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Pairwise semicontinuous mappings in smooth bitopological spaces

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Abstract

We introduce (τ_i, τ_j)-fuzzy (r, s)-semiclosures and (τ_i, τ_j)-fuzzy (r, s)-semiinteriors. Using the notions, we investigate some of characteristic properties of fuzzy pairwise (r, s)-semicontinuous, fuzzy pairwise (r, s)-semiopen and fuzzy pairwise (r, s)-semiclosed mappings.

Key words : smooth bitopological spaces, semiclosures, semiinteriors, pairwise semicontinuous mappings

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [12] in his classical paper. Using the concept of fuzzy sets, Chang [2] introduced fuzzy topological spaces and several other authors continued the investigation of such spaces. Chattopadhyay et al. [4] and Ramadan [9] introduced new definition of smooth topological spaces as a generalization of fuzzy topological spaces. Kandil [6] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. Lee et al. [7] introduced and studied the concept of smooth bitopological spaces as a generalization of smooth topological spaces and Kandil's fuzzy bitopological spaces.

In this paper, we introduce (τ_i, τ_j)-fuzzy (r,s)-semiclosures and (τ_i, τ_j)-fuzzy (r,s)-semiinteriors. Using the notions, we investigate some of characteristic properties of fuzzy pairwise (r,s)-semicontinuous, fuzzy pairwise (r,s)-semiopen and fuzzy pairwise (r,s)-semiclosed mappings.

2. Preliminaries

In this paper, I denotes the unit interval $[0,1]$ of the real line and $I_0=(0,1]$. For a set X, I^X denotes the collection of all mappings from X to I. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1}-\mu$. All other notations are standard notations of fuzzy set theory.

A *Chang's fuzzy topology* on X [2] is a family T of fuzzy sets in X which satisfies the following properties:

(1) $\tilde{0}, \tilde{1} \in T$.

(2) If $\mu_1, \mu_2 \in T$ then $\mu_1 \wedge \mu_2 \in T$.

(3) If $\mu_k \in T$ for each k, then $\bigvee \mu_k \in T$.

The pair (X, T) is called a *Chang's fuzzy topological space.* Members of T are called T-fuzzy open sets of X and their complements T-fuzzy closed sets of X.

A *smooth topology* on X [4,9] is a mapping $\tau : I^X \to I$ which satisfies the following properties:

(1) $\tau(\tilde{0}) = \tau(\tilde{1}) = 1$.

(2) $\tau(\mu_1 \wedge \mu_2) \ge \tau(\mu_1) \wedge \tau(\mu_2)$.

(3) $\tau(\bigvee \mu_k) \ge \bigwedge \tau(\mu_k)$.

The pair (X, τ) is called a smooth topological space. For $r \in I_0$, we call μ a τ-fuzzy r-open set of X if $\tau(\mu) \ge r$ and μ a τ-fuzzy r-closed set of X if $\tau(\mu^c) \ge r$.

A system (X, τ_1, τ_2) consisting of a set X with two smooth topologies τ_1 and τ_2 on X is called a smooth bitopological space. Throughout this paper the indices i, j take values in $\{1,2\}$ and $i \ne j$.

Let (X, τ) be a smooth topological space. For each $r \in I_0$, an r-cut

$$\tau_r = \{\mu \in I^X \mid \tau(\mu) \ge r\}$$

is a Chang's fuzzy topology on X.

Let (X, T) be a Chang's fuzzy topological space and $r \in I_0$. Then a smooth topology $T^r : I^X \to I$ is defined by

$$T^r(\mu) = \begin{cases} 1 & \text{if } \mu = \tilde{0}, \tilde{1}, \\ r & \text{if } \mu \in T - \{\tilde{0}, \tilde{1}\}, \\ 0 & \text{otherwise}. \end{cases}$$

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Definition 2.1 [8] Let (X,τ) be a smooth topological space. For $r\in I_0$ and for each $\mu\in I^X$, the *fuzzy r-closure* is defined by

$$\tau-\mathrm{Cl}(\mu,r)=\bigwedge\{\rho \mid \mu\le\rho,\ \tau(\rho^c)\ge r\}$$

and the *fuzzy r-interior* is defined by

$$\tau-\mathrm{Int}(\mu,r)=\bigvee\{\rho \mid \mu\ge\rho,\ \tau(\rho)\ge r\}.$$

Theorem 2.2 [8] For a fuzzy set μ of a smooth topological space (X,τ) and $r\in I_0$, we have:

(1) $\tau-\mathrm{Int}(\mu,r)^c=\tau-\mathrm{Cl}(\mu^c,r)$.

(2) $\tau-\mathrm{Cl}(\mu,r)^c=\tau-\mathrm{Int}(\mu^c,r)$.

Definition 2.3 [7] Let μ be a fuzzy set of a smooth bitopological space (X,τ_1,τ_2) and $r,s\in I_0$. Then μ is said to be

(1) a (τ_i,τ_j)-fuzzy (r,s)-semiopen set if there is a τ_i-fuzzy r-open set ρ in X such that $\rho\le\mu\le\tau_j-\mathrm{Cl}(\rho,s)$,

(2) a (τ_i,τ_j)-fuzzy (r,s)-semiclosed set if there is a τ_i-fuzzy r-closed set ρ in X such that $\tau_j-\mathrm{Int}(\rho,s)\le\mu\le\rho$.

Theorem 2.4 [7] Let μ be a fuzzy set of a smooth bitopological space (X,τ_1,τ_2) and $r,s\in I_0$. Then the following statements are equivalent:

(1) μ is a (τ_i,τ_j)-fuzzy (r,s)-semiopen set.

