On Delay-Dependent Stability of Neutral Systems with Mixed Time-Varying Delay Arguments

J.H. Park

Abstract - This paper focuses on the asymptotic stability of a class of neutral linear systems with mixed time-varying delay arguments. Using the Lyapunov method, a delay-dependent stability criterion to guarantee the asymptotic stability for the systems is derived in terms of linear matrix inequalities (LMIs). The LMIs can be easily solved by various convex optimization algorithms. Two numerical examples are given to illustrate the proposed methods.
Keywords - neutral differential systems, time-varying delay, Lyapunov method, linear matrix inequality.

1. Introduction

During the decades, the stability analysis of various neutral delay-differential systems has received considerable attention (see Gopalsamy [5], Hale and Verduyn Lunel [7], and references therein). The theory of neutral delay-differential systems is of both theoretical and practical interest. For a large class of electrical networks containing lossless transmission lines, the describing equations can be reduced to neutral delay-differential equations; such networks arise in high speed computers where nearly lossless transmission lines are used to interconnect switching circuits. Also, the neutral systems often appear in the study of automatic control, population dynamics, and vibrating masses attached to an elastic bar. In the literature, several analysis techniques such as Lyapunov technique, characteristic equation method, or state solution approach have been utilized to derive stability criteria for asymptotic stability of the systems. Depending on whether the stability criterion itself contains the delay argument as a parameter, the developed stability criteria are often classified into two categories, namely delay-independent criteria and delay-dependent criteria. In the literature, many delay-independent sufficient conditions for the asymptotic stability of neutral delay-differential systems are presented by many researchers (Chen [3], Hale et al. [6], Hu and Hu [8], Hui and Hu [9], Kuang et al. [12], Li [13], Park and Won [15]). Also, a few delay-dependent sufficient conditions have been exploited in Brayton and Willoughby [2], Khusainov and Yun'Kova [10], and Park and Won [16]. In general, delay-independent criteria are very conservative. This limits the applicability of delay-independent stability results. Above all, the time-delay considered in these works is constant and their results are only applicable to the systems with same delay arguments.

In this paper, the stability analysis for neutral delay-differential systems with mixed time-varying delay arguments is considered. Then, the goal of this paper is to find the delay-dependent criterion for asymptotic stability of the system using the Lyapunov method. In most of previous works, the stability criteria are expressed in terms of the matrix norm and matrix measure of system matrices. Unfortunately, the matrix norms and matrix measure operations usually make the criteria more conservative. However, the stability criterion derived in this paper will be expressed in terms of LMIs to find the less conservative criterion. The solutions of the LMIs can be easily solved by various effective optimization algorithms (Boyd et al. [1]).

Through the paper, the following notations are used. R^n denotes n dimensional Euclidean space, $R^{n\times m}$ is the set of all $n\times m$ real matrices, I denotes identity matrix of appropriate order, and $*$ denotes the symmetric part. $\|\cdot\|$ and $\mu(\cdot)$ denote the induced matrix 2-norm and corresponding matrix measure, respectively. The notation $X > Y$ (respectively, $X \geq Y$), where X and Y are matrices of same dimensions, means that the matrix $X-Y$ is positive definite(respectively, positive semi-definite).

2. Problem Formulation and Main Results

We are interested in the following linear systems of neutral type with mixed time-varying delay arguments described by:

This work was supported by Korea Research Foundation Grant (KRF-2001-003-E00278).
Manuscript received: June 18, 2001 accepted: Feb. 20, 2002.
J.H. Park is with School of Electrical Engineering and Computer Science, Yeungnam University, 214-1 Dae-Dong, Kyongsan 712-749, Korea.

$$\dot{x}(t)=Ax(t)+Bx(t-\tau(t))+C\dot{x}(t-h(t)), \tag{1}$$

with the initial condition function

$$x(t_0+\theta)=\phi(\theta), \quad \forall\ \theta\in[-\rho,0] \tag{2}$$

where $x(t)\in R^n$ is the state vector, A, B and $C\in R^{n\times n}$ are constant real matrices, $\tau(t)$ and $h(t)$ are positive time-varying bounded delays satisfying

$$\begin{aligned} &0<\tau(t)\le\bar{\tau}<\infty, \quad \dot{\tau}(t)\le\bar{\tau}_d<1\\ &0<h(t)\le\bar{h}<\infty, \quad \dot{h}(t)\le\bar{h}_d<1, \end{aligned}\tag{3}$$

$\rho=\max\{\bar{\tau},\bar{h}\}$, and $\phi(\cdot)$ is the given continuously differentiable function on $[-\rho,0]$. In the paper, the matrix $A+B$ is assumed to be a Hurwitz matrix. For the basic theory of various neutral systems, the reader is referred to the books by Hale and Verduyn Lunel [7]. Here, the goal of this paper is to find the delay-dependent criterion for asymptotic stability of the system using Lyapunov method with LMI technique.

