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Measurement of Complex Modulus of Acoustic Materials by Using Transfer Function Method

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Abstract

Two improvements are discussed in measurement of the complex Young's modulus of the acoustic materials by using the transfer function method. It is found that the accelerometer misalignment might result in the severe measurement error, particularly in high frequency range. The supporting structure is modified to attach the upper and lower accelerometers along the vertical axis. Secondly, the method for solving the equation associated with wave model is described. The solution of the lumped mass-spring model is chosen as the starting value for low frequency range, while in the mid and high frequency, the solution to the previous frequency step is used as the initial values. Measurements are done for hard and soft rubber specimens. It is shown that the erroneous peaks in the transfer function, due to the measurement error, cause highly incorrect Young's modulus and loss factors.

Keywords: *Complex modulus, Transfer function method, Loss factor*

I. Introduction

Acoustic materials such as rubbers, foams, and mineral wools are of great importance in reducing noise and vibration. The frequency range of interest often covers few kilo-Hertz where the Young's modulus and loss factor, or the complex modulus, of the acoustic materials must be determined as the functions of the frequency. Several measurement techniques have been developed to measure the complex modulus, among which impedance[1] and transfer function methods[2-5] are widely used. In applying these methods, a specimen is prepared in the rod-like shape, and one end is excited by an electromagnetic shaker. The complex modulus is determined by assuming that the motion of the specimen is governed by the one-dimensional wave equation in the longitudinal direction. While the impedance method needs precise measurement of the force and acceleration, the transfer function method needs only the ratio of accelerations at both ends of the specimen. The complex modulus is affected by several factors, such as specimen shapes, Poisson's ratios, dynamic strain of the specimen, bonding effects at the ends, etc. Pritz[4,5] discussed the transfer function method in great details.

In this paper, we apply the transfer function method to measure the complex modulus of the acoustic materials. The aim of our study is to propose two things for improving the measurement accuracy and efficiency of the numerical method associated with the transfer function method, but not to develop more complicated theories for the transfer function method.

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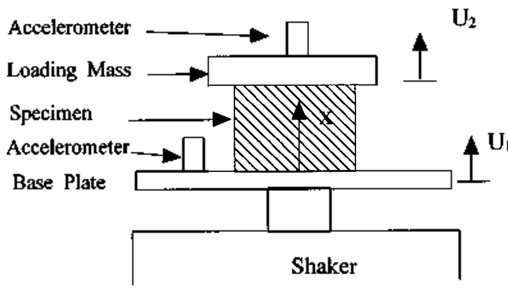

Figure 1. A typical measurement setup.

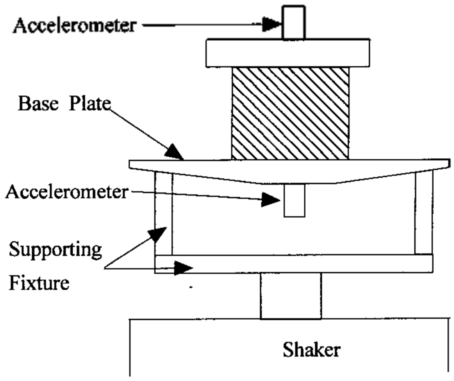

Figure 2. Modified base plate with the supporting fixture.

II. Transfer Function Method

A typical measurement setup for the transfer function method that are found in previous works is shown in Figure 1. The specimen is of a rod-like shape with a short length. The bottom of the specimen is bonded rigidly to the base plate, while the upper end is bonded to the rigid loading mass. The loading mass provides a place for attaching the accelerometer, as well as making the specimen pre-compressed, which is important to ensure the uniform contact between the specimen and the base plate and the loading mass.

In Figure 1, the lower accelerometer on the base plate is located off the shaker axis due to the specimen, while the upper accelerometer is located along the shaker axis. Misalignment of the accelerometers in vertical direction might cause error in transfer function. However, effects of accelerometer misalignment have never been discussed in the previous works. Intuitively, the best location of the lower accelerometer is the center of the base plate (or on the axis of the shaker). We modified the supporting system as shown in Figure 2, in which the base plate was lifted from the shaker table by adding supporting fixture so that the lower accelerometer can be located at the center.

Figure 3 shows comparisons of two accelerations, Q_0 (at the center) and Q_1 (30 mm apart from the center) on the modified base plate. Figures 4 and 5 show magnitude and phase of the transfer function of two accelerations, Q_1/Q_0. It is found that the magnitude ratio shows more distortion as the frequency increases.

