section 3 presents dynamic output feedback controllers without velocity measurements. Robustness analysis of the proposed control law is carried out in Section 4. Simulation results are presented in Section 5. Some conclusions are given in Section 6.

Notations: I_n is an $n\times n$ identity matrix. When x is a vector, x_i represents its i -th element and $|x|$ means sqrt $\sqrt{x^Tx}$. $\lambda_m(K)$ (or $\lambda_M(K)$) stands for the minimum(or maximum) eigenvalue of a matrix K, and $\|K\|$ means its largest singular value.

2. Preliminaries

2.1 Passive and feedback passive systems

Consider the following nonlinear system

$$(P):\begin{cases}\dot{x}=f(x)+g(x)u, & x\in R^n,\ u\in R^m\\ y=h(x), & y\in R^m\end{cases} \quad (1)$$

where x is the state ; u is the control input ; y is the output and the vector fields f, g and h are smooth with $f(0)=0$ and $h(0)=0$.

Definition 1. (Hill and Moylan, 1977) The system (1) is said to be output strictly passive (OSP) if there exist a $\rho>0$ and a real constant β such that, $\forall t\geq 0$, the following dissipation inequality holds :

$$\int_0^t u^T(\tau)y(\tau)d\tau+\beta\geq\rho\int_0^t y^T(\tau)y(\tau)d\tau \quad (2)$$

More basically, if $\rho=0$ in (2), the system is said to be passive.

Systems that can be rendered passive by a feedback will be called feedback passive systems. Moreover, the procedure is called passification. Byrnes et al. (1991) showed that, under some mild assumptions, the given system is globally feedback equivalent to a C^2-passive system with a positive definite storage function if and only if the system has relative degree one and is globally weakly minimum phase.

If (1) has relative degree one, under some conditions, it can be represented as the normal form

$$\begin{cases} \dot{z}=q_0(z)+q_1(z,\ y)y, & z\in R^{n-m} \\ \dot{y}=b(z,\ y)+a(z,\ y)u, & y\in R^m \end{cases} \tag{3}$$

where $a(z,\ y)=L_g h(x)$ is nonsingular for all $x\in R^n$. The zero dynamics (ZD) of (3) is described by

$$\dot{z}=q_0(z),\ z\in R^{n-m}. \tag{4}$$

The system (1) is minimum phase if the equilibrium $z=0$ of its ZD subsystem (4) is asymptotically stable.

In this paper, we consider a parallel connection of (1) and the following additional system (see Fig. 1) to get the effect of the differentiators

$$(V):\begin{cases} \dot{\eta}=k(\eta)+u,\ \eta\in R^m,\ u\in R^m \\ y_\eta=\eta. \end{cases} \tag{5}$$

The function $\nu(\cdot)$ is used to make some modification of the output y. Note also that the system (5) has the same dimension with the input of (1). In this case, (5) is called input-dimensional system.

In the next section, with a properly designed system (5), new control laws are provided which suffice to globally stabilize the elastic joint robots

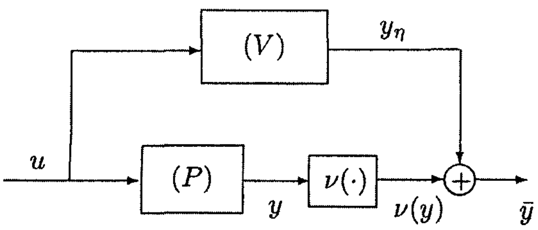


Fig. 1 Parallel connection

about a reference position. This scheme is an alternative way of replacing the PD controller when the velocity measurements are not available. For the complete proof of our approach, we briefly introduce the PD law in (Tomei, 1991b).

2.2 Tomei's PD control system

Consider the elastic joint robot model (Ailon and Ortega, 1993; Kelly *et al.*, 1994)

$$\begin{cases} D(q_1)\ddot{q}_1+C(q_1,\ \dot{q}_1)\dot{q}_1+g(q_1)=K(q_2-q_1) \\ J\ddot{q}_2+K(q_2-q_1)=u \\ y=q_2 \end{cases} \tag{6}$$

where $q_1\in R^n$ and $q_2\in R^n$ represent the link angles and motor angles, respectively; $u\in R^n$ is the torque vector; y is the measurable position output; $D(q_1)$ is the inertia matrix for the rigid links; J is a diagonal matrix of actuator inertias reflected to the link side; $C(q_1,\ \dot{q}_1)\dot{q}_1$ represents the Coriolis and centrifugal forces; $g(q_2)$ represents the gravitational terms, and K is a diagonal matrix containing the joint stiffness coefficients. One of the important structural properties of the model (6) is that the matrix $\dot{D}-2C$ is skew-symmetric. More detailed discussions can be found in many other references e.g. (Spong and Vidyasagar, 1989; Ortega *et al.*, 1998).

