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Unsteady Interaction of the Surface Gravity Waves with the Nonuniform Current

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ABSTRACT: *본 논문에는 수면파(Surface wave)와 수중파 (Internal wave)간의 동적 상관관계에 관하여 수행된 연구결과를 정리하였다. 표면파의 비선형 문제는 파의 경사매개변수를 2차원으로 가정하여 해석하였으며, Cauchy 문제는 불균일 조류상의 균일 수면중력파에 대하여 해석하였다. 또한, 파의 경사, 주기의 범위(Frequency range) 그리고 자유표면하의 조류의 분포들간의 조화에 대한 연구가 수행되었으며 해류 및 이동파와 연계되어 수중파의 최전 후방에 형성될 수 있는 정적 파형(Steady wave pattern)이 수면파형에 포함되었다.*

1. Introduction

Surface wave propagation in the presence of large scale currents was considered in numerous works (Philips, 1966; Craik, 1985; Kwak, 1999). The nonuniformity of the surface current can be conditioned by a variable depth, long internal or surface waves, tides, etc. Most papers deal with the geometrical optics approximation, which is associated with the first terms of Stokes expansion in the wave steepness $x = ak$ (a and k are typical wave amplitude and number) and with a condition of a constant wave frequency $\omega = \text{const}$, which corresponds to the stationary pattern of wave-current interaction. Most intriguing property of the solutions here is the possibility of waves reflection in the blockage points, where the wave propagation is blocked by the opposite current.

Several models (Gerber, 1986; Shugan and Voliak, 2000) include into consideration the nonlinear properties of surface waves and as a result have the behaviour of interaction very different from the linear waves modulation. Most interesting solutions describe connected nonlinear surface wave packets propagating with phase velocity of internal waves. But stationary character of solution can not give an answer to a very important question about stability and realization of such kind of wave phenomena.

Thus the necessity of considering the nonstationary modelis apparent. Series of works were made on the subject (Basovich, 1979; Basovich, Bahanov and Talanov, 1982).

More details of the wave behavior were revealed, for example the existence of the forerunner of the internal wave, the decrease of the period of the surface wave envelope in the zone of periodic short waves. The kinematics of the wave packets behavior and effect of wind stresses in energy balance of surface motion were analyzed in more detail.

Cornelis and Van (1999) recently carried out Benjamin-Feir instability analysis of surface waves in the presence of variable current.

The modification of nonlinear Schrodinger equation for the envelope of surface waves in the presence of internal waves where considered by Stocker and Peregrine (1999). Variations of length and wave number of surface waves where assumed to be relatively small compared to changes of amplitude and so carrier wave and envelope can be introduced. Presented in the work analysis is valid for the range of main parameters far from blockage points with strong changing of surface wave number and amplitude.

The work (Semenov and Shugan, 1997) was devoted to the linear nonstationary model. It was shown there that moving points of discontinuity occur in the time dynamics of the wave number function. More detailed analysis was given in Voliak, Lossov and Lossov (1992). The type of singularity in the evolution of the amplitude and the wave number were found out. The found solution was typical of the zones of zero surface wave amplitude, when the water surface becomes absolutely flat.

This paper is devoted to the analysis of nonlinear and nonstationary model. We constricted our attention to the

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problem of interaction between surface and internal waves in the ocean.

The second-order nonlinear approximation with respect to ak to the nonstationary initial Cauchy's problem is considered for the following reasons. The first and obvious one is to extend the area of applying the model for greater ak (steep waves). The second reason is related to a very irregular character of the solution in the first approximation. As shown by Voliak and Shermenev (1998), there are two kinds of singularity in evolution of Cauchy's data (a_0, k_0). This fact contradicts our physical intuition and, therefore, we hope to improve the situation while considering the terms of higher order in wave steepness. The last part of interest connected with nonstationary nonlinear waves packet dynamics and realization of some special regimes predicted by stationary models.