The longitudinal vibration of the specimen is described by the wave equation

$$E(1+i\eta)\frac{\partial^2 u}{\partial x^2}-\rho\frac{\partial^2 u}{\partial t^2}=0, \qquad (1)$$

where E is the Young's modulus, η the loss factor, ρ the density, and $i^2=-1$. The length of the specimen is L. It is assumed that the lateral vibration motion is negligible and the specimen is only governed by the one-dimensional wave equation in the longitudinal direction. If the length is not greater than lateral dimension, the assumption of the one-dimensional wave motion may not be valid. In addition, the both ends of the specimen are bonded rigidly so that the specimen may show bulged shape. It is known that the bulging effect makes the

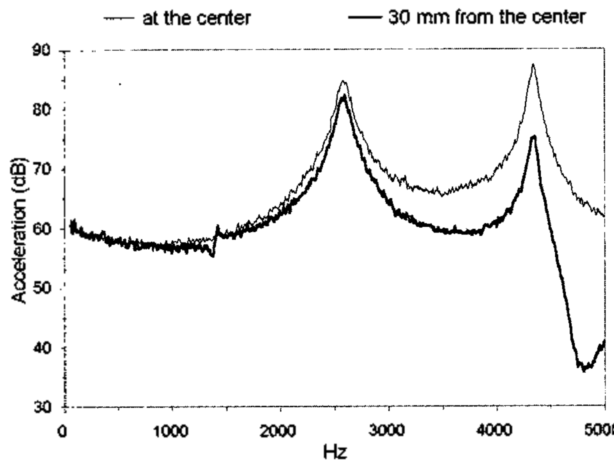

Figure 3. Comparison of the accelerations on the base plate: at the center (thin line) and 30mm apart from the center (thick line).

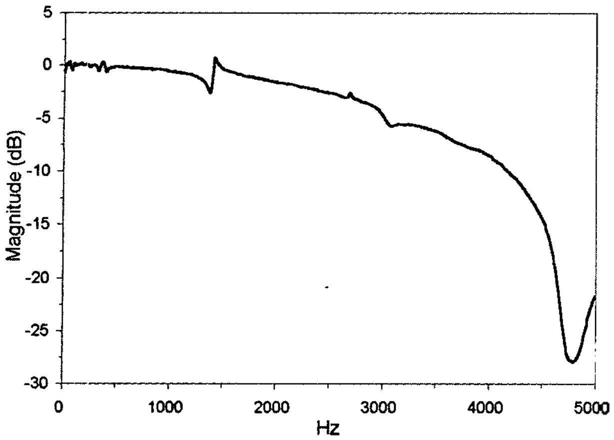

Figure 4. Magnitude ratio of the accelerations: Q_1/Q_0 (Q_1: 30mm apart from the enter; Q_0: at the center).

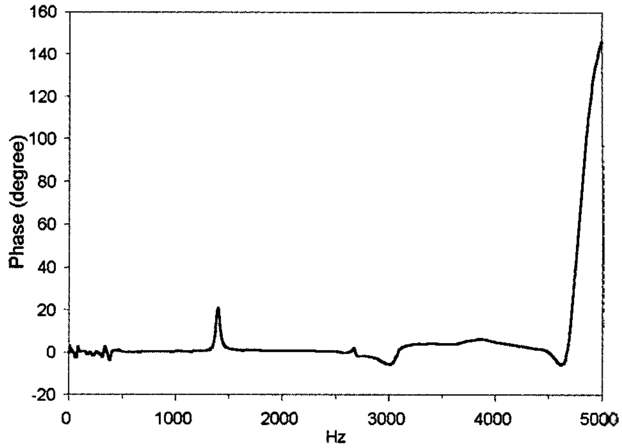

Figure 5. Phase difference of the accelerations: Q_1/Q_0 (Q_1: 30mm apart from the enter; Q_0: at the center).

measured Young's modulus larger than the true value[2]. Detailed description about the bulging effect and its correction by introducing the shape factor may be found in Refs[2,3].

According to Ref.[3], the one-dimensional wave model can be used even in the stumpy specimen whose length is quite short compared to the lateral dimension, although the measured young's modulus must be corrected by the relatively large shape factor.