Before we develop our main result, we introduce an inequality (Moon [14]), which is necessary to establish the criterion.

Assume that $a(\alpha)\in R^{n_x}$, $b(\alpha)\in R^{n_y}$ and $N\in R^{n_x\times n_y}$ are defined on the interval Ω. Then, for any positive definite matrix $X\in R^{n_x\times n_x}$, and for any matrix $Y\in R^{n_x\times n_y}$ and for any symmetric matrix $W\in R^{n_y\times n_y}$ satisfying

$$\begin{bmatrix} X & Y\\ Y^T & W\end{bmatrix}\ge 0,$$

the following inequality holds:

$$\begin{aligned} &-2\int_\Omega b^T(\alpha)\ N^T a(\alpha)d\alpha\\ &=\int_\Omega\begin{bmatrix}a(\alpha)\\ b(\alpha)\end{bmatrix}^T\begin{bmatrix}0 & -N\\ -N^T & 0\end{bmatrix}\begin{bmatrix}a(\alpha)\\ b(\alpha)\end{bmatrix}d\alpha\\ &\le\int_\Omega\begin{bmatrix}a(\alpha)\\ b(\alpha)\end{bmatrix}^T\left(\begin{bmatrix}0 & -N\\ -N^T & 0\end{bmatrix}+\begin{bmatrix}X & Y\\ -Y^T & W\end{bmatrix}\right)\begin{bmatrix}a(\alpha)\\ b(\alpha)\end{bmatrix}d\alpha\\ &=\int_\Omega\begin{bmatrix}a(\alpha)\\ b(\alpha)\end{bmatrix}^T\begin{bmatrix}X & Y-N\\ Y^T-N^T & W\end{bmatrix}\begin{bmatrix}a(\alpha)\\ b(\alpha)\end{bmatrix}d\alpha \end{aligned}\tag{4}$$

Then the following theorem gives a delay-dependent criterion for asymptotic stability of the system (1).

Theorem 1: For given scalars ρ, $\bar{\tau}_d$ and $\bar{h}_d$, the system (1) is asymptotically stable for any time-delays $\tau(t)$ and $h(t)$ satisfying (3), if there exist $n\times n$ matrices $P>0$, $Q>0$, $R>0$, $X>0$, $Z\ge 0$, and an $n\times n$ matrix Y satisfying the following LMIs:

$$\Omega(P,Q,X,Y,Z)=\begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13}\\ * & \Omega_{22} & \Omega_{23}\\ * & * & \Omega_{33}\end{bmatrix}<0 \tag{5}$$

and

$$\begin{bmatrix}X & Y\\ * & (1-\bar{\tau}_d)Z\end{bmatrix}\ge 0 \tag{6}$$

where the entries of the symmetric matrix $\Omega(\cdot)$ are

$$\begin{aligned} \Omega_{11}&\equiv PA+A^TP+\rho X+Y+Y^T+\rho A^TZA+R+A^TQA\\ \Omega_{12}&\equiv PB-Y+\rho A^TZB+A^TQB\\ \Omega_{13}&\equiv PC+\rho A^TZC+A^TQC\\ \Omega_{22}&\equiv \rho B^TZB-(1-\bar{\tau}_d)R+B^TQB\\ \Omega_{23}&\equiv \rho B^TZC+B^TQC\\ \Omega_{33}&\equiv \rho C^TZC+C^TQC-(1-\bar{h}_d)Q. \end{aligned}\tag{7}$$

Proof: Choose a legitimate Lyapunov functional [7] for the system (1) as

$$V=V_1+V_2+V_3+V_4 \tag{8}$$

where

$$V_1=x^T(t)Px(t) \tag{9}$$

$$V_2=\int_{-\tau(t)}^0\int_{t+\beta}^t\dot{x}^T(\alpha)Z\dot{x}(\alpha)\,d\alpha d\beta \tag{10}$$

$$V_3=\int_{t-\tau(t)}^t x^T(\alpha)Rx(\alpha)d\alpha \tag{11}$$

$$V_4=\int_{t-h(t)}^t\dot{x}^T(\alpha)Q\dot{x}(\alpha)d\alpha \tag{12}$$

where the positive definite matrices P, Q, R and the positive semi-definite matrix Z are to be found later, and recall that we are assuming (6).

