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ISOCOMPACTNESS AND RELATED TOPICS OF WEAK COVERING PROPERTY

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Abstract. In this paper, we study the concepts of isocompactness and *cl*-isocompactness. We generalize a pure space defined by Arhangelskii and get some results on initially κ-compact spaces. We also consider open problems related to isocompactness and *cl*-isocompactness.

1. Introduction

It is a fundamental question in general topology that when a countably compact space is compact.

The main concepts that we are going to study are the following.

Definition 1.1 (P. Bacon [2]). A space X is called *isocompact* if every countably compact closed subspace of X is compact.

Definition 1.2 (M. Sakai [26]). A space X is called *cl-isocompact* if the closure of every countably compact subspace is compact.

It is clear that every *cl*-isocompact space is isocompact. It is known from [13] that there exists an isocompact Tychonoff space which is not *cl*-isocompact.

A Lindelöf space is the easiest example of an isocompact space. A number of well-known covering properties imply isocompactness, for example, $\mathcal{F}$-spaces ([10]), spaces satisfying property θL ([9]), weakly $[\omega_1, \infty)^r$-refinable spaces ([29]), $\delta\theta$-penetrable spaces ([4]), almost realcompact spaces ([12]), weakly $\delta\theta$-refinable spaces ([30]), weakly Borel complete spaces ([26]), pure spaces ([1]), and neat spaces ([27]). Indeed,

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almost realcompact, $\delta\theta$-penetrable, spaces having G_δ-diagonal ([7]), symmetrizable spaces ([24]), weakly Borel complete, and weakly $\delta\theta$-refinable spaces are cl-isocompact.

A weakly star reducible space is another space which is in the class of isocompact spaces ([8]). For more information about isocompactness, see the article by D. K. Burke [5] and J. E. Vaughan [29].

In [6], J. Cao proved that $C_p(X)$ is isocompact provided that X is a $\mathcal{G}$-space (the proof from [6] shows that $C_p(X)$ is cl-isocompact).

A property that is needed for an isocompact space be cl-isocompact is the following.

DEFINITION 1.3 ([13]). A space X has property $\mathcal{CC}$ if whenever D is a countable closed discrete subset of X and C is a countably compact subspace of X such that $D \cap C = \emptyset$, then there is a cofinite subset E of D that is completely separated from $cl_X C$.

Notice that every countably compact space has property $\mathcal{CC}$. Hence, a space X is countably compact if and only if X has property $\mathcal{CC}$ and a dense countably compact subspace. The Tychonoff plank $[0,\omega_1] \times [0,\omega] \backslash \{(\omega_1,\omega)\}$ is a locally compact space containing a dense countably compact subspace and is not countably compact.

LEMMA 1.4. *Every cl-isocompact Tychonoff space has property $\mathcal{CC}$.*

Proof. Let X be a cl-isocompact Tychonoff space. Suppose that D is a countable closed discrete subset of X which does not intersect the countably compact subspace C of X. Since $cl_X C$ is compact, $D \cap cl_X C$ is finite and hence $E = D \backslash cl_X C$ can be completely separated from $cl_X C$. □

THEOREM 1.5 ([15]). *A Tychonoff space X is cl-isocompact if and only if X is an isocompact space with property $\mathcal{CC}$.*

Proof. The necessity is Lemma 1.4. Let X be an isocompact Tychonoff space with property $\mathcal{CC}$. Assume that there is a countably compact subset C of X such that $cl_X C$ is not compact. In particular, $cl_X C$ cannot be countably compact. Hence, $cl_X C \backslash C$ contains a countable closed discrete subset D of X. But, this is impossible because $D \subseteq cl_X C$ and a cofinite subset of D would be completely separated from C. □

A space is called *weakly normal* [11] if two disjoint closed sets one of which is countable can be separated by disjoint open sets. We have that every weakly normal space has property $\mathcal{CC}$ (see [17, Theorem 13]) and hence every weakly normal isocompact space is cl-isocompact. By a result from [19], we also have that every isocompact sequential space is *cl*-isocompact.