If a harmonic motion is assumed as $u = U\exp(i\omega t)$, in which U is the amplitude, and ω is the frequency, the solution of Eq. (1) is given by

$$U = A_1 \sin\beta x + A_2 \cos\beta x, \tag{2}$$

where the complex wavenumber $\beta = \sqrt{\rho\omega^2/E(1+i\eta)}$ and A_1, A_2 are constants. The boundary condition at $x = L$ is given by

$$E(1+i\eta)S\frac{\partial U_2}{\partial x} = \omega^2 M U_2, \tag{3}$$

where S is the cross-sectional area of the specimen and M is the mass of the loading block. After removing one constant by using the boundary condition, and rearranging the result, we have the transfer function as

$$T = U_2/U_1 = 1/(\cos\beta L - \gamma\beta L \sin\beta L), \tag{4}$$

where γ is the ratio of the loading block mass to the specimen mass, $\gamma = M/\rho AL$. Details of derivation and related cases may be found in Ref[6]. In Eq. (4), β is the unknown, whereas the transfer function T is given by measurement. After solving Eq. (4) for β by using the numerical method such as the Newton-Raphson method, we can determine the complex modulus. However, Eq. (4) has an infinite number of solutions and determination of the physically meaningful solution is not a simple matter. Buchanan[7] described the procedures for obtaining the physical solution in details. The solution we seek is the one whose real part is smallest, while satisfying the conditions: $Re(\beta) > 0$ and $Im(\beta) < 0$.

The problem to be solved by using the Newton-Rapshon method is given by

$$\beta_{n+1} = \beta_n - f(\beta_n)/f'(\beta_n), \tag{5}$$

where

$$f(\beta) = \cos\beta L - \gamma\beta L \sin\beta L - 1/T. \tag{6}$$

The selection of the starting value is very important. Starting with $\beta = 0$, for instance, does not always converge to the physical solution. Furthermore, if Eq. (6) is rewritten as

$$f(\beta) = \sin\beta L - \frac{1}{\gamma}\frac{\cos\beta L}{\beta L} + \frac{1}{T\beta L}\frac{1}{\gamma}, \tag{7}$$

the same starting values in Eqs. (6) and (7) may result in different solutions when $\gamma \gg 1$. If the solution to the previous frequency step is known, it is a good choice as the starting value, since Young's modulus and loss factor are not likely to change abruptly as the frequency increases. Still, we have to find the physical solution for the very first frequency step.

If the mass of the loading block is large compared to the that of the specimen, i.e., $\gamma \gg 1$, and the wavelength is larger than the specimen length for low frequency, the system can be modeled as a lumped mass-spring system [3,4] whose equation of motion is given by

$$M(1+\alpha)\frac{\partial^2 u_2}{\partial t^2}+(A/L)E_0(1+i\eta_0)(u_2-u_1)=0, \tag{8}$$

where α is the fraction of the specimen mass included in the loading mass, $\alpha=1/2$. In Eq. (8), we expressed the Young's modulus and loss factor as " E_0" and " η_0" to denote the difference from the solution of the wave equation (4). According to Ref.[3], the solutions of the lumped mass-spring system can be derived explicitly as

$$E_0=\omega^2 M(1+\alpha)\frac{L}{A}\frac{[(Re^2+Im^2)-Re]}{(Re-1)^2+Im^2}, \tag{9}$$

$$\eta_0=-\frac{Im}{(Re^2+Im^2)-Re}, \tag{10}$$

where Re and Im denote real and imaginary parts of the transfer function T respectively. It was reported in the previous work[4] that the Young's modulus E_0 in Eq. (9) approaches the solution of the wave equation (4) when $\gamma \gg 1$ and L is small compared with the wavelength.

In this paper, we use the solutions (9) and (10) as the initial values in finding the solution of Eq. (4) for the first few low frequency steps. As frequency increases, the solutions E_0 and η_0 start to diverge from the solutions of the wave equation. Therefore, for mid and higher frequency ranges, we use the solution to the previous frequency step as the initial values. In addition, we choose Eq. (6) when $\gamma < 10$ and Eg. (7) when $\gamma \geq 10$.

III. Experimental Results

Measurements have been done for hard (density: 1100 kg/m^3) and soft (density: 130 kg/m^3) rubber specimens. The dimensions and properties are in the Table 1, where the soft rubber has been prepared in block (Case II) and cylinder (Case III) shape.

Figures 6 and 7 show the transfer function of the Case I specimen, while Figures 8 and 9 show the measured Young's modulus and loss factor.

