Flux Linkage Calculation for 3-D Finite Element Analysis

Chang-Hwan Im, Hyun-Kyo Jung and Hong-Kyu Kim

Abstract - Novel method to calculate flux linkage for 3-D finite element analysis is proposed. It does not require any integral path if the current direction in a coil is known. The flux linkage can be calculated very easily using simple volume based integration. The current direction is calculated based on the recently developed technique by the authors. The novel method for flux linkage calculation is verified by applying to a very complicated deflection yoke coil. The simulation result is compared to the experimental one. From the simulation, it is shown that the proposed method is very accurate and effective to calculate the flux linkage of a coil.

Keywords - current density distribution, deflection yoke(DY), finite element methods(FEM), flux linkage

1. Introduction

Flux linkage calculation is a very essential part in analyzing electric devices using 3-dimensional finite element method (FEM). The flux linkage should be calculated for the back-EMF analysis, inductance calculation, voltage source FEM, and so on. Although there are some methods that can be easily thought, a general method applied to an arbitrary shaped coil has not been developed yet.

The easiest way to calculate flux linkage is a direct line integration using

$$\lambda = N\int_C A\cdot dl \tag{1}$$

where λ is flux linkage, A vector potential, l directional vector, N the number of coil turns and C integration path.

An integration path along a solidly modeled coil is generated as shown in Fig. 1. Then, the vector potentials at each sampling point are calculated by interpolation. The term, dl, is evaluated by using two successive sampling points. Then the flux linkage is calculated by summing the values of $A\cdot dl$ at all the sampling points. However, there are some problems in applying the method to practical cases. First, finding the current paths and sampling points of a complicated coil is very difficult. Second, multiple paths that contain different turns are needed in case of a separated coil such as a deflection yoke. Third, if the cross-sectional area of a coil is large or varies along the coil direction, additional paths are required to averaging the vector potential values. After synthesizing these disadvantages, it can be concluded that the method is not practical enough to apply to general coil systems.

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In this paper, a novel method to calculate flux linkage for 3-dimensional finite element analysis is proposed. The proposed method does not require any integral path if the direction of current in a coil is known. Using the method, the line integration of vector potential is changed into the volume integration. Current direction and cross-sectional areas are calculated based on the novel technique developed by the authors [1]. The method proposed in this paper is verified by applying to a very complicated deflection yoke model. The simulation result is verified by the comparison with the experimental one. From the case study, it will be proved that the proposed method is very accurate and effective to calculate the flux linkage of a coil.

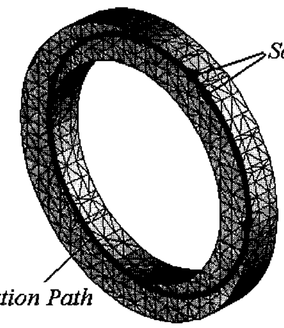

Fig. 1 Direct integration method to calculate flux linkage

2. Novel Method to Calculate Flux Linkage

2.1 Case I – Cross-sectional Area of Coil is Uniform

The basic equation to calculate flux linkage is given as (1). In the magnetic field analysis with vector potential formulation, coil parts are usually modeled as solid forms. To calculate the flux linkage more effectively, the line integration in (1) will be transformed into volume integration. First of all, following three assumptions are needed before the transformation.

(*Assumption 1*) The current direction of an element that belongs to coil region is uniform.

(*Assumption 2*) The value of vector potential of the element is assumed to be uniform (average value).

(*Assumption 3*) Coil region includes enough elements to satisfy the above two assumptions.

Assume an element (e) in the coil region as shown in Fig. 2(a). At the first stage, the direction of current should be determined as in the figure. The current direction can be determined by using a recently proposed technique by the authors [1]. The detailed description is contained in the APPENDIX. Then, in the element (e), n surfaces that are perpendicular to the current direction are determined as shown in Fig. 2(b). In fact, those surfaces correspond to equip-potential surfaces when electrostatic finite element analysis is performed. It is because current density J is in proportion to electric field intensity E and E is perpendicular to the equip-potential surfaces.