QUESTION 1.6. *Is every isocompact space with countable tightness cl-isocompact?*

QUESTION 1.7 ([15]). *Is there a space X such that $C_p(X)$ is isocompact but it is not cl-isocompact?*

QUESTION 1.8 ([15]). *Is there a locally compact, isocompact space which is not cl-isocompact?*

For the rest of this paper, κ denotes an infinite cardinal number and all spaces are assumed to be T_1. We use the following notation: For any set $A \subset X$ and a collection $\mathcal{U}$ of subsets of X, $st(A,\mathcal{U})$ (the *star* of $\mathcal{U}$ about A) denotes the set $\bigcup\{U \in \mathcal{U} : U \cap A \neq \emptyset\}$.

If $x \in A$, $st(\{x\},\mathcal{U})$ is simply denoted by $st(x,\mathcal{U})$. $ord(x,\mathcal{U}) = |\{U \in \mathcal{U} : x \in U\}|$, $[\mathcal{U}]^{<\omega} = \{\mathcal{K} \subset \mathcal{U} : \mathcal{K}$ is finite$\}$, and $[\mathcal{U}]^{\leq\omega} = \{\mathcal{K} \subset \mathcal{U} : \mathcal{K}$ is countable$\}$.

Recall from [30, 31] that a space X is said to be *weakly θ-refinable* (resp. *weakly $\delta\theta$-refinable*) if for every open cover $\mathcal{U}$ of X there is an open refinement $\mathcal{G} = \bigcup_{n\in\omega} \mathcal{G}_n$ of $\mathcal{U}$ such that if $x \in X$ there is some $n \in \omega$ with $0 < ord(x,\mathcal{G}_n) < \omega$ (resp. $0 < ord(x,\mathcal{G}_n) \leq \omega$). Moreover, if each $\mathcal{G}_n$ covers X, then X is said to be *θ-refinable* (resp. *$\delta\theta$-refinable* or *submeta-Lindelöf*).

A space X is called *κ-refinable* if every open cover $\mathcal{G}$ of X has an open refinement $\mathcal{H}$ such that $\mathcal{H} = \bigcup_{\alpha<\kappa} \mathcal{H}_\alpha$ and for each $x \in X$ there exists $\alpha < \kappa$ such that $1 \leq ord(x,\mathcal{H}_\alpha) \leq \kappa$. Note that the ω-refinable spaces are the same as the weakly $\delta\theta$-refinable spaces.

A topological space X is called *initially κ-compact* if every open cover of X of cardinality $\leq \kappa$ has a finite subcover. In particular, if $\kappa = \omega$, then it is countably compact. A reason for studying initially κ-compact and related spaces is that the theory in this area provides a means for answering fundamental questions which arise in other areas of topology. It is known ([20]) that an initially κ-compact κ-refinable space is compact. For more details, see [20].

In this paper, we define a new space, a κ-pure space, which is a generalization of a pure space defined by Arhangelskii ([1]). We prove two main theorems that every κ-pure space is κ-neat (Theorem 3.5) and every ultrapure space X with countable spread is Lindelöf (Theorem 4.6).

We also study interrelationships among κ-pure spaces, κ-neat spaces, and weakly star reducible spaces. We give an example (Example 4.1) of an ω-neat space which is neither ω_1-neat nor weakly star reducible. We also answer the questions in [8]. In fact, we prove Theorem 4.3 that if X is a regular weakly star reducible space of cardinality ω_1 and every discrete subspace of X is countable, then X is realcompact (and hence α-realcompact). And we prove Theorem 4.4 that if a topological space X is ω_1-compact and weakly star reducible, then it is α-realcompact. Also, we pose open problems related to isocompactness.