Depending on the measurement conditions, the same material can show quite different transfer functions. Figures 10 and 11 compare the transfer functions of the Case II and III specimens. A mini B&K shaker (Model 4810) was used for the Case II, while much bigger shaker (about three times bigger than B&K 4810) was used for the Case

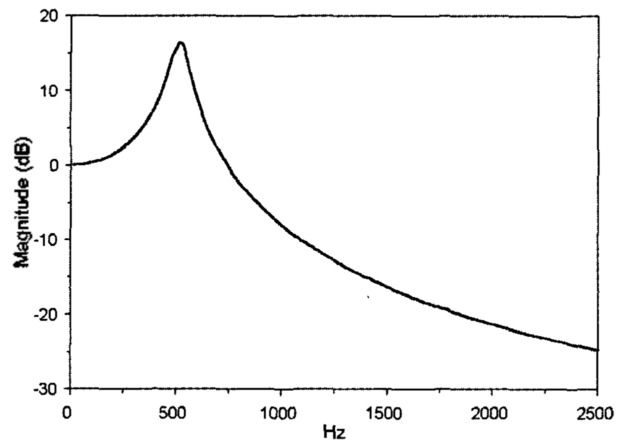

Figure 6. Magnitude of the transfer function for the Case I.

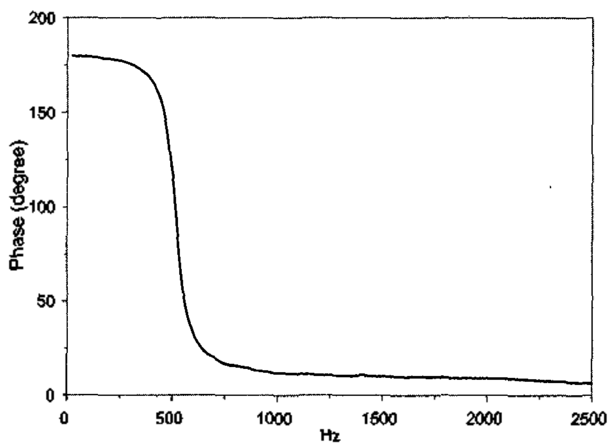

Figure 7. Phase of the transfer function for the Case I.

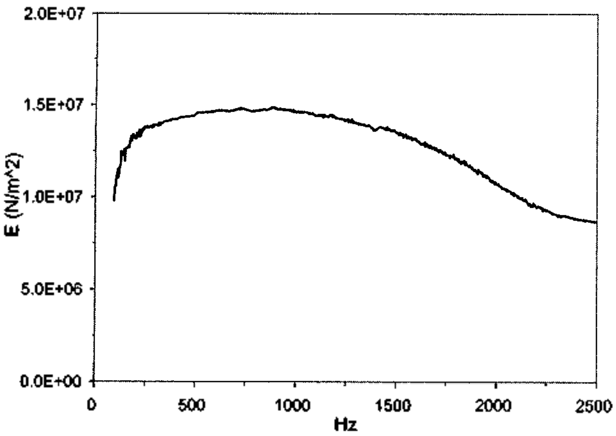

Figure 8. Young's modulus of the Case I.

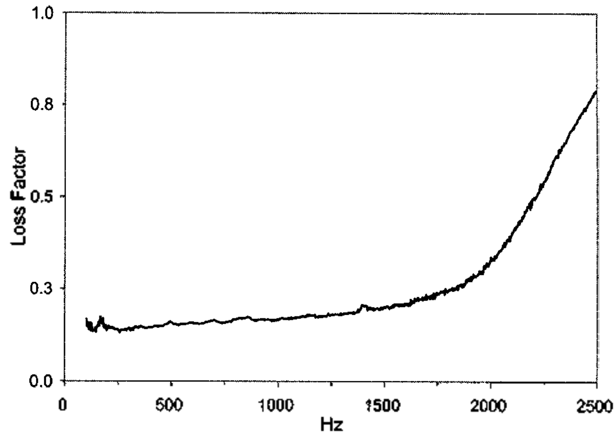

Figure 9. Loss factor of the Case I.

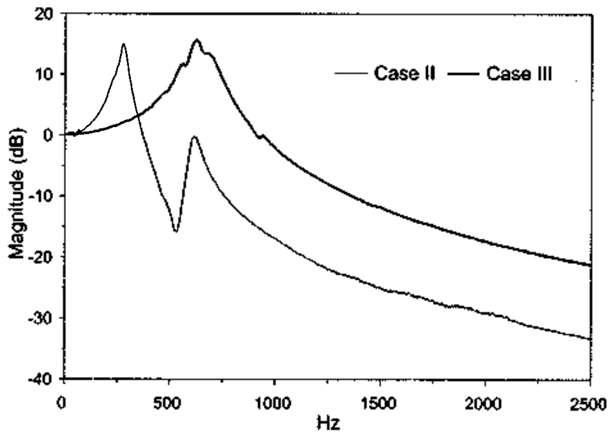

Figure 10. Magnitude of the transfer functions for the Case II and III.