In case that the cross-sectional area of a whole coil is uniform as the case of Fig. 1, the number of the coil turns passing through an unit area T_{eff} can be written as

$$T_{eff} = \frac{N}{S_{cross}} \tag{2}$$

where, N is the total number of coil turns and S_{cross} is the cross-sectional area. Then, the number of the coil turns passing through the i-th surface of the element (e) $T_i^{(e)}$ can be evaluated by

$$T_i^{(e)} = T_{eff} S_i^{(e)} = \frac{N}{S_{cross}} S_i^{(e)} \tag{3}$$

where, $S_i^{(e)}$ is the area of the i-th surface.

Then, the flux linkage in (1) can be modified as

$$\lambda = \sum_{(e)} \sum_{i=1}^{n} \boldsymbol{A}^{(e)} \cdot \boldsymbol{l}^{(e)} \; T_i^{(e)} d_i^{(e)} \tag{4}$$

where, $A^{(e)}$ is the average vector potential of an element (e), $l^{(e)}$ is the unit directional vector of current flow, and $d_i^{(e)}$ the distance between i and $i+1$ th surface.

Substituting (3) for (4) yields

$$\lambda = \sum_{(e)} \sum_{i=1}^{n} \boldsymbol{A}^{(e)} \cdot \boldsymbol{l}^{(e)} \; \frac{N}{S_{cross}} S_i^{(e)} d_i^{(e)} \; . \tag{5}$$

This equation can be rewritten as follows:

$$\lambda = \frac{N}{S_{cross}} \sum_{(e)} \boldsymbol{A}^{(e)} \cdot \boldsymbol{l}^{(e)} \sum_{i=1}^{n} S_i^{(e)} d_i^{(e)} \tag{6}$$

From the assumption that all the surfaces in the element are parallel each other, following equation can be deduced:

$$\lim_{n \to \infty} \sum_{i=1}^{n} S_i^{(e)} d_i^{(e)} = V^{(e)} \tag{7}$$

where $V^{(e)}$ means the volume of the element.

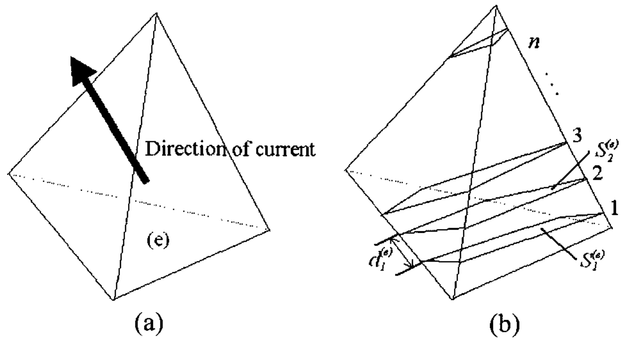

Fig. 2 Definitions of variables in an element: (a) direction of current (b) surfaces perpendicular to the direction

Finally, the flux linkage can be calculated by

$$\lambda = \frac{N}{S_{cross}} \sum_{(e)} \boldsymbol{A}^{(e)} \cdot \boldsymbol{l}^{(e)} \; V^{(e)} \; . \tag{8}$$

From the last equation, it is found that the flux linkage can be calculated very easily by using the values of current direction, vector potential, and elemental volume.

2.2 Case II – Cross-sectional Area of Coil is not Uniform

Practically, in case of the deflection yoke, coil winding is highly pressed and deformed with maintaining its thickness, which makes the cross-sectional areas perpendicular to the current directions to be non-uniform [4]. In this case, equations (2) and (3) are not valid because the number of the coil turns passing through unit area varies from an element to another element. The problem can be resolved by using a concept adopted in [1]. First, two arbitrary potentials are imposed as a boundary condition on the cross sections *I* and *II* in Fig. 3. By the electrostatic analysis using FEM, electric field intensity vector E can be evaluated for each element. The direction of E is the same as that of current density, as mentioned in the previous section. Between two potentials imposed on the cross sections *I* and *II*, m potentials are determined (ϕ_j, $j = 1,2,\ldots, m$) and equipotential surfaces for each potential are found. Then the areas of each equipotential surface, A_j ($j = 1,2,\ldots, m$) are calculated. Because the current direction is perpendicular to the equipotential surface, the number of the coil turns passing through unit area of each surface can be defined as