Tomei (1991b) proposed the following PD controller to solve the set-point regulation problem

$$u=-K_p(q_2-q_{2d})-K_d\dot{q}_2+g(q_{1d}) \tag{7}$$

where K_p and K_d are diagonal positive definite matrices; q_{1d} is the desired link position which is assumed to be constant, and q_{2d} is a constant vector defined by

$$K(q_{1d}-q_{2d})=-g(q_{1d}). \tag{8}$$

Let $H(q_1):=\dfrac{\partial g(q_1)}{\partial q_1}$ and $\tilde{q}=q-q_d=[\tilde{q}_1{}^T\ \tilde{q}_2{}^T]^T$ when $q_d=[q_{1d}^T\ q_{2d}^T]^T$. Global asymptotic stability (GAS) of the closed-loop system (6)-(7) is established if the following inequality holds

$$\lambda_m(\chi):=\lambda_m\begin{bmatrix} K & -K \\ -K & K+K_p \end{bmatrix}>\alpha \tag{9}$$

or, equivalently,

$$\min\{\lambda_m(K),\ \lambda_m(K_p)\} > \frac{3+\sqrt{5}}{2}\alpha \tag{10}$$

where $\alpha \geq \| H(q_1) \|\ \forall q_1 \in R^n$. Indeed, by virtue of (9) or (10), the closed-loop system has a unique equilibrium point $q=q_d$(or $\tilde{q}=0$) which is the solution of the following equation

$$\chi\begin{pmatrix}\tilde{q}_1\\ \tilde{q}_2\end{pmatrix}=\begin{pmatrix}q(q_{1d})-g(q_1)\\ 0\end{pmatrix} \tag{11}$$

It is also an absolute minimum point for the following function

$$P_1=\frac{1}{2}\tilde{q}^T\chi\tilde{q}+U_1(q_1)-q_1^Tg(q_{1d}) \tag{12}$$

where $g(q_1)=\frac{\partial U_1(q_1)}{\partial q_1}$. With (12), the following function

$$V(q,\ \dot{q})=\frac{1}{2}\dot{q}_1^TD(q_1)\dot{q}_1+\frac{1}{2}\dot{q}_2^TJ\dot{q}_2 + P_1(q)-P_1(q_d) \tag{13}$$

is proper and positive definite with respect to tilde $\tilde{q}=\dot{q}=0$. The time derivative of (13), along (6)-(7), is given by

$$\dot{V}(q,\ \dot{q})=-\dot{q}_2^TK_d\dot{q}_2. \tag{14}$$

By applying the LaSalle's Invariance principle, GAS of the closed-loop system is obtained.

We complete this section by the following corollary which provides another sufficient condition for GAS of the closed-loop system (6)-(7).

Corollary 1. If the following inequality holds

$$\min\{\lambda_m(K),\ \lambda_m(K_p)\}>2\alpha \tag{15}$$

where $\alpha \geq \| H(q_1) \|$, then the closed-loop system (6)-(7) has a unique equilibrium point $q=q_d$ which is GAS.

Proof. It will be shown that by (15), instead of by (10), (11) has a unique solution $q=q_d$ which is also the absolute minimum point for (12). The rest of the proof is illustrated in the above or in (Tomei, 1991b).

If we multiply $\begin{bmatrix}I_n & 0\\ I_n & I_n\end{bmatrix}$ to both sides of (11), then

$$\begin{bmatrix}K & -K\\ 0 & K_p\end{bmatrix}\begin{pmatrix}\tilde{q}_1\\ \tilde{q}_2\end{pmatrix}=\begin{pmatrix}g(q_{1d})-g(q_1)\\ g(q_{1d})-g(q_1)\end{pmatrix}. \tag{16}$$

This implies $\tilde{q}_2=K_p^{-1}(g(q_{1d})-g(q_1))$ and

$$\chi_1\tilde{q}_1=g(q_{1d})-g(q_1) \tag{17}$$

when $\chi_1=(I_n+KK_p^{-1})^{-1}K$. Let $K=:\ \mathrm{diag}\{k_i\}$ and $K_p=:\ \mathrm{diag}\{p_i\}$. χ_1 is a diagonal matrix and its i-th diagonal element is $(k_ip_i)(k_i+p_i)^{-1}$. By (15), $(k_1+p_i)(k_ip_i)^{-1}<\alpha^{-1}$, which implies $\lambda_m(\chi_1)>\alpha$. Therefore,

$$\begin{aligned}|\chi_1\tilde{q}_1| &\geq \lambda_m(\chi_1)|\tilde{q}_1|>\alpha|\tilde{q}_1|\\ &\geq |g(q_{1d})-g(q_1)|\end{aligned} \tag{18}$$

for all $q_1 \neq q_{1d}$. This implies (17) has the unique solution $q_1=q_{1d}$. Hence, (11) has the unique solution.

