Design of A Controller Using Successive Approximation for Weakly Coupled Bilinear Systems

Jae-Won Chang, Young-Joong Kim, Beom-Soo Kim and Myo-Taeg Lim

Abstract - In this paper, the infinite time optimal regulation problem for weakly coupled bilinear systems with quadratic performance criteria is obtained by a sequence of algebraic Lyapunov equations. This is the new approach is based on the successive approximation. In particular, the order reduction is achieved by using suitable state transformation so that the original Lyapunov equations are decomposed into the reduced-order local Lyapunov equations. The proposed algorithms not only solve optimal control problems in the weakly coupled bilinear system but also reduce the computation time. This paper also includes an example to demonstrate the procedures.
Keywords - weakly coupling, bilinear system, reduced-order, successive approximation, Lyapunov equation

1. Introduction

Weakly coupled systems were introduced to the control audience by [1], and then many researchers have examined various control aspects for linear weakly coupled system [2]. In between linear and nonlinear systems lies a very large class of so called bilinear systems [3]. In fact, bilinear models can explain a lot of real physical systems. However, there are few results of optimal control in the explicit feedback form for general regulation problem for bilinear systems. Thus, many researchers have studied numerical methods to find optimal controls for bilinear systems and recently the common approach is to solve the state and costate equations numerically on the basis of a Hamiltonian formulation of the optimal control problems. Although there have been a few attempts on the optimal control for the bilinear system [4],[5], previous studies have not dealt in the context of weakly coupled bilinear systems. The main result in [6] indicates that the application of the linear optimal control theory to the bilinear system with an iteration step should be carried out with Riccati equations. An infinite optimal control for the weakly coupled bilinear system is derived from a sequence of Riccati equations as illustrated in [8]. In [4], bilinear systems can be solved a sequence of the time- varying algebraic Lyapunov equations instead of a sequence of the time-varying algebraic Riccati equations.

In this paper, we derive a new optimal control algorithm for infinite-time weakly coupled bilinear systems using the successive approximation that is the approximate procedure is presented in [6]. We can solve the problem from the algebraic Lyapunov equations, instead of the algebraic Riccati equations as required in [6]. As a result, the Riccati equations are replaced by Lyapunov equations, which simplifies the actual computations and speeds up the convergence. In addition, the order-reduction is achieved by the weak coupling theory, so that original Lyapunov equations are decomposed into the reduced-order Lyapunov equations. Therefore, the optimal control problem can be solved with less computational efforts. This paper deals with the optimal control for bilinear system in section 2 and derives main results in section 3. With an example in section 4, the conclusion is followed in the last section.

2. Optimal Control of Weakly Coupled Bilinear Systems

The weakly coupled bilinear systems are represented by

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} A_1 & \varepsilon A_2 \\ \varepsilon A_3 & A_4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \left(\begin{bmatrix} B_1 & \varepsilon B_2 \\ \varepsilon B_3 & B_4 \end{bmatrix} + \{xM\} \right) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{1}$$

with initial condition

$$\begin{bmatrix} y_1(t_0) \\ y_2(t_0) \end{bmatrix} = \begin{bmatrix} y_1^0 \\ y_2^0 \end{bmatrix}$$

where $y_1 \in R^{n_1}$, $y_2 \in R^{n_2}$, and $x = [y_1^T \; y_2^T]^T \in R^n$ is the state vector with $n_1 + n_2 = n$, $u_1 \in R^{m_1}$, $u_2 \in R^{m_2}$, $u = [u_1^T \; u_2^T]^T \in R^m$ is a control vector with $m_1 + m_2 = m$ and ε is a small positive coupling parameters, with

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$$\{xM\} = \left\{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\begin{bmatrix} M_a & \varepsilon M_b \\ \varepsilon M_c & M_d \end{bmatrix}\right\} = \sum_{i=1}^{n_1} y_{1i}\begin{bmatrix} M_{ai} & \varepsilon M_{bi} \\ \varepsilon M_{ci} & M_{di} \end{bmatrix} + \sum_{j=n_1+1}^{n_1+n_2} y_{2(j-n_1)}\begin{bmatrix} M_{aj} & \varepsilon M_{bj} \\ \varepsilon M_{cj} & M_{dj} \end{bmatrix} \quad (2)$$