$$T_{equip}^{j} = \frac{N}{A_j} \tag{9}$$

where, T_{equip}^{j} is the number of the coil turns passing through unit area of j-th equipotential surface and N is the total number of turns. If the average electric scalar potential of an element (e) is $\phi^{(e)}$ and the value is between ϕ_j and ϕ_{j+1}, the number of turns passing through the element $T_{eff}^{(e)}$ can be calculated by

$$T_{eff}^{(e)} = \frac{\phi^{(e)} - \phi_j}{\phi_{j+1} - \phi_j} T_{equip}^{j+1} - \frac{\phi^{(e)} - \phi_{j+1}}{\phi_{j+1} - \phi_j} T_{equip}^{j} \tag{10}$$

Then, the number of the coil turns passing through i-th surface in the element (e) $T_i^{(e)}$ can be evaluated by (11) instead of (3).

$$T_i^{(e)} = T_{eff}^{(e)} S_i^{(e)} \tag{11}$$

Then, the flux linkage in (5) is rewritten as

$$\lambda = \sum_{(e)} \sum_{i=1}^{n} \boldsymbol{A}^{(e)} \cdot \boldsymbol{l}^{(e)} T_{eff}^{(e)} S_i^{(e)} d_i^{(e)} . \tag{12}$$

Applying (7) to the equation (12) yields the final equation as follows:

$$\lambda = \sum_{(e)} \boldsymbol{A}^{(e)} \cdot \boldsymbol{l}^{(e)} T_{eff}^{(e)} V^{(e)} . \tag{13}$$

From (13), it can be seen that the only difference between two cases lies in describing $T_{eff}^{(e)}$. Equation (8) is just a special case when the term is constant for all elements.

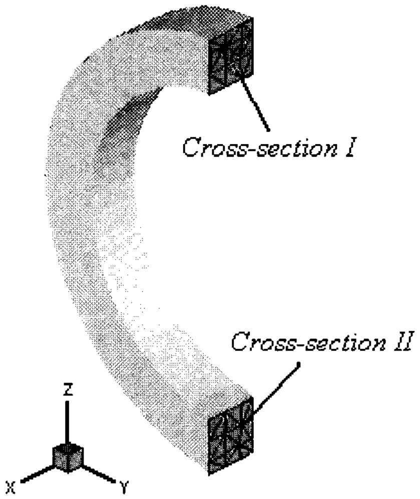

Fig. 3 Model to explain the concept of electrostatic field analysis

3. Simulation and Results

The proposed method is applied to a very complicated deflection yoke model. The simulation result is verified by the comparison with the experimental one.

A deflection yoke (DY) is usually used for deflecting electrons in color display tubes (CDTs) or color picture tubes (CPTs) [3]. The DY consists of three parts – horizontal coil, vertical coil and ferrite core. Because of a pressing process during the winding process, some parts of the coils are highly distorted. Therefore, the cross-sectional areas perpendicular to the winding direction are not uniform.

Among various characteristics of the DY, the sensitivity of a deflection coil is a very important factor that is closely related to energy saving. The sensitivity is defined as $L \times I^2$ (Inductance × square of maximum current). To save energy consumption, it is obvious that smaller value of the sensitivity is better. As seen in the definition, the inductance calculation of a coil is the main part to calculate the sensitivity. Because the sensitivity of a horizontal coil is much larger than that of a vertical coil and the mutual inductance between two coils is zero, only the horizontal coil is considered in this paper.