2. Preliminaries

We establish some convenient terminology used throughout the rest of this paper. As far as topological concepts are concerned, we follow [14]. Let us first consider the isocompactness properties defined by A.V. Arhangel'skii in [1]. A countable family $\mathcal{V} = \{\mathcal{V}_n : n \in \omega\}$ of collections of subsets of a space X is called an *interlacing* on X if $\bigcup\bigcup\mathcal{V} = X$ and for each $n \in \omega$, each $V \in \mathcal{V}_n$ is open in $\bigcup\mathcal{V}_n$.

An interlacing $\mathcal{V}$ is called *suspended* (resp. *δ-suspended*) from a family $\mathcal{H}$ of subsets of a space X if for every $n \in \omega$ and $x \in \bigcup\mathcal{V}_n$, there is a finite family $\mathcal{K} \in [\mathcal{H}]^{<\omega}$ (resp. a countable family $\mathcal{K} \in [\mathcal{H}]^{\leq\omega}$) such that $st(x, \mathcal{V}_n) \cap (\bigcap\mathcal{K}) = \emptyset$.

A space X is called *ultrapure* if for each free closed collection $\mathcal{F}$ on X there is an interlacing which is δ-suspended from $\mathcal{F}$.

A space X is called *astral* if for every countably prime free closed filter $\mathcal{F}$ on X with c.i.p. there exists an interlacing which is δ-suspended from $\mathcal{F}$.

A space X is called *pure* if for each free closed ultrafilter $\mathcal{F}$ on X there is an interlacing which is δ-suspended from $\mathcal{F}$.

It is easy to see that ultrapure implies astral implies pure.

Arhangelskii ([1]) showed that every weakly $\delta\theta$-refinable space X is ultrapure.

Realcompactness was introduced by Hewitt in [18] and he gave a characterization of it in Theorem 18 of [18]. We use it here as the definition for convenience.

A Tychonoff space X is realcompact if every prime z-filter on X is fixed.

A space X is α-realcompact if every maximal collection of its closed subsets with c.i.p. on X is fixed.

We can easily see that every realcompact space is astral. However, not all ultrapure spaces are realcompact ([33]). For example, every Moore space is subparacompact and therefore weakly $\delta\theta$-refinable and so ultrapure, but the Moore space is not realcompact ([16]). H. Wicke in [33, Theorem 1] described a general class of spaces which are not ultrapure.

Arhangelskii ([1]) also showed that every countably compact pure space is compact. This theorem implies that every pure space is isocompact.

Now we consider a class of isocompact spaces defined by M. Sakai ([27]). Define for each free closed ultrafilter $\mathcal{H}$ on X with c.i.p.,

$$\lambda(\mathcal{H}) = \min\{|\mathcal{F}| : \mathcal{F} \subset \mathcal{H}, \bigcap \mathcal{F} = \emptyset\}.$$

Note that $\lambda(\mathcal{H})$ is an uncountable regular cardinal.

DEFINITION 2.1 ([27]). Let $\mathcal{H}$ be a free closed ultrafilter on X with c.i.p. and κ be a cardinal number. A system $\langle\{X_\gamma\}, \{\mathcal{V}_\gamma\}, \{f_\gamma\}\rangle_{\gamma\in\Gamma}$ is called a *κ-neat system* for $\mathcal{H}$ if the following are satisfied:

(1) $|\Gamma| < \lambda(\mathcal{H})$.
(2) $\{X_\gamma\}_{\gamma\in\Gamma}$ is a cover of X and $\mathcal{V}_\gamma$ is an open collection of X such that $X_\gamma \subset \bigcup \mathcal{V}_\gamma$ for each $\gamma \in \Gamma$.
(3) Each f_γ is a function from X_γ to $\mathcal{V}_\gamma$ such that if $A \subset X_\gamma$, $|A| \leq \kappa$ and $f_\gamma|A$ is injective, then the closure of A in $\bigcup \mathcal{V}_\gamma$ is contained in $\bigcup_{x\in A} f_\gamma(x)$.
(4) For each $\gamma \in \Gamma$ and $x \in X_\gamma$, there exists $H \in \mathcal{H}$ such that $f_\gamma(x) \cap X_\gamma \cap H = \emptyset$.