Moreover,

$$\begin{aligned}\frac{\partial^2 P_1}{\partial q^2}&=\chi+\frac{\partial}{\partial q}\begin{pmatrix}g(q_1)\\ 0\end{pmatrix}\\ &=\begin{bmatrix}K+H(q_1) & -K\\ -K & K+K_p\end{bmatrix}.\end{aligned} \tag{19}$$

Since $x_1^TH(q_1)x_1 \geq -\alpha x_1^Tx_1,\ \forall x_1,\ q_1 \in R^n$,

$$\begin{aligned}&(x_1^T\ x_2^T)\begin{bmatrix}K+H(q_1) & -K\\ -K & K+K_p\end{bmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}\\ &\geq x_1^T(K-\alpha I_n)x_1-2x_1^TKx_2+x_2^T(K+K_p)x_2\end{aligned} \tag{20}$$

where $x_2 \in R^n$. Let $K_2:=K-\alpha I_n=K_2^{\frac{1}{2}}K_2^{\frac{1}{2}}$. K_2 is positive definite by (15), and the right side of (20) is rewritten as

$$|K_2^{\frac{1}{2}}x_1-K_2^{-\frac{1}{2}}Kx_2|^2+x_2^T(K+K_p-KK_2^{-1}K)x_2. \tag{21}$$

Since $K+K_p-KK_2^{-1}K$ is diagonal and its i-th diagonal element is positive by (15), (19) is positive definite. This means $q=q_d$ is an absolute minimum point for P_1. ■

In the next section, a method for designing the additional system (5) is provided.

3. Point-to-Point Control of Elastic Joint Robots

In this section, we show that the velocity measurements in Tomei's scheme can be replaced by using an additional input-dimensional system. Notice that the system (6) has relative degree of two and the standard passivity-based control law cannot be applied directly. We will take two steps in the procedure : first an additional system and a new output are determined such that the composite system has relative degree of one and its ZD become the system (6)-(7). Then the regulation of the overall system is performed.

Since (7) is a linear control, the additional dynamics are chosen to be the following linear system

$$\begin{cases} \dot{\eta}=E\eta+u,\ \eta\in R^n \\ y_\eta=\eta. \end{cases} \tag{22}$$

The new output and its time derivative are given by

$$\bar{y}=\nu(q_2)+\eta, \text{ and } \dot{\bar{y}}=\frac{d}{dt}\nu(q_2)+E\eta+u. \tag{23}$$

If we choose

$$\nu(q_2)=Fq_2+G$$

with $F=K_d$, $G=-K_dq_{2d}-K_dK_p^{-1}g(q_{1d})$ and $E=-K_pK_d^{-1}$, then

$$-\frac{d}{dt}\nu(q_2)+E\nu(q_2)=-K_p\tilde{q}_2-K_d\dot{q}_2+g(q_{1d}). \tag{25}$$

Note that (25) is the very PD law (7). Consequently, the composite system (see Fig. 1) is determined as

$$\begin{cases} D(q_1)\ddot{q}_1+C(q_1,\dot{q}_1)\dot{q}_1+g(q_1)=K(q_2-q_1) \\ J\ddot{q}_2K(q_2-q_1)=u \\ \dot{\eta}=-K_pK_d^{-1}\eta+u \\ \bar{y}=K_dq_2+\eta+G. \end{cases} \tag{26}$$

Recall that the main feature of the normal form is that the control input does not appear in the dynamic equations of the ZD. With this in mind, in order to preserve the physical meaning of the system Eq. (26), only two of the state variables are transformed into the new variables by the following CT

$$\begin{pmatrix} \xi \\ \bar{y} \end{pmatrix}=\begin{pmatrix} \dot{q}_2-J^{-1}\bar{y} \\ K_dq_2+\eta+G \end{pmatrix}. \tag{27}$$

By (27), the system (26) is rewritten as

$$\begin{cases} D(q_1)\ddot{q}_1+C(q_1,\dot{q}_1)\dot{q}_1+g(q_1)=K(q_2-q_1) \\ J\dot{\xi}=-K(q_2-q_1)-K_p\tilde{q}_2-K_d\xi+g(q_{1d}) \\ \quad+(K_pK_d^{-1}-K_dJ^{-1})\bar{y} \\ \dot{\bar{y}}=K_p\tilde{q}_2+K_d\xi-g(q_{1d}) \\ \quad+(K_dJ^{-1}-K_pK_d^{-1})\bar{y}+u. \end{cases} \tag{28}$$

It is obvious that the system (28) has relative degree of one and is minimum phase because $\xi\equiv\dot{q}$ when $\bar{y}\equiv 0$.

We take a positive definite Lyapunov function $V(q,\dot{q}_1,\xi,\bar{y})=V_1(q,\dot{q}_1,\xi)+V_2(\bar{y})$ as follows:

$$\begin{aligned} V_1&=\frac{1}{2}\dot{q}_1^TD(q_1)\dot{q}_1+\frac{1}{2}\xi^TJ\xi+P_1(q)-P_1(q_d) \\ V_2&=\frac{1}{2}\bar{y}^T\bar{y}. \end{aligned} \tag{29}$$

Let $K_d^{-1}K_p-J^{-1}K_d+K_d=:Q$ and $J^{-1}(K+K_p)+K_p=:R$. Then, the time derivative of V along (28) gives

$$\begin{aligned} \dot{V}=&-\xi^TK_d\xi+\bar{y}^TQ\xi+\bar{y}^T(K_dJ^{-1}-K_pK_d^{-1})\bar{y} \\ &+\bar{y}^T(R\tilde{q}_2-J^{-1}K\tilde{q}_1-g(q_{1d})+u). \end{aligned} \tag{30}$$