2. Statement of Problem

We construct the model of the internal waves effect on propagation of the narrow-band weakly nonlinear packet of gravity surface waves based on the following assumptions:

(i) surface and internal waves propagate along a common x-direction;

(ii) the characteristic length of surface waves is much shorter than the scale of internal wave;

(iii) the depth of an oceanic pycnocline or a layer of maximum background current shear much exceeds the surface waves length, thus the internal wave in the upper layer of a homogeneous (by density) fluid is presented by a horizontal current varying along the direction x and reproducing internal wave shape;

(iv) vertical displacement of particles has a negligible impact on internal waves manifestation (Philips, 1966);

(v) long internal waves disperse weakly, therefore, the current horizontal velocity in the subsurface layer is set as the traveling wave $U(x-ct)$, where c is the phase velocity of the internal wave

Let us consider the one-dimensional Cauchy problem for the surface gravity waves dynamics with wave number k and frequency on the nonuniform current U in an infinitely deep fluid. The wave propagation in the moving with phase velocity c of internal wave frame of reference is governed by the equation of phase conservation (subscripts stand for partial differentiation)

$$k_t + \omega_x = 0, \tag{1}$$

together with the equation of energy conservation

$$\left(\frac{a^2}{k^{1/2}}\right)_t + \left[\frac{a^2}{k^{1/2}}\left(\frac{\alpha}{2k^{1/2}} + v(x)\right)\right]_x = 0, \tag{2}$$

and the initial conditions

$$k(x,0) = k_0 = \text{ const },$$
$$a(x,0) = a_0 = \text{ const }, \tag{3}$$

here a is the wave amplitude, $\alpha = \sqrt{g}$, and g is the gravity-caused acceleration. The key element of the analysis is taking into account the effects of the so-called amplitude (quadratic) wave dispersion together with the weak surface current. In this case the dispersion relation has the form

$$\omega = \alpha k^{1/2}\left(1 + \frac{1}{2}a^2k^2\right) + kv\ (x) \tag{4}$$

$$v(x) = -c + \varepsilon U\ (x) \tag{5}$$

where U(x) is the velocity of the submarine current, and $\varepsilon = U(x)/c$ is a small parameter. Experimental data usually confirmed a relative weakness of subsurface current compared with the phase speed of internal wave U(x)/c << 1.

Equations (1) and (2) after substitution of (4) and (5) present a couple of quasilinear equations for the functions k(x, t), a(x, t) which should be solved with initial conditions (3). We will investigate these equations using the series expansions with respect to ε.

3. Reduction and Scaling of the Governing Equations

Substituting new variables

$$\Omega = k^{1/2},\ A = \frac{a^2}{k^{1/2}} \tag{6}$$

which have the physical meaning of reduced wave frequency and wave action, into the nonlinear dispersion equation (4) one can have

$$\omega = \alpha\Omega\left(1 + \frac{1}{2}A\Omega^5\right) + \Omega^2\left[-c + e\,U(x)\right]. \tag{7}$$

As was mentioned, it is appropriate to seek solution to set (1), (2) in the form of expansion in terms of the small parameter ε,

$$\Omega = \Omega_0 + \Omega_1\varepsilon + \Omega_2\varepsilon^2 + \cdots + \Omega_m\varepsilon^m + \cdots,$$
$$A = A_0 + A_1\varepsilon + A_2\varepsilon^2 + \cdots + A_m\varepsilon^m + \cdots, \tag{8}$$

where Ω_i and A_i are unknown functions of x and t.

Let us put down our set as the coupled system of linear differential equations

$$\Omega_{1t} + a_{11}\Omega_{1x} + a_{12}A_{1x} + \frac{\varepsilon}{2}\Omega_0 U'(x) = 0, \quad A_{1t} + a_{21}\Omega_{1x} + a_{22}A_{1x} + \varepsilon A_0 U'(x) = 0, \tag{9}$$

where the coefficients are given by expansions

$$a_{ij} = a^0_{ij}(A_0, \Omega_0) + a^1_{ij}(A_0, \Omega_0, A_1, \Omega_1)\varepsilon + \cdots + a^m_{ij}(A_0, \Omega_0, \cdots, A_m, \Omega_m)\varepsilon^m + \cdots \tag{10}$$