A quadratic cost functional to be minimized is associated with (1) and it has the following form

$$J = \frac{1}{2}\int_{t_0}^{\infty}(x^TQx + u^TRu) \quad (3)$$

with $Q \geq 0$, $R > 0$ possessing the weak coupling structures, that is

$$Q = \begin{bmatrix} Q_1 & \varepsilon Q_2 \\ \varepsilon Q_2^T & Q_3 \end{bmatrix}, \quad R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \quad (4)$$

Corresponding Hamiltonian is given by

$$H(x, u, \frac{\partial J}{\partial x}, t) = \frac{1}{2}(x^TQx + u^TRu) + \frac{\partial J}{\partial x}^T[Ax + (B + \{xM\})u] \quad (5)$$

This optimization problem has been studied in [8] under following assumption.

Assumption 1 The pair (A, B) is stabilizable, x stays in the stabilizability domain $X_s = \{x \in R^n | (A, B + \{xM\})$ is stabilizable$\}$, and the pair $(A, \sqrt{Q})$ is detectable.

The dynamic programming approach applied to (1)-(3) results in a steady-state Hamilton-Jacobi-Bellman equation of the form [6].

$$\frac{1}{2}x^TQx + J_x^TAx - \frac{1}{2}J_x^T(B + \{xM\})R^{-1}(B + \{xM\})^TJ_x = 0 \quad (6)$$

Since it is known in [9] for the infinite-time case $J = J_x$, the solution of the Hamilton-Jacobi-Bellman equation can be sought in the form $J_x = P(x)x$ where the matrix-valued function $P(x)$ is symmetric [9]. Equation (6) is now reduced to

$$Q + P(x)A + A^TP(x) - P(x)(B + \{xM\})R^{-1}(B + \{xM\})^TP(s) = 0 \quad (7)$$

The required optimal control is in the form

$$u^{opt} = -R^{-1}(B + \{xM\})^TP(x)x \quad (8)$$

Unfortunately, there is no analytical solution to Equation (7). Thus, we need to find an approximate method to solve the optimal control problem of bilinear systems.

It was shown in [10] that the solutions of a sequence of linear differential equation.

$$\dot{x}^{(0)} = Ax^{(0)} + Bu, \quad x^{(0)}(t_0) = x^0 \quad (9)$$

$$\dot{x}^{(i)} = Ax^{(i)} + \{x^{(i-1)}(t)M\}u^{(i)} + Bu^{(i)}$$
$$x^{(i)}(t_0) = x^0, \quad i = 1, 2, \cdots \quad (10)$$

uniformly converge the same control input u to the solution x of (1)-(2). Applying the above result and Assumption, the optimization problem biliear system (1)-(2), subject to (3), is replaced by a sequence of linear-quadratic optimization problems in [8],

$$\dot{x}^{(i)} = Ax^{(i)} + \tilde{B}^{(i-1)}(t)u^{(i)} \quad (11)$$

where

$$\tilde{B}^{(i-1)}(t) \equiv B + \{x^{(i-1)}(t)M\}$$

with the algebraic Riccati equation

$$Q + P^{(i)}(t)\tilde{A}^{(i)}(t) + \tilde{A}^{(i)^T}(t)P^{(i)}(t) - P^{(i)}(t)\tilde{B}^{(i-1)}(t)R^{-1}\tilde{B}^{(i-1)^T}(t)P^{(i)}(t) = 0 \quad (12)$$

and

$$\dot{x}^{(i)}(t) = (A - \tilde{B}^{(i-1)}(t)R^{-1}\tilde{B}^{(i-1)^T}(t)P^{(i)}(t))x^{(i)}(t)$$
$$x^{(i)}(t_0) = x^0, \quad i = 1, 2, \cdots \quad (13)$$

The sequence (12)-(13) is initialized by $P^{(0)}(t)$, and the real symmetric positive definite matrix is acquired by the following algebraic Riccati equation.

$$Q + P^{(0)}(t)A + A^TP^{(0)}(t) - P^{(0)}(t)BR^{-1}B^TP^{(0)}(t) = 0 \quad (14)$$

This solution stabilizes the system

$$\dot{x}^{(0)}(t) = (A - BR^{-1}B^TP^{(0)}(t))x^{(0)}(t)$$
$$x^{(0)}(t_0) = x^0 \quad (15)$$

After only a few iteration steps $P^{(i)}(t)$, the solution of the time-varying algebraic Riccati equation in the iterate scheme (12)-(13), gets equally close to the optimal solution $P(t)$ of (7).