If the control input is

$$u=-K_dJ^{-1}\bar{y}-R\tilde{q}_2+g(q_{1d})+u_2, \tag{31}$$

$$\begin{aligned} V=&-\xi^TK_d\xi+\bar{y}^TQ\xi-\bar{y}^TK_pK_d^{-1}\bar{y} \\ &+\bar{y}^T(-J^{-1}K\tilde{q}_1+u_2). \end{aligned} \tag{32}$$

When Q is positive definite diagonal, by application of Schwarz's inequality,

$$\bar{y}^TQ\xi=\bar{y}^T\sqrt{Q^T}\sqrt{Q}\xi\le\frac{1}{4\gamma}\bar{y}^TQ\bar{y}+\gamma\xi^TQ\xi \tag{33}$$

with a constant $\gamma>0$ such that $K_d-\gamma Q>0$. With (32) and (33),

$$\begin{aligned} \dot{V}\le&-\xi^T(K_d-\gamma Q)\xi+\frac{1}{4\gamma}\bar{y}^TQ\bar{y}-\bar{y}^TK_pK_d^{-1}\bar{y} \\ &+\bar{y}^T(-J^{-1}K\tilde{q}_1+u_2). \end{aligned} \tag{34}$$

At this point, we consider the following two cases. In the first case, only the rotor position q_2 is available for feedback. The link position q_1 is supposed to be also available in the other case.

3.1 Case 1: control law using only q_2

Note that, especially when the set-point regulation problem is concerned, there exists $b_i>0$ such that $|(\tilde{q}_1)_i|=|(q_1)_i-(q_{1d})_i|\le b_i$, $1\le i\le n$. Let $B_q:=\text{diag}\{b_i\}$ and $(\text{sgn}(\bar{y}))_i=\text{sgn}(\bar{y}_i)$.

The next result provides a control law with which the closed-loop system is GAS.

Proposition 1. Consider the system (26) with the following control law

$$\begin{aligned} u=&-R\tilde{q}_2+g(q_{1d})-\left(\frac{1}{4\gamma}Q+K_dJ^{-1}\right)\bar{y} \\ &-J^{-1}KB_q\text{sgn}(\bar{y})+r. \end{aligned} \tag{35}$$

When Q is positive definite and $K_d-\gamma Q>0$ with $\gamma>0$, the closed-loop system is OSP from r to $\bar{y}$ if the inequality (15) is satisfied. Moreover, when $r=0$, it has the unique equilibrium point $[\tilde{q}^T\ \dot{q}^T\ \bar{y}^T]^T=0$ which is GAS.

Proof. By (15) the closed-loop system has the unique equilibrium point $[\tilde{q}^T\ \dot{q}^T\ \bar{y}^T]^T=0$.

Since $-\bar{y}^T J^{-1} K\tilde{q}_1 \le \sum_{i=1}^{n} k_i b_i J_i^{-1} |\bar{y}_i|$,

$$\dot{V} \le -\xi^T(K_d-\gamma Q)\xi - \bar{y}^T K_p K_d^{-1}\bar{y} + \bar{y}^T v. \quad (36)$$

This implies the OSP of the closed-loop system (28)-(35) from r to $\bar{y}$. Moreover, by the La-Salle's Invariance principle and similar analysis in (Tomei, 1991b), the GAS is obtained when $r=0$.

Remark 2. Instead of the function sgn(·) in (35), by using the technique in (Su and Xie, 1996), the OSP property can be obtained by the following control

$$u=-R\tilde{q}_2+g(q_{1d})-\left(\frac{1}{4\gamma}Q+K_dJ^{-1}\right)\bar{y}-v+r \quad (37)$$

where $v_i=k_ib_i\bar{y}_iJ_i^{-1}(|\bar{y}_i|+\varepsilon)^{-1}$ and $\varepsilon=\varepsilon_0 e^{-c_1 t}$ with arbitrary constants ε_0, $c_1>0$. Indeed, by (37)

$$\begin{aligned}\dot{V} \le &-\xi^T(K_d-\gamma Q)\xi-\bar{y}^TK_pK_d^{-1}\bar{y}\\ &+\sum_{i=1}^{n}k_ib_iJ_i^{-1}\varepsilon_0 e^{-c_1 t}+\bar{y}^T r.\end{aligned} \quad (38)$$

The integration over both sides of (38) results in

$$\int_0^t \bar{y}^T(\tau)r(\tau)d\tau+\beta \ge \rho\int_0^t \bar{y}^T(\tau)\bar{y}(\tau)d\tau \quad (39)$$

where $\rho=\min\{(K_pK_d^{-1})_i\}$ and

$$\beta=V(0)+\sum_{i=1}^{n}k_ib_i\varepsilon_0J_i^{-1}c_1^{-1}.$$

The 'sgn' -function is usually approximated by a differentiable function for the practical use (Edwards and Spurgeon, 1998). The following proposition formalizes the result when $\gamma=1$.