Fig. 4 shows the deflection yoke that consists of horizontal coil and ferrite core. The complicated coil is modeled by using a very accurate coil winding modeler that is presented in [4]. Fig. 5 shows the manufactured one. Fig. 6 shows the generated meshes of the horizontal coil and the ferrite core. Total numbers of tetrahedral elements and nodes are 85383 and 14252, respectively.

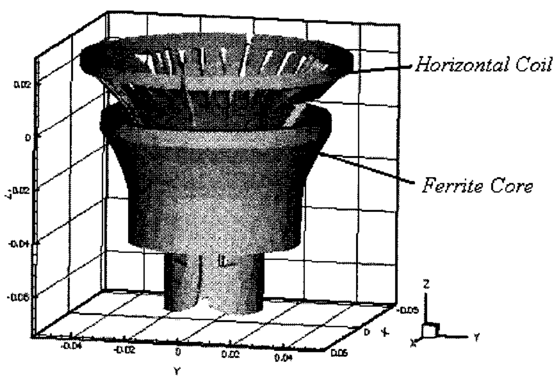

Fig. 4 Analysis model – deflection yoke with horizontal coil and ferrite core

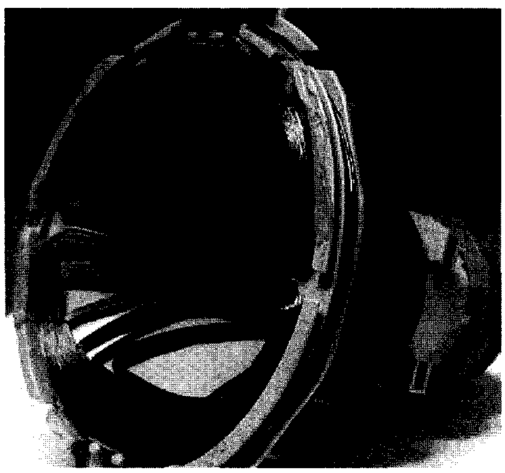

Fig. 5 Manufactured deflection yoke

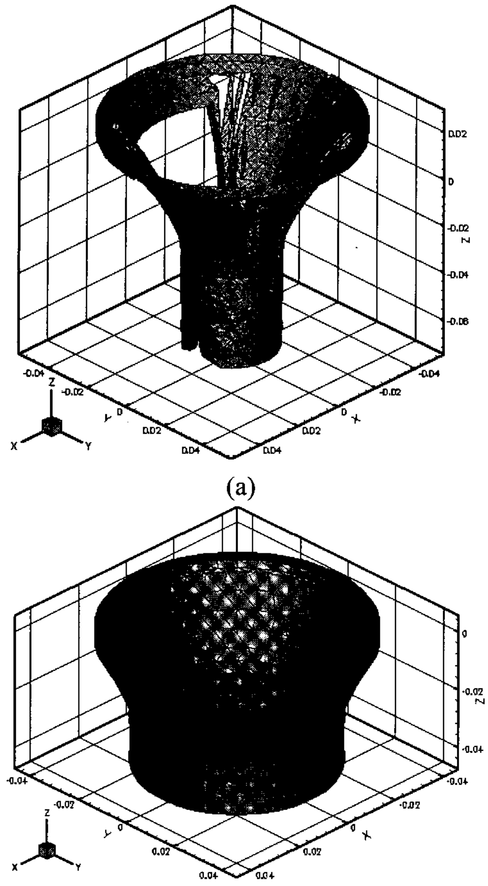

Fig. 6 Generated meshes (a) meshes of horizontal coil (b) mesh of ferrite core

In the analysis of the DY, the accurate calculation of current density distribution is the most important. The current density distribution of the coil is analyzed based on the electric field analysis as the previous case. Calculated electric potential and current density distribution will be used for the flux linkage calculation again. Fig. 7 shows a quarter of a whole coil system. To perform an electrostatic FEM, different boundary conditions (two arbitrary potentials, e.g. $0V$ and $100V$) are imposed on the cross-sections *I* and *II*. From the electrostatic analysis, the equipotential surfaces for calculating the current density distribution are obtained as shown in Fig. 8. Fig. 9 shows the current density vector distribution calculated by the proposed method.