3. Weakly Coupled Bilinear Systems Using Successive Approximation

The method of successive approximations is the main tool in solving the functional equation of dynamic programming. It has been used in several control theory papers [11]. We derive a new algorithms using successive approximation method. The successive approximation technique applied to (1)-(4) is composed of the following steps

Step 1 Obtain $P^{(0)}(t)$ from the following algebraic Riccati equation

$$Q+P^{(0)}(t)A+A^TP^{(0)}(t)-P^{(0)}(t)BR^{-1}B^TP^{(0)}(t)=0 \quad (16)$$

along the constraint of dynamical system

$$\dot{x}^{(0)}(t)=(A-BR^{-1}B^TP^{(0)}(t))x^{(0)},\ x^{(0)}(t_0)=x^0 \quad (17)$$

Therefore, we can obtain the stabilizing linear control law

$$u^{(0)}(x(t))=-R^{-1}B^TP^{(0)}(t)x(t)$$

with $P^{(0)}(t)$ being symmetric.

Step 2 Convert the weakly coupled bilinear systems (1) into the following linear time varying form (18).

$$\dot{x}^{(i)}(t)=Ax^{(i)}(t)+\tilde{B}^{(i)}(t)u^{(i)}(t) \quad (18)$$

where

$$\tilde{B}^{(i)}(t)=B+\{x^{(i)}(t)M\}$$
$$u^{(i)}(t)=-R^{-1}(t)\ \tilde{B}^{(i)^T}(t)P^{(i)}(t)x^{(i)}(t),\quad i=1,2,\cdots$$

Find the expression for $\partial J^{(i)}/\partial x(t)$ of the performance criterion

$$J^{(i)}(t) = \frac{1}{2}\int_t^\infty x^{(i)^T}(\tau)[Q+P^{(i)}(\tau) \cdot \tilde{B}^{(i)}(\tau)R\ \tilde{B}^{(i)^T}(\tau)P^{(i)}(\tau)]x^{(i)}(\tau)d\tau \quad (19)$$

Step 3 For the known value of $\partial J^{(i)}/\partial x(t)$, we derive Lyapunov Equation.

$$\frac{dJ(t)}{dt^{(i)}} = \frac{\partial J(t)}{\partial x(t)}^{(i)}\frac{dx(t)}{dt}^{(i)} = -\frac{1}{2}(x^{(i)}(t)Qx^{(i)}(t)+u^{(i)^T}(t)Ru^{(i)}(t)) \quad (20)$$

Applying the known $(\partial J(t)/\partial x(t))^{(i)}$, (19) and the results of [12] are expressed in (22).

$$J^{(i)}=\frac{1}{2}x^{(i)^T}(t)P^{(i+1)}(t)x^{(i)}(t) \quad (21)$$

We bring out the following Lyapunov equation

$$\tilde{A}^{(i)^T}(t)P^{(i+1)}(t)+P^{(i+1)}(t)\ \tilde{A}^{(i)}(t)+\tilde{Q}^{(i)}(t)=0 \quad (22)$$

In (22), $\tilde{A}^{(i)}(t)$ and $\tilde{Q}^{(i)}(t)$ are given by

$$\tilde{A}^{(i)}(t) = A-S^{(i)}(t)P^{(i)}(t) = \begin{bmatrix} \tilde{A}_1^{(i)}(t) & \varepsilon\tilde{A}_2^{(i)}(t) \\ \varepsilon\tilde{A}_3^{(i)}(t) & \tilde{A}_4^{(i)}(t)\end{bmatrix} \quad (23)$$

$$\tilde{Q}^{(i)}(t) = Q+P^{(i)}(t)S^{(i)}(t)P^{(i)}(t) = \begin{bmatrix} \tilde{Q}_1^{(i)}(t) & \varepsilon\tilde{Q}_2^{(i)}(t) \\ \varepsilon\tilde{Q}_2^{(i)T}(t) & \tilde{Q}_3^{(i)}(t)\end{bmatrix} \quad (24)$$

where

$$S^{(i)}(t) = \tilde{B}^{(i)}(t)R^{-1}\tilde{B}^{(i)^T}(t) = \begin{bmatrix} \tilde{S}_1^{(i)}(t) & \varepsilon\tilde{S}_2^{(i)}(t) \\ \varepsilon\tilde{S}_2^{(i)^T}(t) & \tilde{S}_3^{(i)}(t)\end{bmatrix}$$