The first-order (in the absence of current) solutions $\Omega_0(x,t)$ and $A_0(x,t)$ are constant and, therefore, a^0_{ij} are constant, too. We express them explicitly as

$$a^0_{11} = \frac{\alpha}{2}, \Omega_0^{-1} + \frac{3\alpha}{2}A_0\Omega_0^4 - c, \quad a^0_{12} = \frac{\alpha}{4}\Omega_0^5, \quad a^0_{21} = -\frac{\alpha}{2}\Omega_0^{-2}A_0, \quad a^0_{22} = \frac{\alpha}{2}, \Omega_0^{-1} - c. \tag{11}$$

Let λ_1 and λ_2 be the eigenvalues of the matrix a^0_{ij} (characteristic velocities of the solution to system, (9)), (l^1_1, l^1_2) and (l^2_1, l^2_2) be its left eigenvectors. These eigenvectors can be chosen so that $l^i_1 = 1$ and further denoted as $(1, L^1_2)$ and $(1, L^2_2)$.

The characteristic polynomial of the system takes on the quadratic form

$$P(\lambda) = \lambda^2 - \left(\alpha\Omega_0^{-1} + \frac{3}{2}\Omega_0^4 A_0 - 2c\right)\lambda + \frac{\alpha^2}{4}\Omega_0^{-2} + \frac{7\alpha^2}{8}\Omega_0^3 A_0 - \alpha c\Omega_0^{-1} - \frac{3\alpha c}{2}\Omega_0^4 A_0. \tag{12}$$

The roots of (12) are

$$\lambda_{1.2} = \frac{\alpha}{2}\Omega_0^{-1} + \frac{3\alpha}{4}\Omega_0^4 A_0 - c \pm \frac{1}{2}\sqrt{\Delta} \tag{13}$$

with the determinant

$$\Delta = \frac{9\alpha^2}{4}\Omega_0^8 A_0^2 - \frac{\alpha^2}{2}\Omega_0^3 A_0 \tag{14}$$

By using the eigenvelocities λ_1 and λ_2, we introduce new independent variables

$$x_1 = \frac{x - \lambda_2 t}{\lambda_1 - \lambda_2}, \qquad x_2 = -\frac{x - \lambda_1 t}{\lambda_1 - \lambda_2}. \tag{15}$$

Some useful transformations with scaling (15) are listed below,

$$x = \lambda_1 x_1 + \lambda_2 x_2, \qquad t = x_1 + x_2, \quad \partial_x = \frac{1}{\lambda_1 - \lambda_2}(\partial_{x_1} - \partial_{x_2}), \quad \partial_t = \frac{1}{\lambda_1 - \lambda_2}(-\lambda_2\partial_{x_1} + \lambda_1\partial_{x_2}). \tag{16}$$

3.1. Second-order solution

Multiplying the first equation of system (9) by Li (i = 1, 2) and adding it to the second one we get in the first order in ε,

$$\Omega_{1t} + L^i A_{1t} + \left(a^0_{11} + L^i a^0_{21}\right)\Omega_{1x} + \left(a^0_{12} + L^i a^0_{22}\right)A_{1x} = -L^i A_0 U'(x) - \frac{1}{2}\Omega_0 U'(x) \tag{17}$$

From (15), we derive the set

$$\left(\Omega_1 + L^i A_1\right)_{x_i} = -L^i A_0 U'(x) - \frac{1}{2}\Omega_0 U'(x). \tag{18}$$

Integrating these equations, we have the system for determining the second-order solutions Ω_1 and A_1,

$$\Omega_1 + L^1 A_1 = \frac{1}{\lambda_1}\left(-L^1 A_0 - \frac{1}{2}\Omega_0\right)\left\{U(\lambda_1 x_1 + \lambda_2 x_2) - U\left[(\lambda_2 - \lambda_1)x_2\right]\right\}, \quad \Omega_1 + L^2 A_1 = \frac{1}{\lambda_2}\left(-L^2 A_0 - \frac{1}{2}\Omega_0\right)\left\{U(\lambda_1 x_1 + \lambda_2 x_2) - U\left[(\lambda_1 - \lambda_2)x_1\right]\right\}. \tag{19}$$