Now, taking the time derivative of V, we obtain $\dot{V}=\dot{V}_1+\dot{V}_2+\dot{V}_3+\dot{V}_4$. First, from (9), we have

$$\dot{V}_1=\dot{x}^T(t)Px(t)+x^T(t)P\dot{x}(t). \tag{13}$$

Since (1) can be rewritten as

$$\begin{aligned}\dot{x}(t)=(A+B)x(t)-B\int_{t-\tau(t)}^t\dot{x}(\alpha)d\alpha\\ +C\dot{x}(t-h(t)),\end{aligned}\tag{14}$$

we obtain

$$\begin{aligned}\dot{V}_1&=2x^T(t)P(A+B)x(t)-2x^T(t)PB\int_{t-\tau(t)}^t\dot{x}(\alpha)d\alpha\\ &\quad+2x^T(t)PC\dot{x}(t-h(t))\end{aligned}\tag{15}$$

In order to apply the inequality (4) to the second term of right-hand side of (15), let

$a(\alpha)\equiv x(t)$, $b(\alpha)\equiv\dot{x}(\alpha)$, $N\equiv PB$, $W\equiv(1-\bar{\tau}_d)Z\ \ \forall\ \alpha\in[t-\bar{\tau},t]$.

Then, we have

$$
\begin{aligned}
&-2x^T(t)PB\int_{t-\tau(t)}^{t}\dot{x}(\alpha)\,d\alpha \\
&\le \int_{t-\tau(t)}^{t}\begin{bmatrix} x(t) \\ \dot{x}(\alpha)\end{bmatrix}\begin{bmatrix} X & Y-PB \\ * & (1-\bar{\tau})Z\end{bmatrix}\begin{bmatrix} x(t) \\ \dot{x}(\alpha)\end{bmatrix}d\alpha \\
&= \int_{t-\tau(t)}^{t}[x^T(t)Xx(t)+2x^T(t)(Y-PB)\,\dot{x}(\alpha) \\
&\quad +(1-\bar{\tau}_d)\,\dot{x}^T(\alpha)Z\dot{x}(\alpha)]d\alpha \\
&= \tau(t)x^T(t)Xx(t)+2x^T(t)(Y-PB)\int_{t-\tau(t)}^{t}\dot{x}(\alpha)\,d\alpha \\
&\quad + (1-\bar{\tau}_d)\int_{t-\tau(t)}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha. \qquad (16)
\end{aligned}
$$

Using the inequality (16), a new bound of $\dot{V}_1$ is

$$
\begin{aligned}
\dot{V}_1 &\le 2x^T(t)P(A+B)x(t)+\tau(t)x^T(t)Xx(t) \\
&\quad +2x^T(t)(Y-PB)\int_{t-\tau(t)}^{t}\dot{x}(\alpha)\,d\alpha \\
&\quad +2x^T(t)PC\,\dot{x}(t-h(t)) \\
&\quad +(1-\bar{\tau}_d)\int_{t-\tau(t)}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)\,d\alpha \\
&\le x^T(t)(A^TP+PA+Y+Y^T+\rho X)x(t) \\
&\quad +(1-\bar{\tau}_d)\int_{t-\tau(t)}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)\,d\alpha \\
&\quad -2x^T(t)(Y-PB)x(t-\tau(t)) \\
&\quad +2x^T(t)PC\,\dot{x}(t-h(t)). \qquad (17)
\end{aligned}
$$

Next, from (10)-(12), we obtain the time derivative of V_2, V_3 and V_4 as:

$$
\begin{aligned}
\dot{V}_2 &= \tau(t)\,\dot{x}^T(t)Z\dot{x}(t) \\
&\quad -(1-\dot{\tau}(t))\int_{t-\tau(t)}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)d\alpha \\
&\le \rho\,\dot{x}^T(t)Z\dot{x}(t)-(1-\bar{\tau}_d)\int_{t-\tau(t)}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)\,d\alpha \\
&= \rho[Ax(t)+Bx(t-\tau(t))+C\dot{x}(t-h(t))]^TZ[Ax(t) \\
&\quad +Bx(t-\tau(t))+C\dot{x}(t-h(t))] \\
&\quad -(1-\bar{\tau}_d)\int_{t-\tau(t)}^{t}\dot{x}^T(\alpha)Z\dot{x}(\alpha)\,d\alpha \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_3 &= x^T(t)Rx(t)-(1-\dot{\tau}(t))x^T(t-\tau(t))Rx(t-\tau(t)) \\
&\le x^T(t)Rx(t)-(1-\bar{\tau}_d)x^T(t-\tau(t))Rx(t-\tau(t)) \qquad (19)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_4 &= \dot{x}^T(t)Q\dot{x}(t) \\
&\quad -(1-\dot{h}(t))\,\dot{x}^T(t-h(t))Q\dot{x}(t-h(t)) \\
&\le \dot{x}^T(t)Q\dot{x}(t)-(1-\bar{h}_d)\,\dot{x}^T(t-h(t))Q\dot{x}(t-h(t)) \\
&= [Ax(t)+Bx(t-\tau(t))+C\dot{x}(t-h(t))]^TQ \\
&\quad \cdot[Ax(t)+Bx(t-\tau(t))+C\dot{x}(t-h(t))] \\
&\quad -(1-\bar{h}_d)\,\dot{x}^T(t-h(t))Q\dot{x}(t-h(t)) \qquad (20)
\end{aligned}
$$