$$\tilde{A}_1^{(i)} = A_1-S_1^{(i)}(t)P_1^{(i)}(t)-\varepsilon^2S_2^{(i)}(t)P_2^{(i)^T}(t)$$
$$\tilde{A}_2^{(i)} = A_2-S_1^{(i)}(t)P_2^{(i)}(t)-S_2^{(i)}(t)P_3^{(i)}(t)$$
$$\tilde{A}_3^{(i)} = A_3-S_2^{(i)^T}(t)P_1^{(i)}(t)-S_3^{(i)}(t)P_2^{(i)^T}(t)$$
$$\tilde{A}_4^{(i)} = A_4-S_3^{(i)}(t)P_3^{(i)}(t)-\varepsilon^2S_2^{(i)T}(t)P_2^{(i)}(t)$$

and

$$\tilde{Q}_1^{(i)}=Q_1+P_1^{(i)}(t)S_1^{(i)}(t)P_1^{(i)}(t)+\varepsilon^2P_2^{(i)}(t)S_2^{(i)^T}(t)P_1^{(i)}(t)+\varepsilon^2P_1^{(i)}(t)S_2^{(i)}(t)P_2^{(i)^T}(t)+\varepsilon^2P_2^{(i)}(t)S_3^{(i)}(t)P_2^{(i)^T}(t)$$
$$\tilde{Q}_2^{(i)}=Q_2+P_1^{(i)}(t)S_1^{(i)}(t)P_2^{(i)}(t)+\varepsilon^2P_2^{(i)}(t)S_2^{(i)^T}(t)P_2^{(i)}(t)+P_1^{(i)}(t)S_2^{(i)}(t)P_3^{(i)}(t)+P_2^{(i)}(t)S_3^{(i)}(t)P_3^{(i)}(t)$$
$$\tilde{Q}_3^{(i)}=Q_3+P_3^{(i)}(t)S_3^{(i)}(t)P_3^{(i)}(t)+\varepsilon^2P_3^{(i)}(t)S_2^{(i)^T}(t)P_2^{(i)}(t)+\varepsilon^2P_2^{(i)^T}(t)S_1^{(i)}(t)P_2^{(i)}(t)+\varepsilon^2P_2^{(i)^T}(t)S_2^{(i)}(t)P_3^{(i)}(t)$$

Step 4 Applying the transformation to block diagonalize matrix $\tilde{A}^{(i)}(t)$, we will obtain the decoupled Lyapunov equations. The corresponding nonsingular transformation is given by

$$T_2=\begin{bmatrix} I & -\varepsilon L_2 \\ \varepsilon H_2 & I-\varepsilon^2H_2L_2\end{bmatrix},\ T_2^{-1}=\begin{bmatrix} I-\varepsilon^2L_2H_2 & \varepsilon L_2 \\ -\varepsilon H_2 & I\end{bmatrix} \quad (25)$$

where matrices L_2 and H_2 satisfy the following algebraic equations

$$\tilde{A}_1^{(i)}L_2-L_2\tilde{A}_4^{(i)}+\tilde{A}_2^{(i)}-\varepsilon^2L_2\tilde{A}_3^{(i)}L_2=0 \quad (26)$$

and

$$H_2(\tilde{A}_1^{(i)}-\varepsilon^2L_2\tilde{A}_3^{(i)}) -(\tilde{A}_4^{(i)+}\varepsilon^2\tilde{A}_3^{(i)}L_2)H_2+\tilde{A}_3^{(i)}=0 \quad (27)$$

Multiplying the full-order Lyapunov equation (22) from left by T_2^{-T} and from right by T_2^{-1}, (22) can be rewritten as

$$T_2^{-T}\tilde{A}^{(i)T}(t)P^{(i+1)}(t)T_2^{-1}+T_2^{-T}P^{(i+1)}(t) \cdot \tilde{A}^{(i)}(t)T_2^{-1}+T_2^{-T}\tilde{Q}^{(i)}T_2^{-1}=0 \quad (28)$$