A space X is called a *κ-neat space* if for each free closed ultrafilter $\mathcal{H}$ on X with c.i.p. there exists a κ-neat system for $\mathcal{H}$. An ω-neat space is merely called a *neat space*.

A cover $\mathcal{U}$ of a space X is called *regularly rigid* ([32]) if no subcollection of $\mathcal{U}$ of cardinality less than $|\mathcal{U}|$ covers X and $|\mathcal{U}|$ is regular or $1 < |\mathcal{U}| < \omega$.

A space X is called *star reducible* if for every regularly rigid open cover $\mathcal{H}$ of X, there exists a sequence $\langle \mathcal{G}_n : n \in \omega\rangle$ of open covers of X such that for all $p \in X$ there exist $n \in \omega$ and $\mathcal{H}' \subset \mathcal{H}$ such that $|\mathcal{H}'| < |\mathcal{H}|$ and $\mathcal{H}'$ covers $st(p, \mathcal{G}_n)$.

The following definition is essentially based on Definition 4.8 in [32].

DEFINITION 2.2 ([8]). A space X is called *weakly star reducible* if for every uncountable regularly rigid open cover $\mathcal{U}$ of X there is a collection $\mathcal{V}$ of collections of subsets of X such that:

(i) $\bigcup\bigcup\mathcal{V} = X$,
(ii) $|\mathcal{V}| < |\mathcal{U}|$,
(iii) for all $\mathcal{G} \in \mathcal{V}$ and for all $G \in \mathcal{G}$, G is open in $\bigcup\mathcal{G}$, and
(iv) for all $p \in X$, there exist $\mathcal{G} \in \mathcal{V}$ and $\mathcal{U}' \subset \mathcal{U}$ such that $|\mathcal{U}'| < |\mathcal{U}|$ and $st(p, \mathcal{G}) \subset \bigcup\mathcal{U}'$.

Clearly, weakly star reducibility is obviously weaker than star reducibility. It follows from Remark 1.5 in [32] that every developable space is star reducible and thus weakly star reducible. In [8], the author showed that every ultrapure space is weakly star reducible and every weakly star reducible space is κ-neat for every cardinal κ, and M. Sakai showed in Theorem 2.6 of [27] that every neat space is isocompact. Therefore every weakly star reducible space is isocompact (see Corollary 3.6 in [8]).

3. Initially κ-compact and related new spaces

THEOREM 3.1 ([27, Theorem 2.6]). *Every neat space is isocompact.*

We note that every initially κ-compact space is, by the definition, countably compact and every κ-neat space is neat. So the following corollary follows from Theorem 3.1.

COROLLARY 3.2. *Every initially κ-compact, κ-neat space is compact.*

One needs to make a generalization of pure to κ-pure because pure involves ω or countability. So we need to generalize to $\kappa > \omega$.

DEFINITION 3.3. A family $\mathcal{V} = \{\mathcal{V}_\alpha : \alpha < \kappa\}$ of collections of subsets of a space X is called *κ-interlacing* on X provided $\bigcup\mathcal{V}$ is a cover of X and for each $\alpha < \kappa$ each $V \in \mathcal{V}_\alpha$ is open in $\bigcup\mathcal{V}_\alpha$.

DEFINITION 3.4. A space X is said to be *κ-pure* (resp. *κ-ultrapure*) if for each free closed ultrafilter (resp. filter) $\mathcal{H}$ on X with the κ-intersection property (i.e., for every $\mathcal{H}' \in [\mathcal{H}]^{\leq\kappa}$, $\bigcap\mathcal{H}' \neq \emptyset$), there exists a κ-interlacing on X that is κ-suspended from $\mathcal{H}$ (i.e., for each $\alpha < \kappa$ and for each $x \in \bigcup\mathcal{V}_\alpha$, there is a family $\mathcal{F}' \in [\mathcal{F}]^{\leq\kappa}$ such that $st(x, \mathcal{V}_\alpha) \cap (\bigcap\mathcal{F}') = \emptyset$).