Using (17)-(20), we have

$$
\begin{aligned}
\dot{V} &= \dot{V}_1+\dot{V}_2+\dot{V}_3+\dot{V}_4 \\
&\le \chi^T(t)\Omega(P,Q,R,X,Y,Z)\chi(t) \qquad (21)
\end{aligned}
$$

where $\chi(t) = [x^T(t)x^T(t-\tau(t))\ \dot{x}^T(t-h(t))]^T$.

Therefore, $\dot{V}$ is negative if the LMI conditions (5) and (6) hold, which guarantees the asymptotic stability of the system (1) [7]. This completes the proof.

Remark 1: Most of the criteria for asymptotic stability of neutral delay-differential systems are expressed in terms of matrix norm or matrix measure of the system matrices. Unfortunately, the matrix norm operations usually make the criteria more conservative. Also the criteria in recent studies (Chen [3], Hu and Hu [8], Hui and Hu [9], Li [13]), require strong assumptions such as the matrix measures of system matrices have to be negative. These assumptions often make it difficult to apply the criteria to various neutral systems.

Remark 2: For stability analysis of various control systems, the following inequality given two vectors $a, b \in R^n$,

$$-2a^Tb \le \eta \equiv \inf_{X>0}\{a^TXa+b^TX^{-1}b\}$$

has been widely used. In this case, the upper bound of $(-2a^Tb)$, $a^TXa+b^TX^{-1}b$ is always greater than or equal to zero. Therefore, if $(-2a^Tb) < 0$, the upper bound η is not a good estimate. So, the inequality given in (4) is utilized to obtain a good estimate [14].

Remark 3: In order to solve the LMIs given in Theorem 1, we can utilize Matlab's LMI Control Toolbox (Gahinet et al. [4]), which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithms (Boyd et al. [1]).

To illustrate the usefulness of the proposed method, we present the following examples.

Example 1: Consider the following neutral system with time-invariant delays

$$\dot{x}(t)=Ax(t)+Bx(t-2)+C\dot{x}(t-2)$$

where

$$A=\begin{bmatrix} 0 & 1 \\ -2 & -3\end{bmatrix},\quad B=\begin{bmatrix} 0 & 1 \\ 1 & 0\end{bmatrix},\quad C=\begin{bmatrix} 0.2 & 0 \\ 0 & 0.3\end{bmatrix}$$

By solving the LMIs given in (5) and (6), we obtain the solutions of the LMIs as

$$
\begin{aligned}
P &= \begin{bmatrix} 6.6413 & 4.3362 \\ 4.3362 & 4.8417\end{bmatrix},\quad Q=\begin{bmatrix} 2.0138 & 1.3761 \\ 1.3761 & 1.3861\end{bmatrix}, \\
R &= \begin{bmatrix} 4.0418 & 4.2134 \\ 4.2134 & 7.0629\end{bmatrix},\quad X=\begin{bmatrix} 1.5806 & 0.9060 \\ 0.9060 & 0.8938\end{bmatrix}, \\
Y &= \begin{bmatrix} -1.0080 & -0.2356 \\ -0.2356 & -0.0093\end{bmatrix},\quad Z=\begin{bmatrix} 1.6494 & 0.7997 \\ 0.7997 & 0.5645\end{bmatrix}.
\end{aligned}
$$

This implies the system is asymptotically stable for the time-invariant delays. However, note that since $\mu(A)=0.0811>0$, the criteria of Li [13] and Hu and Hu [8] are not applicable for this system.

Example 2: Consider the following system:

$$\dot{x}(t)=\begin{bmatrix}-2 & 0\\ 1 & -1\end{bmatrix}x(t)+\begin{bmatrix}-1 & 0\\ 0.5 & -1\end{bmatrix}x(t-\tau(t))+\begin{bmatrix}0.2 & 0.05\\ 0.05 & 0.2\end{bmatrix}\dot{x}(t-h(t))$$

where it is assumed that the time derivative of delays $\tau(t)$ and $h(t)$ are bounded as 0.25, i.e., $\bar{\tau}_d=0.25$ and $\bar{h}_d=0.25$.