we obtain the following Lyapunov equation

$$K^{(i+1)}(t)a^{(i)}(t)+a^{(i)T}(t)K^{(i+1)}(t)+q^{(i)}(t)=0$$
$$K^{(i+1)}(t_0)=K^{(0)}=T_2^{-T}P^{(0)}T_2^{-1} \quad (29)$$

where

$$\begin{aligned} a^{(i)}(t) &= T_2 \tilde{A}^{(i)}(t) T_2^{-1} = \begin{bmatrix} \tilde{A}_{10}{}^{(i)}(t) & 0 \\ 0 & \tilde{A}_{40}{}^{(i)}(t) \end{bmatrix} \\ q^{(i)}(t) &= T_2^{-T} \tilde{Q}^{(i)}(t) T_2^{-1} \\ K^{(i+1)}(t) &= T^{-T} P^{(i+1)}(t) T_2^{-1} \\ i &= 1,2,3,\cdots \end{aligned}$$

The following partitioning and scaling are used for matrices $K^{(i+1)}(t)$ and $q^{(i)}(t)$.

$$K^{(i+1)}(t) = \begin{bmatrix} K_1^{(i+1)}(t) & \varepsilon K_2^{(i+1)}(t) \\ \varepsilon K_2^{(i+1)^T}(t) & K_3^{(i+1)}(t) \end{bmatrix} \tag{30}$$

$$q^{(i)}(t) = \begin{bmatrix} q_1^{(i)}(t) & \varepsilon q_2^{(i)}(t) \\ \varepsilon q_2^{(i)^T}(t) & q_3^{(i)}(t) \end{bmatrix} \tag{31}$$

Partitioning (29) according to (30)-(31) reveals completely decoupled reduced-order equations

$$\begin{aligned} K_1^{(i+1)}(t)\ \tilde{A}_{10}{}^{(i)}(t) + \tilde{A}_{10}{}^{(i)^T}(t) K_1^{(i+1)}(t) + q_1^{(i)}(t) = 0 \\ K_2^{(i+1)}(t)\ \tilde{A}_{40}{}^{(i)}(t) + \tilde{A}_{10}{}^{(i)^T}(t) K_2^{(i+1)}(t) + q_2^{(i)}(t) = 0 \\ K_3^{(i+1)}(t)\ \tilde{A}_{40}{}^{(i)}(t) + \tilde{A}_{40}{}^{(i)^T}(t) K_3^{(i+1)}(t) + q_3^{(i)}(t) = 0 \end{aligned} \tag{32}$$

Having obtained $K^{(i+1)}(t), i=1,2,\cdots$, from (32), we can reach the solution of the Lyapunov equation in the original coordinates as

$$P^{(i+1)}(t) = T_2^T K^{(i+1)}(t) T_2, \quad i=1,2,\cdots.$$

Step 5 Obtained $P^{(i+1)}(t)$, we get following approximated linear control law

$$u^{(i+1)}(t) = -R^{-1}\ \tilde{B}^{(i+1)^T}(t) P^{(i+1)}(t) x^{(i+1)}(t) \tag{33}$$

and systems are expressed by

$$\dot{x}^{(i+1)}(t) = [A - \tilde{B}^{(i+1)}(t) R^{-1} \tilde{B}^{(i+1)^T}(t) P^{(i+1)}(t)] x^{(i+1)}(t) \tag{34}$$

Step 6 Iterate the Step2-5 until convergence is satisfied. △

This algorithm provide explicit control laws. Moreover, we can obtain the control law easily by solving algebraic Lyapunov equations.

4. The Numerical Example

In order to demonstrate the efficiency of the proposed algorithm for closed-loop control of weakly coupled bilinear systems, we simulate a fourth-order real world example, a paper making machine control problem [14]. The bilinear mathematical model of this system is formulated according to (1) and (3) as

$$A = \begin{bmatrix} -1.93 & 0 & 0 & 0 \\ 0.394 & -0.426 & 0 & 0 \\ 0 & 0 & -0.63 & 0 \\ 0.095 & -0.103 & 0.413 & -0.426 \end{bmatrix},$$

$$B = \begin{bmatrix} 1.274 & 1.274 \\ 0 & 0 \\ 1.34 & -0.65 \\ 0 & 0 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.755 & 0.366 \\ 0 & 0 \end{bmatrix},$$

$$M_2 = M_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.718 & -0.718 \\ 0 & 0 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1 & 0 & 0.13 & 0 \\ 0 & 1 & 0 & 0.09 \\ 0.13 & 0 & 0.1 & 0 \\ 0 & 0.09 & 0 & 0.2 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The simulation results are presented in the figure 1-6.