Proposition 2. Consider the system (26) with the following control law

$$\begin{aligned}u=&-R\tilde{q}_2+g(q_{1d})-\left(\frac{1}{4}Q+K_dJ^{-1}\right)\bar{y}\\ &-J^{-1}KB_q\tanh(\sigma\bar{y})\end{aligned} \quad (40)$$

where $\sigma\ge 1$. If the inequality (15) is satisfied, the closed-loop system has the unique equilibrium point $[\tilde{q}^T\ \dot{q}^T\ \bar{y}^T]^T=0$ which is globally bounded. Moreover, if there exists a time t_1 such that for all time $t\ge t_1$ the following inequality is satisfied

$$-\bar{y}^TJ^{-1}KB_q\tanh(\sigma\bar{y})-\bar{y}^TJ^{-1}K\tilde{q}_1\le 0 \quad (41)$$

then the equilibrium point is GAS.

Proof. The closed-loop system (28)-(40) is given by

$$\begin{cases} D(q_1)\ddot{q}_1+C(q_1,\dot{q}_1)\dot{q}_1+g(q_1)=K(q_2-q_1)\\ J\dot{\xi}=K\tilde{q}_1-(K+K_p)\tilde{q}_2-K_d\xi\\ \quad +(K_pK_d^{-1}-K_dJ^{-1})\bar{y}\\ \dot{\bar{y}}=-\frac{1}{4}(5K_d^{-1}K_p-J^{-1}K_d+K_d)\bar{y}+K_d\xi\\ \quad -J^{-1}(K+K_p)\tilde{q}_2-J^{-1}KB_q\tanh(\sigma\bar{y}). \end{cases} \quad (42)$$

Since $\xi=-J^{-1}\bar{y}$ when $\dot{q}\equiv 0$, the equilibrium points are the solutions of

$$\begin{cases} K\tilde{q}_1-K\tilde{q}_2=g(q_{1d})-g(q_1)\\ -K\tilde{q}_1+(K+K_p)\tilde{q}_2=K_pK_d^{-1}\bar{y}\\ S\bar{y}+J^{-1}KB_q\tanh(\sigma\bar{y})=-J^{-1}(K+K_p)\tilde{q}_2 \end{cases} \quad (43)$$

where $S=\frac{1}{4}(5K_pK_d^{-1}+3J^{-1}K_d+K_d)$. Since the left hand side of the third equation in (43) is smooth and strictly increasing function of $\bar{y}$, there exists a positive diagonal matrix $\Delta_1(\bar{y})>0$ such that $\bar{y}=-\Delta_1(\bar{y})\tilde{q}_2$. Hence, the other two equations in (43) are rewritten as

$$\begin{bmatrix} K & -K\\ -K & K+K_p+\Delta\end{bmatrix}\begin{pmatrix}\tilde{q}_1\\ \tilde{q}_2\end{pmatrix}=\begin{pmatrix}g(q_{1d})-g(q_1)\\ 0\end{pmatrix} \quad (44)$$

where $\Delta(\bar{y})=K_pK_d^{-1}\Delta_1(\bar{y})$. Equation (44) has the unique solution $q=q_d$ by (15) because $\lambda_m(K_p+\Delta)>\lambda_m(K_p)$. Consequently, the closed-loop system has the unique equilibrium point.

With the control law (40),

$$\begin{aligned}\dot{V}\le&-\xi^T(J^{-1}K_d-K_d^{-1}K_p)\xi-\bar{y}^TK_pK_d^{-1}\bar{y}\\ &-\bar{y}^TJ^{-1}KB_q\tanh(\sigma\bar{y})-\bar{y}^TJ^{-1}K\tilde{q}_1,\end{aligned} \quad (45)$$

$$\begin{aligned}\le&-\xi^T(J^{-1}K_d-K_d^{-1}K_p)\xi-\bar{y}^TK_pK_d^{-1}\bar{y}\\ &+\sum_{i=1}^{n}k_ib_iJ_i^{-1}|\bar{y}_i|(1-|\tanh(\sigma\bar{y}_i)|).\end{aligned} \quad (46)$$

The first three terms in the right hand side of (45) are always negative except $\xi=\bar{y}=0$, and $\dot{V}<0$ for all $|\bar{y}|\ge\delta$ with an arbitrary small $\delta\ll 1$ and some $\sigma\ge 1$. This guarantees the boundedness of the closed-loop system. Moreover, if the inequality (41) is satisfied, (36) holds for all $t\ge t_1$. (This is probable because $\tilde{q}_1$ converges to zero when $\bar{y}\equiv 0$.) This implies the GAS of the closed-loop system.

3.2 Case 2: Using the Measurements q_1, q_2

If the link position q_1 is also available, we consider the following control law

$$u=-R\tilde{q}_2+g(q_{1d})-\left(\frac{1}{4}Q+K_dJ^{-1}\right)\bar{y} + J^{-1}K\tilde{q}_1+r. \quad (47)$$

Then, the closed-loop system satisfies the inequality (36) and is OSP from r to $\bar{y}$.