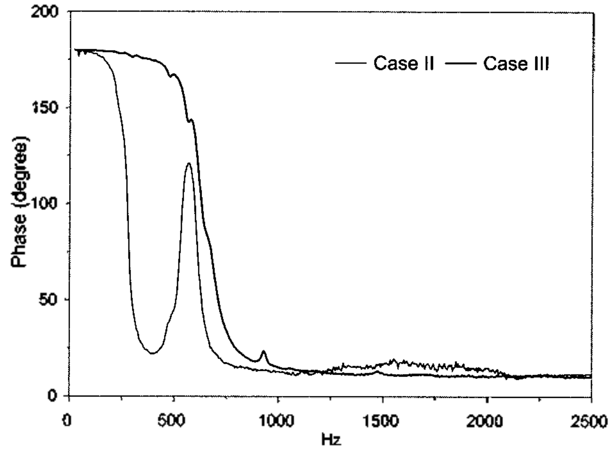

Figure 11. Phase of the transfer functions for the Case II and III.

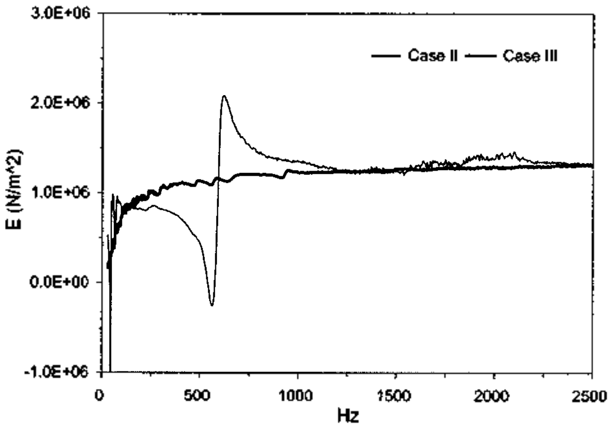

Figure 12. Young's modulus of the Case II and III.

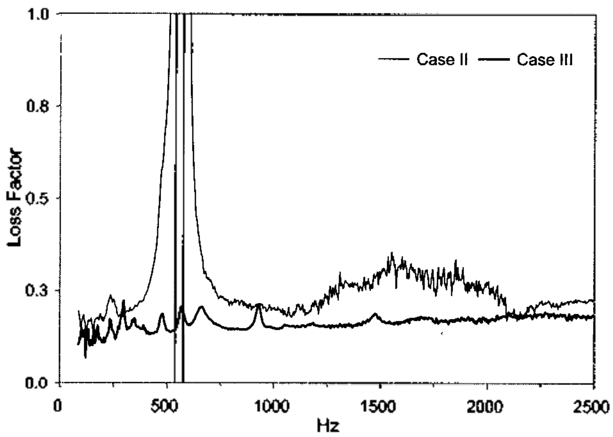

Figure 13. Loss factor of the Case II and III.

I and III. The additional peaks found in the Case II (at 612 Hz in Figure 10 and at 569 Hz in Figure 11) are measurement error that may be caused possibly by the flexibility of the supporting structure, accelerometer position, etc. In the Case II, the supporting structure as shown in Figure 2 was not used. The base plate (diameter of 32 mm and thickness of 1.5 mm) was simply lifted from the shaker by the thin circular rod (diameter of 2 mm and length of 30 mm) connecting the center of the base plate and the shaker. In the Case I and III, the diameter and thickness of the base plate are 120 mm and 5 mm respectively.

The erroneous peaks in the transfer function in Figures 10 and 11 result in highly incorrect Young's modulus and loss factor for case II as found in Figure 12 and 13. However, the transfer function for the Case III, which does not have erroneous peaks, leads to more reliable Young's modulus and loss factor.

IV. Conclusions

Locating upper and lower accelerometers along the vertical axis is important to avoid measurement errors. Comparison of base plate accelerations showed that the magnitude ratio becomes more distorted as the frequency increases.

The equation associated with wave model has an infinite number of solutions, where only physically meaningful solution is that the real part of the wavenumber is smallest as well as its imaginary part is negative. In solving the equation by using the Newton-Rapshon method, selection of the starting value is of prime importance. It was shown that the solution of the lumped mass-spring model is a good choice as the starting value for low frequency range, while in the mid and high frequency, the solution to the previous frequency step can be used as the initial values.

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