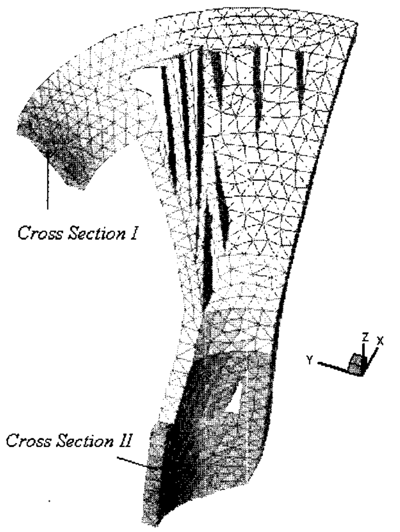

Fig. 7 A quarter model of horizontal coil for the electric field analysis

Magnetic field analysis is performed using the 3-D FEM formulated with vector potentials. Nonlinear analysis is not required because the current is too small to saturate the ferrite core. The calculated flux density vector is shown in Fig. 10. It is found, from the figure, that a uniform field is formed inside the ferrite core.

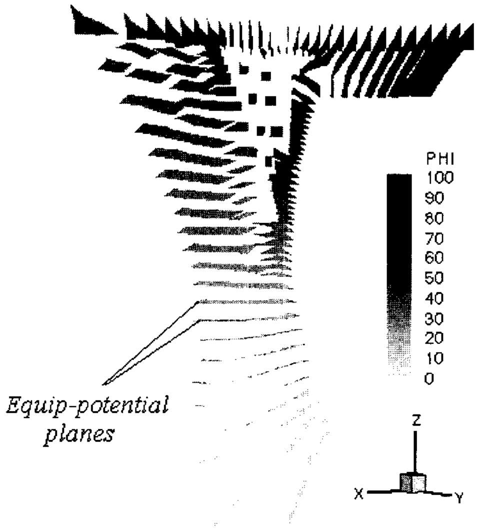

Fig. 8 Calculated equip-potential planes

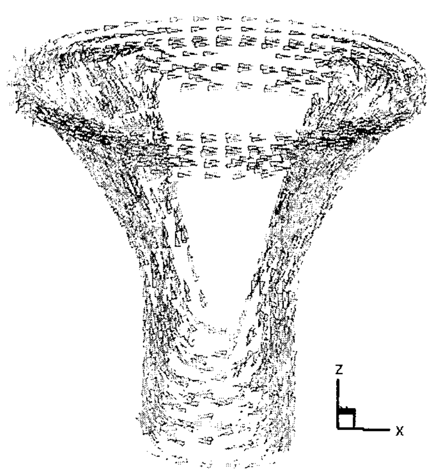

Fig. 9 Calculated current density vector

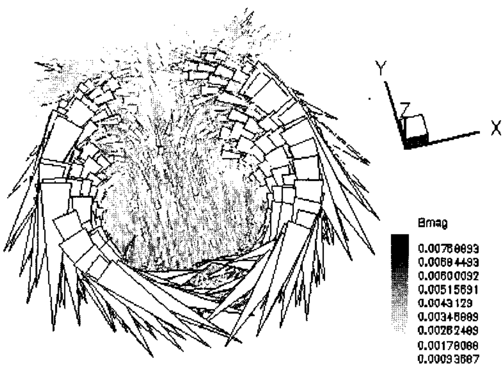

Fig. 10 Plot of calculated flux density vector

To calculate the flux linkage of the horizontal coil, the cross-sectional areas perpendicular to the current direction are calculated for each electric scalar potential. Fig. 11 shows the cross-sectional area versus the electric scalar potential. Then, the flux linkage of the coil is calculated using (13) with the calculated vector potential, direction of electric field intensity vector, volume of each element, and the cross-sectional areas. The calculated inductance is compared to that by an experimental one measured by using an LCR meter. The results are shown in Table 1, where the numerical result shows very good agreement with the experimental one.