Proposition 3. Consider the system (26) with the control law (47). If the inequality (15) is satisfied, the closed-loop system (28)-(47) has the unique equilibrium point $[\tilde{q}^T\ \dot{q}^T\ \bar{y}^T]^T=0$ which is GAS when $r=0$.

Proof. The equilibrium points are the solutions of

$$\begin{cases} K\tilde{q}_1-K\tilde{q}_2=g(q_{1d})-g(q_1) \\ -K\tilde{q}_1+(K+K_p)\tilde{q}_2-K_pK_d^{-1}\bar{y}=0 \\ J^{-1}K\tilde{q}_1-J^{-1}(K+K_p)\tilde{q}_2-S\bar{y}=0. \end{cases} \quad (48)$$

From the last two equations of (48),

$$\tilde{q}_2=(K+K_p)^{-1}K\tilde{q}_1. \quad (49)$$

By (49), the first equation of (48) is rewritten as

$$[I_n-K(K+K_p)^{-1}]K\tilde{q}_1=g(q_{1d})-g(q_1). \quad (50)$$

Since $[I_n-K(K+K_p)^{-1}]K=(I_n+KK_p^{-1})^{-1}$, (50) coincides with (17). Hence, (50) has the unique solution by (15). Consequently, by (49) and (48), the closed-loop system has the unique equilibrium point. Moreover, since (36) is satisfied, the equilibrium point is GAS.

4. Robustness Analysis for Parameter Uncertainties

If there exist uncertainties on the gravitational and elastic parameters, the equilibrium point is different from the desired one. In that case, the result of Proposition 2 is still valid for the unknown new equilibrium point. Let $\hat{g}(q_1)$ and $\hat{K}$ be, respectively, the available estimates. Then,

$$\bar{y}=K_dq_2+n+\hat{G} \quad (51)$$

where $\hat{G}=-K_d\hat{q}_{2d}-K_dK_p^{-1}\hat{g}(q_{1d})$, and $\hat{q}_{2d}$ is defined by $\hat{K}(\hat{q}_{2d}-q_{1d})=\hat{g}(q_{1d})$. The system equation (28) is replaced by

$$\begin{cases} D(q_1)\ddot{q}_1+C(q_1,\dot{q}_1)\dot{q}_1+g(q_1)=K(q_2-q_1) \\ J\dot{\xi}=-K(q_2-q_1)-K_p(q_2-\hat{q}_{2d})-K_d\xi \\ \quad +\hat{g}(q_{1d})+(K_pK_d^{-1}-K_dJ^{-1})\bar{y} \\ \dot{\bar{y}}=K_p(q_2-\hat{q}_{2d})+K_d\xi-\hat{g}(q_{1d}) \\ \quad +(K_dJ^{-1}-K_pK_d^{-1})\bar{y}+u. \end{cases} \quad (52)$$

Moreover, with $\hat{B}_q=2B_q$ the control law (40) is modified to

$$u=-(J^{-1}(\hat{K}+K_p)+K_p)(q_2-\hat{q}_{2d})+\hat{g}(q_{1d}) -\left(\frac{1}{4}Q+K_dJ^{-1}\right)\bar{y}-J^{-1}\hat{K}\hat{B}_q\tanh(\sigma\bar{y}). \quad (53)$$

The closed-loop system (52)-(53) becomes

$$\begin{cases} D(q_1)\ddot{q}+C(q_1,\dot{q}_1)\dot{q}_1+g(q_1)=K(q_2-q_1) \\ J\dot{\xi}=-K_d\xi+K(q_1-q_2)-K_p(q_2-\hat{q}_{2d}) \\ \quad +\hat{g}(q_{1d})+(K_pK_d^{-1}-K_dJ^{-1})\bar{y} \\ \dot{\bar{y}}=-\frac{1}{4}(5K_d^{-1}K_p-J^{-1}K_d+K_d)\bar{y}+K_d\xi \\ \quad -J^{-1}(\hat{K}+K_p)(q_2-\hat{q}_{2d}) \\ \quad -J^{-1}\hat{K}\hat{B}_q\tanh(\sigma\bar{y}) \end{cases} \quad (54)$$

The equilibrium points are the solutions of

$$\begin{cases} K\tilde{q}_1-K\tilde{q}_2=g(q_{1d})-g(q_1) \\ -K\tilde{q}_1+K\tilde{q}_2+K_p(q_2-\hat{q}_{2d})-K_pK_d^{-1}\bar{y} \\ \quad =\hat{g}(q_{1d})-g(q_{1d}) \\ S\bar{y}+J^{-1}\hat{K}\hat{B}_q\tanh(\sigma\bar{y}) \\ \quad =-J^{-1}(\hat{K}+K_p)(q_2-\hat{q}_{2d}) \end{cases} \quad (55)$$