By iteratively solving the LMIs (5) and (6) of Theorem 1 with respect to ρ, an upper bound on ρ that guarantees the asymptotic stability of the above system is 1.529. This implies the system is stable under the time delays, $\tau(t)\le 1.529$ with $\dot{\tau}(t)\le 0.25$ and $h(t)\le 1.529$ with $\dot{h}(t)\le 0.25$. Note that since $\mu(A)+\|B\|+\frac{\|CA\|+\|CB\|}{1-\|C\|}=1.2488>0$, the criteria of Hu and Hu [8] and Li [13] cannot be used to determine the stability of the system.

3. Conclusion

In this paper, a delay-dependent stability criterion for asymptotic stability of a class of neutral systems with mixed time-varying delay arguments has been proposed using the Lyapunov method. The criterion is expressed in terms of linear matrix inequalities, which can be easily solved by various convex optimization algorithms. Finally, two numerical examples are illustrated the proposed results, and gives that our obtained result is less conservative.

Acknowledgement

This work was supported by Korea Research Foundation Grant (KRF-2001-003-E00278).

References

[1] Boyd, B., Ghaoui, L.E, Feron, E. and Balakrishnan, V., 1994, Linear matrix inequalities in systems and control theory (Philadelphia: SIAM).

[2] Brayton, R.K. and Willoughby, R.A., 1967, On the numerical integration of a symmetric system of difference-differential equations of neutral type, Journal of Mathematical Analysis and Applications, 18, 182-189.

[3] Chen, W., 1999, Comment on 'simple criteria for stability of neutral systems with multiple delays', International Journal of Systems Science, 30, 1247-1248.

[4] Gahinet, P., Nemirovski, A., Laub, A. and Chilali, M.,} 1995, LMI Control Toolbox User's Guide (The Mathworks Inc

[5] Gopalsamy, K., 1992, Stability and Oscillations in Delay Differential Equations of Population Dynamics (Boston: Kluwer Academic Publishers).

[6] Hale, J., Infante, E.F. and FTsen, .-S.P., 1985, Stability in linear delay equations, Journal of Mathematical Analysis and Applications, 105, 533-555.

[7] Hale, J. and Verduyn Lunel, S.M., 1993, Introduction to Functional Differential Equations (New York: Springer-Verlag).

[8] Hu, G.D. and Hu, G.D., 1996, Some simple stability criteria of neutral delay-differential systems. Applied Mathematics and Computation, 80, 257-271.

[9] Hui, G.D. and Hu, G.D., 1997, Simple criteria for stability of neutral systems with multiple delays. International Journal of Systems Science, 28, 1325-1328.

[10] Khusainov, D.Y. and Yun'kova, E.V., 1988, Investigation of the stability of linear systems of neutral type by the Lyapunov function method. Differentsialnye Uravneniya}, 24, 613-621.

[11] Kolmanovskii, V. and Myshkis, A., 1992, Applied Theory of Functional Differential Equations (Dordrecht: Kluwer Academic Publishers).

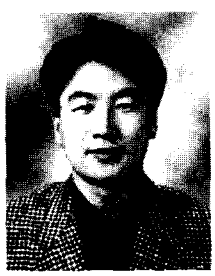
[12] Kuang, J.X., Xiang, J.X. and Tian, H.J., 1994, The asymptotic stability of one-parameter methods for neutral differential equations., 34, 400-408.

[13] Li, L.M.}, 1988, Stability of linear neutral delay-differential systems. Bulletin of Australian Mathematical Society, 38, 339-344.

[14] Moon, Y.S., 1998, Robust Control of Time-Delay System using Linear Matrix Inequalities Ph.D. Thesis, Seoul National University, Korea.

[15] Park, J.H. and Won, S., 1999a, A note on stability of neutral delay-differential systems, Journal of The Franklin Institute, 336, 543-548.

[16] Park, J.H. and Won, S., 1999b, Asymptotic stability of neutral systems with multiple delays, Journal of Optimization Theory and Applications, 103, 187-200.



J.H. Park received the Ph.D. at Pohang University of Science and Technology (POSTECH) in 1997. From 1997 to 2000, he was a research associate at Automation Research Center, POSTECH. In 2000, he joined the Yeungnam University. His research interests center on delay-differential systems and its related control problems, and convex optimization theory and applications.