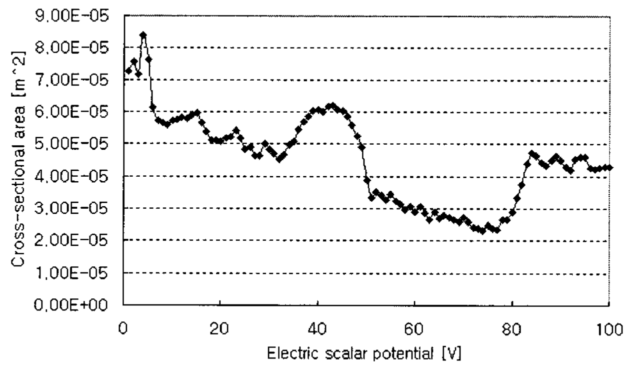

Fig. 11 Cross-sectional area versus electric scalar potential

Table 1 Comparison of Inductance Calculation

Cases	Inductance [*mH*]
Numerical result	0.126
Experimental result	0.13

4. Conclusions

In this paper, a novel method to calculate flux linkage in 3-dimensional finite element analysis is proposed. The proposed method is very easy to implement because it needs no integral path. To calculate the flux linkage of a solidly modeled coil effectively, the line integration of vector potential is changed into the volume integration. The method is verified by the application to a complicated deflection yoke. The result is verified by the comparison with experiments. From the case study, it is shown that the proposed method is very accurate and effective in calculating the flux linkage of an arbitrary shaped coil. It is expected that the proposed method can be applied to all the cases of problems including complex shaped coils. Further studies should be continued for the practical applications such as back-EMF analysis, voltage source FEM, and so on.

Appendix Novel Technique for Current Density Analysis of Solidly Modeled Filamentary Coil [1]

In case of the filamentary coil winding, the current density of the coil is proportional to the density of the filaments per volume. Therefore, when the filament density is constant, the magnitudes of current densities at every point have constant values as well. However, when some part of the coil is pressed and the shape of the coil is highly deformed as the case of a deflection yoke, a novel technique should be introduced. The proposed method is based on the finite element analysis of electrostatics. Using the calculated equipotential surfaces, current density vectors are interpolated on the basis of electric potentials. The main characteristic of the method is the use of equipotential surfaces, which is different from the previous approach in [5,6]. The main idea comes from the fact that the current density is closely related to the electric field intensity, which is perpendicular to the equipotential surfaces.

The procedure for the proposed method is as follows:

A. Electric Field Analysis

Constant potentials are imposed as a boundary condition to the cross sections *I* and *II* in Fig. 11. By the electrostatic analysis using finite element method, electric field intensity vector E can be evaluated for every element. The unit directional vectors of E are found by dividing E with its own magnitude.

B. Calculation of Discrete Current Densities

Between two potentials imposed to the cross sections *I* and *II*, n potentials are determined (ϕ_j, j = 1,2,..., m) and equipotential surfaces for each potential are found. Then the areas of each equipotential surface, S_j (j = 1,2,..., m) are calculated. For each equipotential surface, the magnitudes of current density J_j are calculated by

$$J_j = \frac{N \cdot i}{A_j} \tag{14}$$

where i is the current value of a filament and N is the number of coil turns.

C. Calculation of Current Density for Each Element

Average potential values are evaluated for all elements. If the potential of an element (e) $\phi^{(e)}$ has the value between ϕ_j *and* ϕ_{j+1} , the magnitude of current density $J^{(e)}$ is calculated by

$$J^{(e)} = \frac{\phi^{(e)} - \phi_j}{\phi_{j+1} - \phi_j} J_{j+1} - \frac{\phi^{(e)} - \phi_{j+1}}{\phi_{j+1} - \phi_j} J_j \ . \tag{15}$$

The current density vectors of all elements can be found by multiplying the unit directional vectors of E and the magnitudes of the current densities calculated by (15).

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