Since there exists a positive diagonal matrix $\Delta_2(\bar{y})>0$ such that $\bar{y}=-\Delta_2(\bar{y})(q_2-\hat{q}_{2d})$, the first two equations of (55) are rewritten in the matrix form by

$$\hat{\chi}\begin{pmatrix}\tilde{q}_1\\ \tilde{q}_2\end{pmatrix}=\begin{pmatrix} g(q_{1d})-g(q_1) \\ (K_p+\hat{\Delta})(\hat{q}_{2d}-q_{2d})+\hat{g}(q_{1d})-g(q_{1d}) \end{pmatrix} \quad (56)$$

where $\hat{\Delta}(\bar{y})=K_pK_d^{-1}\Delta_2(\bar{y})$ and

$$\hat{\chi}=\begin{bmatrix} K & -K \\ -K & K+K_p+\hat{\Delta} \end{bmatrix}.$$

By the similar analysis of the proofs of Proposition 2 and Theorem 2 in (Tomei, 1991b), $q=\bar{q}_d$ is the unique solution of (56). An estimate of the difference of $\bar{q}_d$ and q_d is derived as

$$|\bar{q}_d-q_d|\le\frac{1}{\lambda_m(\hat{\chi})-\alpha}(\lambda_M(\hat{\chi})|\hat{q}_{2d}-q_{2d}| +|\hat{g}(q_{2d})-g(q_{1d})|) \quad (57)$$

Define the function

$$P_2=\frac{1}{2}\tilde{q}^T\chi\tilde{q}+U_1(q_1)$$
$$-\tilde{q}^T\begin{pmatrix} g(q_{1d}) \\ K_p(\hat{q}_{2d}-q_{2d})+\hat{g}(q_{1d})-g(q_{1d})\end{pmatrix}. \quad (58)$$

The boundedness of the closed-loop system (54) is obtained with the following Lyapunov function

$$\hat{V}(q,\dot{q}_1,\xi,\bar{y})=\frac{1}{2}\dot{q}_1^TD(q_1)\dot{q}_1+\frac{1}{2}\xi^TJ\xi + P_2(q)-P_2(\bar{q}_d)+\frac{1}{2}\bar{y}^T\bar{y}. \quad (59)$$

Indeed, the time derivative of $\hat{V}$ along (54) is

$$\dot{\hat{V}}\le-\xi^T(J^{-1}K_d-K_d^{-1}K_p)\xi-\bar{y}^TK_pK_d^{-1}\bar{y} +\bar{y}^TJ^{-1}[(K-\hat{K})(q_2-q_1)-\hat{K}B_q\tanh(\sigma\bar{y})] +\bar{y}^TJ^{-1}[-\hat{K}\tilde{q}_1-\hat{K}B_q\tanh(\sigma y\bar{y})] \quad (60)$$

Since $|(q_2)_i-(q_1)_i|$ is also bounded by the same $b_i>0$, $1\le i\le n$, as $\tilde{q}_1$, the boundedness of all the states of (54) is obtained from the analysis of Proposition 2.

5. Simulation Results

In order to test the dynamic performances of the proposed controller, the same numerical simulations as in (Tomei, 1991b) were carried out on a three link manipulator with three rigid bodies interconnected by three revolute elastic joints. All the robot parameters are the same as in (Tomei, 1991a; Tomei, 1991b) except $a_6=0$.

Moreover, the simulation runs were also performed with the control law in (Kelly *et al.*, 1994) to compare the performances of the control law. In the well-known scheme, the following linear filter is used to approximate the velocity measurements (Kelly *et al.*, 1994, Proposition 1)

$$\dot{x}=-Ax-ABq_2$$
$$\vartheta=x+Bq_2, \quad (61)$$
$$u=-K_p(q_2-q_{2d})-K_d\vartheta+g(q_{1d}). \quad (62)$$

In the simulations, $A=30I_3$ and $B=10I_3$ as in (Kelly *et al.*, 1994). It is interesting to note that the closed-loop system (6)-(61)-(62) can be viewed as a feedback connection of two passive subsystems (see also (Lanari and Wen, 1992)). On the other hand, the passive properties in this paper are obtained by the temporary feedforward connection.

The problem considered is that of regulation about the reference position

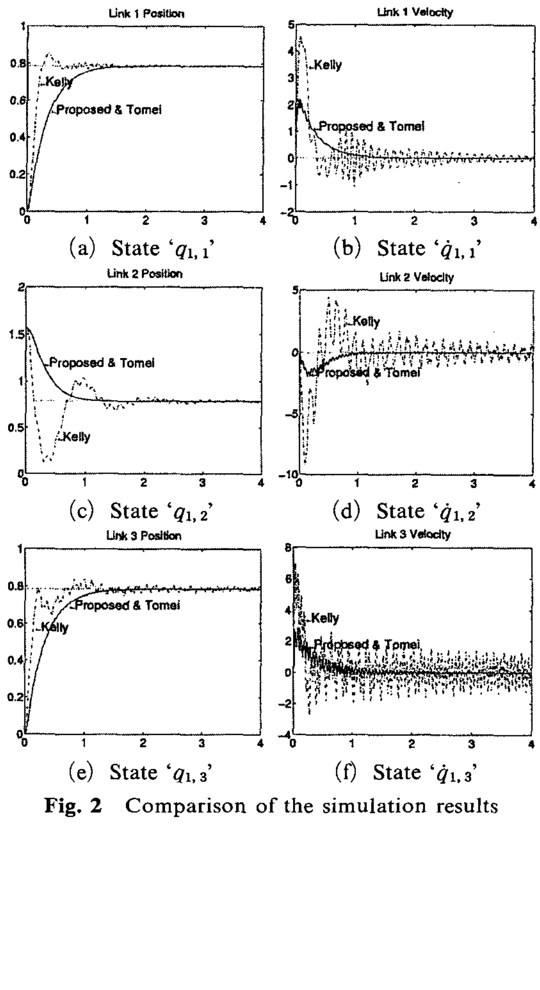
$$q_{1d}=\left[\frac{\pi}{4}\ \frac{\pi}{4}\ \frac{\pi}{4}\right]^T,\ q_{2d}=q_{1d}+K^{-1}g(q_{1d})$$