If $\kappa = \omega$, then an ω-interlacing is just an interlacing, and if $\kappa = \omega$, then an ω-ultrapure space is an ultrapure space defined by Arhangelskii ([1]).

THEOREM 3.5. *Every κ-pure space is κ-neat.*

Proof. Let $\mathcal{H}$ be a free closed ultrafilter on a κ-pure space with c.i.p. Since X is κ-pure, there exists a κ-interlacing $\mathcal{V} = \{\mathcal{V}_\alpha : \alpha < \kappa\}$ which is κ-suspended from $\mathcal{H}$. For each $\alpha < \kappa$ and $V \in \mathcal{V}_\alpha$ we take an open set $U(V)$ of X such that $V = U(V) \cap (\bigcup \mathcal{V}_\alpha)$. Now for $\alpha < \kappa$ put $X_\alpha = \bigcup \mathcal{V}_\alpha$, $\mathcal{U}_\alpha = \{st(x, \mathcal{F}_\alpha) : x \in X_\alpha\}$, where $\mathcal{F}_\alpha = \{U(V) : V \in \mathcal{V}_\alpha\}$, and define a function f_α from X_α to $\mathcal{U}_\alpha$ such that $f_\alpha(x) = st(x, \mathcal{F}_\alpha)$. Then $\langle \{X_\alpha\}, \{\mathcal{U}_\alpha\}, \{f_\alpha\} \rangle_{\alpha<\kappa}$ is a desired κ-neat system. Hence X is κ-neat. □

COROLLARY 3.6. *Every initially κ-compact, κ-pure space is compact.*

Proof. It follows from Corollary 3.2 and Theorem 3.5. □

REMARK. If $\kappa = \omega$, then the above corollary is the theorem of Arhangelskii ([1]).

4. Interrelationships among spaces of weak covering property and problems

A space X is said to be *ω_1-compact* if the cardinality of every closed discrete subset is countable. A space X is called *closed-complete* (resp. *real compact*) if every closed (resp. z-) ultrafilter with c.i.p. is fixed. A space X is called *α-realcompact* (or *a-realcompact*) if every maximal family of its closed subsets with c.i.p. is fixed.

The linearly ordered space ω_1 is collectionwise normal as a countably compact and normal space. The result from 9.4 in [5] shows that ω_1 cannot be weakly $\delta\theta$-refinable and so ω_1 is not ω-refinable. Is then ω_1 weakly star reducible?

It follows from Corollary 3.2 in [8] that every weakly $\delta\theta$-refinable space is weakly star reducible space. Hence every ω-refinable space is weakly star reducible.

We give an example of an ω-neat (= neat) space which is neither ω_1-neat nor weakly star reducible. This example was pointed out to the authors by H. Wicke.

EXAMPLE 4.1. Let Y be the subspace of ω_2 obtained by deleting all nonisolated points having a countable base [16, 9L]. Then Y is a normal ω_1-compact space which is not α-realcompact [4, Remark 2.7]. Moreover, Y is ω-neat [27, Example 3.8] but not ω_1-neat because Theorem 2.8 in [27] says that an ω_1-compact ω_1-neat T_1-space is α-realcompact. Hence by Theorem 3.5 in [8], Y is not weakly star reducible.

The following question related to the above example was given in Question 4.2 in [8].

QUESTION 4.2. *Is there an example of a $\kappa\,(\kappa > \omega)$-neat space which is not weakly star reducible?*

It is known from [1] that every α-realcompact space is pure. It is also known from Example 4.4 in [8] that pure does not imply weak star reducibility. Thus α-realcompactness does not imply weak star reducibility. What about the converse? Let us recall Question 4.5 in [8]: When does weak star reducibility imply α-realcompactness? The following two theorems (Theorem 4.3 and Theorem 4.5 below) answer the above question.