Let us determine the constants α_i as follows:

$$\alpha_i = -L^i A_0 - \frac{1}{2}\Omega_0, \quad \Omega_1 = \frac{1}{(L^2 - L^1)}\left[\left(\frac{\alpha_1 L^2}{2\lambda_1} - \frac{\alpha_2 L^1}{2\lambda_2}\right)U(x) - \frac{\alpha_1 L^2}{2\lambda_1}U(x - \lambda_1 t) + \frac{\alpha_2 L^1}{2\lambda_2}U(x - \lambda_2 t)\right], \tag{20}$$

$$A_1 = \frac{1}{(L^1 - L^2)}\left[\left(\frac{\alpha_1}{2\lambda_1} - \frac{\alpha_2}{2\lambda_2}\right)U(x) - \frac{\alpha_1 L^2}{2\lambda_1}U(x - \lambda_1 t) + \frac{\alpha_2 L^1}{2\lambda_2}U(x - \lambda_2 t)\right] \tag{21}$$

Therefore, the perturbations induced by a suddenly arising weak nonuniform current in the wave action and frequency of a surface gravity wave with initially constant parameters are generally presented in the second approximation by three disturbances proportional to the current: one is immediately fixed to the current, while two others propagate at their own(if real) characteristic velocities λ_1 and λ_2 from the region of the current. Signs of these disturbances are completely defined by the eigenvalues λ_1 and λ_2, if these are real. The case of complex λ_1 and λ_2 should be analyzed separately.

Let us now consider the immobile wave of elevation, which is connected with the current. The coefficients at U(x) in both formulas (21) are of the greatest importance to us. Their signs define whether the immobile wave becomes longer or shorter and whether its wave action grows or becomes smaller. When the eigenvelocities are complex (which is physically more important) we have the following cases.

(i) If

$$c > \frac{3\left(\alpha + 2\alpha A_0 \Omega_0^5\right)}{4\Omega_0}, \tag{22}$$

the coefficients at U(x) in the formula (21) are positive and it means that the wave action grows and the wave becomes shorter.

(ii) If

$$\frac{\alpha - \alpha A_0 \Omega_0^5}{2\Omega_0} < c < \frac{3\left(\alpha + 2\alpha A_0 \Omega_0^5\right)}{4\Omega_0}, \tag{23}$$

then the wave action itself becomes smaller and the wave itself becomes shorter.

(iii) In the case of a comparatively small phase velocity c

$$c < \frac{\alpha - \alpha A_0 \Omega_0^5}{2\Omega_0}. \tag{24}$$

The wave action decreases and the wave becomes longer.

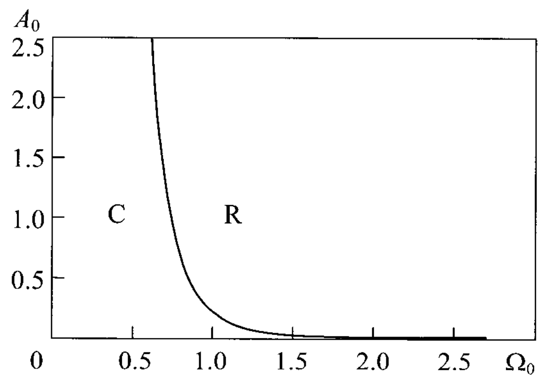

Fig. 1 Domains of real (R) and complex (C) characteristic velocities

4. Types of Solutions

As was mentioned before, the behaviour of Ω_1 and A_1 critically depends on the characteristic velocities λ_1 and λ_2 of outcoming disturbances. As will be shown in Sec. 5, the sign of the real part of $\lambda_{1,2}$ determines the direction of wave propagation. If the velocities are real, then Ω_1 and A_1 are regular and bounded functions of x and t. If they are complex, we have only two disturbances instead of three. One of them is steady and bounded while the other (it depends on the type of the current) is going away from the internal wave location and growing in time. The detailed behavior of Ω_1 and A_1 is discussed in the following sections. Now we decompose the positive orthant of the (Ω_0, A_0) plane into two parts R and C, where λ_1 and λ_2 are real and complex, respectively. Displacement of regions R and C are presented at fig. 1.