starting from the following initial conditions

$$q_{10}=q_{20}=\left[0\ \frac{\pi}{2}\ 0\right]^T,\ \dot{q}_{10}=\dot{q}_{20}=\bar{y}_0=[0\ 0\ 0]^T.$$

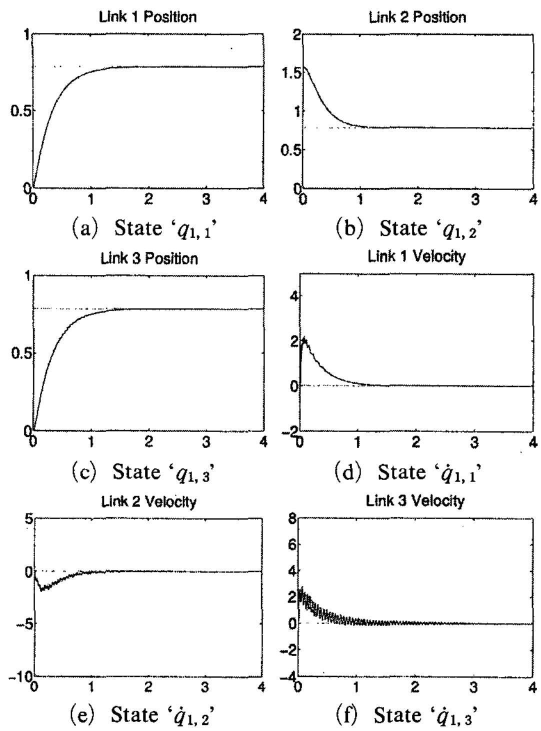
The proportional and derivative gain matrices are chosen as in (Tomei, 1991b) i.e., $K_p=3000I_3$ and $K_d=1000I_3$.

The simulation results by the three control laws i.e., (Tomei, 1991b), (Kelly *et al.*, 1994) and (40) with $B_q=2\pi I_3$, $\sigma=1$ are all presented in Fig. 2. Note that Tomei's scheme requires the velocity measurements. Since the maximum differences between (Tomei, 1991b) and (40) are of order 10^{-4} for q_1 and 10^{-2} for $\dot{q}_1$, respectively, in Fig. 2 it looks as if they had the same trajectories. Hence, it can be said that the proposed control law approximates the velocity measurements more accurately than (Kelly *et al.*, 1994).



(a) State '$q_{1,1}$' (b) State '$\dot{q}_{1,1}$' (c) State '$q_{1,2}$' (d) State '$\dot{q}_{1,2}$' (e) State '$q_{1,3}$' (f) State '$\dot{q}_{1,3}$'

Fig. 2 Comparison of the simulation results



(a) State '$q_{1,1}$' (b) State '$q_{1,2}$'
(c) State '$q_{1,3}$' (d) State '$\dot{q}_{1,1}$'
(e) State '$\dot{q}_{1,2}$' (f) State '$\dot{q}_{1,3}$'

Fig. 3 Robustness for parameter uncertainties

In order to verify the robustness of the proposed controller, another simulation was performed in the same manner as in (Tomei, 1991b). The simulation test was repeated by using a different payload (see (Tomei, 1991b, Table 1)) and the elastic constants that are 10% greater than the nominal ones. Note that the controller was designed with the nominal parameters. Both the result by the exact Tomei's law (7) and the new result are reported in Fig. 3.

6. Conclusion

The set-point (or point-to-point) control problem for an elastic joint robot is considered. We first provide somewhat relaxed sufficient condition with which a Lyapunov candidate function is proper and positive definite. Consequently, the PD control law in (Tomei, 1991b) achieves GAS of the closed-loop robot system.

Though the PD law is a good solution to the problem, the velocity measurements are not always available for the feedback law. To obtain the effect of the velocity measurements, the proposed method makes a parallel connection of the robot system and an input-dimensional additional system. Under some conditions, the proposed control law achieves the effect of the desired derivatives of the output and the problem is solved with only the position measurements. Robust property with regard to the gravitational and elastic parameters are also obtained. In simulation studies of a manipulator with three revolute elastic joints, the proposed controller shows some enhanced performances to the previous result in (Kelly et al., 1994).

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