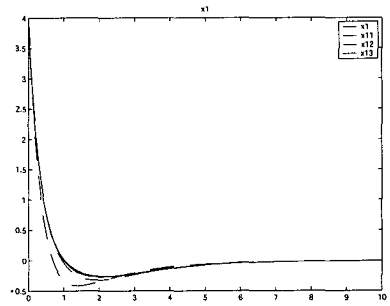

Fig. 1 Trajectories x_1

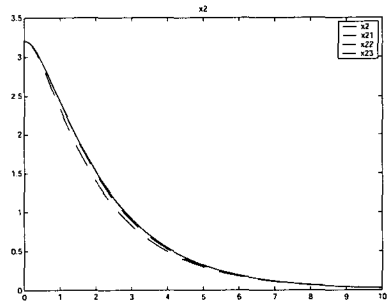

Fig. 2 Trajectories of x_2

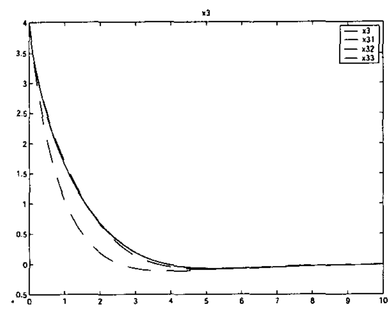

Fig. 3 Trajectories of x_3

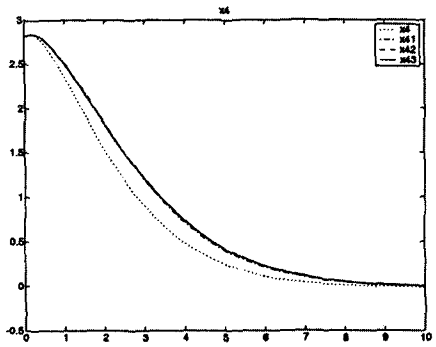

Fig. 4 Trajectories of x_4

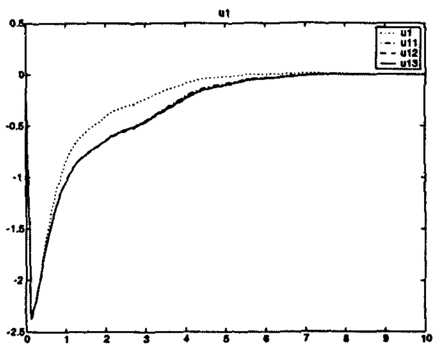

Fig. 5 Trajectories of u_1

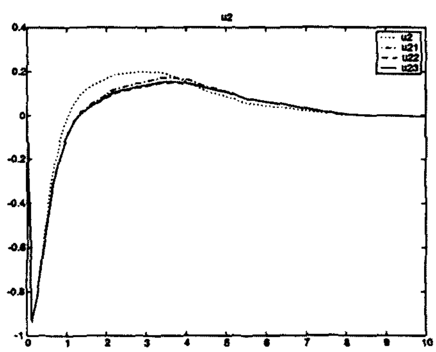

Fig. 6 Trajectories of u_2

Fig. 1-4 represent the approximate and the optimal state trajectories, and the fig. 5-6 represent the optimal controls. The optimal ones are represented by the solid lines. It can be seen from the these plots that the approximate trajectories are very good approximations for the optimal ones. The number of iterations performed is $i=3$, where i represents the number of linear time varying systems in the sequence defined by (18). Each iteration step, FLOPS are expressed as follows:

	P(Riccati)	P(Lyap)	P(reduced Lyap)
FLOPS	19237	5526	1804

Therefore, the number of computations is reduced. All numerical results in this paper are obtained by using MATLAB software and its control tool box.

5. Conclusion

In this paper, the new optimal control algorithm for the infinite-time weakly coupled bilinear systems using the successive approximation method is presented. Since the bilinear system has weakly coupled structure, this method avoids ill-defined numerical problem and reduces the number of computations and speeds up the optimization process by using a sequence of Lyapunov equations.

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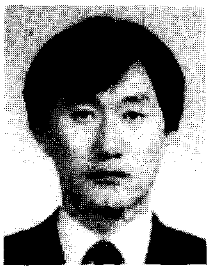

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