THEOREM 4.3. *If X is a regular weakly star reducible space of cardinality ω_1 and every discrete subspace of X is countable, then X is realcompact (and hence α-realcompact).*

Proof. It is easy to see that X is Lindelöf (without using regularity). It is well-known that every regular Lindelöf space is realcompact (and thus α-realcompact). □

REMARK. A space X under the same assumption in Theorem 4.3 is also weakly $\delta\theta$-refinable as being a regular Lindelöf space. Thus the space X is ultrapure. However, H. Wicke showed in [33] that not all realcompact spaces are ultrapure.

In [33], H. Wicke pointed out that the so-called Juhasz-Kunen-Rudin line (or J-K-R line) in [21] is a CH example of a hereditarily realcompact space which is not ultrapure (by Theorem 1 in [33]). Also, he constructed ZFC examples of spaces which are neat in Sakai's sense but not pure in the sense of Arhangel'skii. More precisely, M. Sakai in [27] observed that the product $X \times Y$ of the J-K-R line X with the space Y (which is Example 5L in [16]) is a neat space. H. Wicke replaced X in Sakai's example with a ZFC example A of Tordorčević space (see [33]) to get a ZFC example of a neat space $A \times Y$. The natural question then arises:

Is the space $A \times Y$ weakly star reducible? This question is also closely related to Question 4.2 above.

THEOREM 4.4. *If a space X is ω_1-compact and weakly star reducible, then it is α-realcompact.*

Proof. It is known from Theorem 3.5 in [8] that every weakly star reducible space is κ-neat for every $\kappa \geq \omega$. So by Theorem 2.8 in [27], the space X is α-realcompact. □

QUESTION 4.5 ([22]). *Must a pure space with countable extent be Lindelöf?*

Question 4.5 is negative under CH. Indeed, see Example 2 in [25]. The example is non-Lindelöf realcompact (hence pure) space with countable extent.

On the other hand, if we strengthen pure to ultrapure, but if we weaken countable extent to countable spread, then we have the following:

THEOREM 4.6. *Every ultrapure space X with countable spread is Lindelöf.*

Proof. Let $\mathcal{U}$ be an open cover of X. We may assume that if a subset $V \subset X$ is open and $V \subset U \in \mathcal{U}$ for some $U \in \mathcal{U}$, then $V \in \mathcal{U}$. Let $\mathcal{F} = \{X \backslash U : U \in \mathcal{U}\}$. Since $\mathcal{F}$ is a free closed filter of X, there is an interlacing $\{\mathcal{V}_n : n \in \omega\}$ which is δ-suspended from $\mathcal{F}$. For each $n \in \omega$, let $X_n = \bigcup \mathcal{V}_n$. Let A_n be a maximal element of the collection $\{F \subset X_n : |V \cap F| \leq 1 \text{ for any } V \in \mathcal{V}_n\}$. Since every element of $\mathcal{V}_n$ is open in X_n, A_n is a discrete subspace of X, hence it is countable. Since A_n is maximal, $X_n = \bigcup_{x \in A_n} St(x, \mathcal{V}_n)$. Choose a countable $\mathcal{F}_n \subset \mathcal{F}$ with $X_n \cap (\bigcap \mathcal{F}_n) = \emptyset$. Now the collection $\{X \backslash F : F \in \bigcup_{n \in \omega} \mathcal{F}_n\}$ is a countable subcover of $\mathcal{U}$. □

We present the question posed by the author in [8] again.

QUESTION 4.7. *Is every weakly star reducible space pure (or even ultrapure)?*

QUESTION 4.8. *Is weakly star reducibility inversely preserved under perfect mapping (with regular domain)?*

Is weakly star reducibility inversely preserved under closed mappings with Lindelöf fibers (and regular domain)?

QUESTION 4.9. *Suppose Y is a regular σ-compact space and X is a (regular) weakly star reducible space. Is $X \times Y$ weakly star reducible?*

QUESTION 4.10. *Is every T_3-pure space cl-isocompact?*

QUESTION 4.11. *Is every T_3-weakly star reducible space cl-isocompact?*

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