The border of domain R (a set, where $\Delta = 0$) is given by two curves defined by expressions

$$A_0 = \frac{2}{9\Omega_0^5} \tag{25}$$

From the condition $a^2k^2 \ll 1$ it follows that $\Omega_0^5 \ll 1$, and thus it can be seen that the domain C is of the most important. There velocities $\lambda_{1,2}$ are complex. The sign of their real part is important. It determines the direction of wave propagation. The condition of group resonance define surface wave components that accompany the internal waves:

$$c = \frac{\alpha}{2\Omega_0} + \frac{3\alpha}{4} A_0 \Omega_0^4, \tag{26}$$

which mean that the phase velocity of internal wave c is equal to the nonlinear group velocity of surface waves ($c_g = \alpha/2\Omega_0 + 3\alpha/4 A_0 \Omega_0^4$). For relatively short surface waves

$$c > c_g + \frac{1}{2}\sqrt{\Delta}, \tag{27}$$

where $\sqrt{\Delta}$ is detuning from the group resonance, the eigenvelocities $\lambda_{1,2}$ are negative and, therefore, both waves of elevation move to the left. When

$$c_g - \frac{1}{2}\sqrt{\Delta} < c < c_g + \frac{1}{2}\sqrt{\Delta}, \tag{28}$$

the radiated waves run to the opposite from the current. If the phase velocity c is small enough,

$$c < c_g - \frac{1}{2}\sqrt{\Delta}, \tag{29}$$

both waves travel in the same direction to the right from the current.

5. Examples of the Wave Evolution

Now we will focus our attention on the Gaussian profile of the current

$$U(x) = \exp(-B^2x^2), \tag{30}$$

where B is constant, while in the following examples $B^{1/2}= 0.11/m$. This form of subsurface current can be initiate by the typical solitary internal wave with the same shape and phase speed. Now let us illustrate the modes of surface wave modulation and main effects of internal wave impact on the sea surface.

5.1 Real eigenvelocities

Steep surface waves form three disturbances of the same form. One of them is steady while the other two move with the velocities λ_1 and λ_2. Direction of propagation is defined by difference between linear group velocity of surface waves and phase velocity of internal waves: $\alpha\ \Omega_0^{-1}/2-c$.

If $\lambda_{1,2}>0$, then both waves move to the right and form the forerunner of the internal wave at the sea surface. This case is presented in fig. 2 for the parameters $\Omega_0^2=0,86$ rad/m, $(A_0\Omega_0)^{1/2}=0,61$ m, and $c=0.5$ m/s. The wave steepness ak=0.53. Wave action is shown in Fig. 2a, while the frequency is graphically presented in Fig. 2b. As can be seen from formula (13), the eigenvelocities are positive when the phase velocity c is small enough.

If λ_1 and λ_2 have opposite signs, then the waves run to the opposite sides from the current. When the phase velocity c is high enough, both λ_1 and λ_2 are negative. The waves run to the left from the current and represent the tail of the internal wave.

5.2 Complex eigenvelocities

We have found the wave motion in domain of the plane (Ω_0, A_0) to differ critically from that in domain R, as was predicted. First, the real part of is the velocity of the two disturbances that are connected everywhere on the x axis. Thus, instead of three disturbances we have only two, as was predicted in Sec. 5.1. Second, the amplitude of this

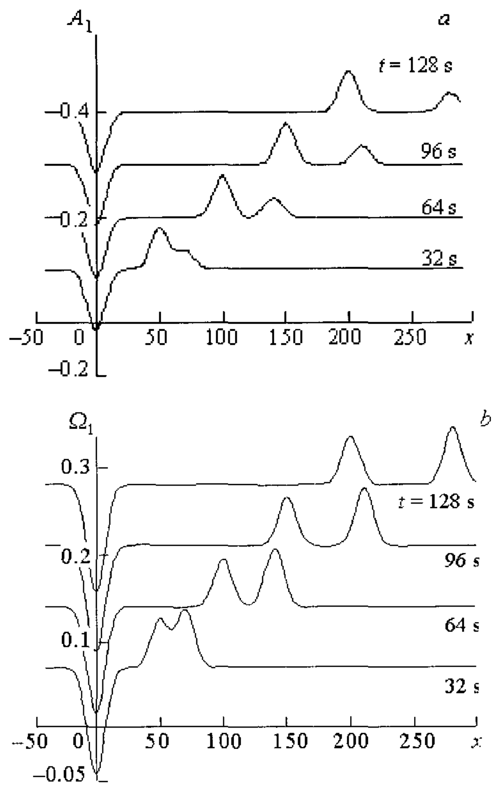

Fig. 2 Wave evolution for positive eigenvelocities $c=0.5$ m/sec, $\Omega_0^2=0.86$ rad/m, $(A_0\Omega_0)^{1/2}=0.61$ m

moving increases in time. The second-order solutions Ω_1 and A_1 are shown on fig.3 for the initial data $\Omega_0^2=1.0$ rad/m, $(A_0\Omega_0)^{1/2}=0.32$ m, and $c=3,0$ m/s. The wave steepness is here $ak=0.32$. Then linear group velocity of relatively short surface waves is less then the phase velocity of internal waves c, so nonlinear wave packet is moving back from the current front. Internal wave propagation is also accompanied by steady surface wave packet and connected with it.

6. Conclusions

Nonlinear nonstationary model of modulation of surface wave field by large scaled internal waves shows a variety of modulation modes for different component of surface waves. Nonlinearity of surface waves and its steepness play a dominant role in the formation of sea surface disturbance.

Three wave packets arise for steep surface waves: one is connected with the internal wave and reproduce its form on the sea surface, and the two others form the forerunner or tail of it in dependence on the relation between the corresponding velocity c_g of the linear group of surface waves and phase velocity c of the internal wave.

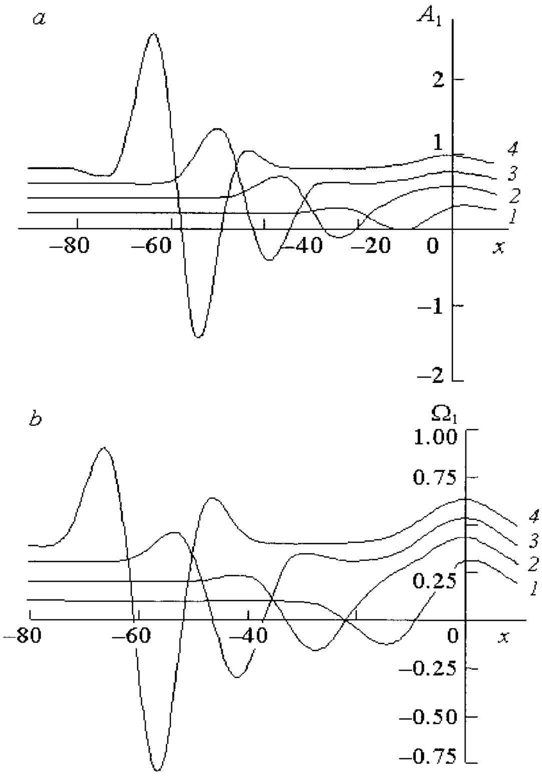

Fig. 3 Time evolution for complex eigenvelocities $\Omega_0^2 = 1.0\,\text{rad/m}$ $(A_0\Omega_0)^{1/2} = 0.32$ m, $c = 3.0$ m/s. Curve (1) corresponds to $t = 12$ sec, (2) $-t = 24$ sec, (3) -36 sec (4) -48 sec. The wave steepness is here $ak = 0.32$

Gently sloping surface waves also form wave disturbance over the current and nonlinear running wavepacket, with amplitude increasing in time. If the group velocity is larger than the phase velocity of internal wave, the moving disturbance forms a forerunner, otherwise it is a tail of the internal wave. Thus we have shown that there is no blockage of waves, though the wave amplitude increases in time.

Movement of surface wavepackets is defined in the mean order by their linear group velocities. Thus, components that are moving with the internal wave speed order accompanied it within a relatively long time and are most visible at the sea surface in the region of subsurface current.

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