(2) μ^c is a (τ_i,τ_j)-fuzzy (r,s)-semiclosed set.

(3) $\tau_j-\mathrm{Cl}(\tau_i-\mathrm{Int}(\mu,r),s)\ge\mu$.

(4) $\tau_j-\mathrm{Int}(\tau_i-\mathrm{Cl}(\mu^c,r),s)\le\mu^c$.

3. (τ_i,τ_j)-fuzzy (r,s)-semiclosures and (τ_i,τ_j)-fuzzy (r,s)-semiinteriors

Definition 3.1 Let (X,τ_1,τ_2) be a smooth bitopological space and $r,s\in I_0$. For each $\mu\in I^X$, the *(τ_i,τ_j)-fuzzy (r,s)-semiclosure* is defined by

$$(\tau_i,\tau_j)-\mathrm{sCl}(\mu,r,s)=\bigwedge\{\rho\in I^X \mid \mu\le\rho,\ \rho \text{ is } (\tau_i,\tau_j)-\text{fuzzy } (r,s)-\text{semiclosed}\}$$

and the *(τ_i,τ_j)-fuzzy (r,s)-semiinterior* is defined by

$$(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s)=\bigvee\{\rho\in I^X \mid \mu\ge\rho,\ \rho \text{ is } (\tau_i,\tau_j)-\text{fuzzy } (r,s)-\text{semiopen}\}.$$

Obviously, $(\tau_i,\tau_j)-\mathrm{sCl}(\mu,r,s)$ is the smallest (τ_i,τ_j)-fuzzy (r,s)-semiclosed set which contains μ and $(\tau_i,\tau_j)-\mathrm{sCl}(\mu,r,s)=\mu$ for any (τ_i,τ_j)-fuzzy (r,s)-semiclosed set μ. Also, $(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s)$ is the greatest (τ_i,τ_j)-fuzzy (r,s)-semiopen set which is contained μ and $(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s)=\mu$ for any (τ_i,τ_j)-fuzzy (r,s)-semiopen set μ. Moreover, we have

$$\begin{aligned}\tau_i-\mathrm{Int}(\mu,r) &\le(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s)\\ &\le\mu\le(\tau_i,\tau_j)-\mathrm{sCl}(\mu,r,s)\\ &\le\tau_i-\mathrm{Cl}(\mu,r).\end{aligned}$$

Also, we have the following results:

(1) $(\tau_i,\tau_j)-\mathrm{sCl}(\tilde{0},r,s)=\tilde{0}$, $(\tau_i,\tau_j)-\mathrm{sCl}(\tilde{1},r,s)=\tilde{1}$.

(2) $(\tau_i,\tau_j)-\mathrm{sCl}(\mu,r,s)\ge\mu$.

(3) $(\tau_i,\tau_j)-\mathrm{sCl}(\mu\vee\rho,r,s)$
$$\ge(\tau_i,\tau_j)-\mathrm{sCl}(\mu,r,s)\vee(\tau_i,\tau_j)-\mathrm{sCl}(\rho,r,s).$$

(4) $(\tau_i,\tau_j)-\mathrm{sCl}((\tau_i,\tau_j)-\mathrm{sCl}(\mu,r,s),r,s)$
$$=(\tau_i,\tau_j)-\mathrm{sCl}(\mu,r,s).$$

(5) $(\tau_i,\tau_j)-\mathrm{sInt}(\tilde{0},r,s)=\tilde{0}$, $(\tau_i,\tau_j)-\mathrm{sInt}(\tilde{1},r,s)=\tilde{1}$.

(6) $(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s)\le\mu$.

(7) $(\tau_i,\tau_j)-\mathrm{sInt}(\mu\wedge\rho,r,s)$
$$\le(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s)\wedge(\tau_i,\tau_j)-\mathrm{sInt}(\rho,r,s).$$

(8) $(\tau_i,\tau_j)-\mathrm{sInt}((\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s),r,s)$
$$=(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s).$$

Theorem 3.2 For a fuzzy set μ of a smooth bitopological space (X,τ_1,τ_2) and $r,s\in I_0$, we have:

(1) $((\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s))^c=(\tau_i,\tau_j)-\mathrm{sCl}(\mu^c,r,s)$.

(2) $((\tau_i,\tau_j)-\mathrm{sCl}(\mu,r,s))^c=(\tau_i,\tau_j)-\mathrm{sInt}(\mu^c,r,s)$.

Proof. (1) Since $(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s)\le\mu$ and $(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s)$ is a (τ_i,τ_j)-fuzzy (r,s)-semiopen set, $\mu^c\le((\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s))^c$ and $((\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s))^c$ is a (τ_i,τ_j)-fuzzy (r,s)-semiclosed set of X. Thus

$$\begin{aligned}&(\tau_i,\tau_j)-\mathrm{sCl}(\mu^c,r,s)\\ &\le(\tau_i,\tau_j)-\mathrm{sCl}(((\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s))^c,r,s)\\ &=((\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s))^c.\end{aligned}$$

Conversely, since $\mu^c\le(\tau_i,\tau_j)-\mathrm{sCl}(\mu^c,r,s)$ and $(\tau_i,\tau_j)-\mathrm{sCl}(\mu^c,r,s)$ is a (τ_i,τ_j)-fuzzy (r,s)-semiclosed set, we have

$$((\tau_i,\tau_j)-\mathrm{sCl}(\mu^c,r,s))^c\le\mu$$

and $((\tau_i,\tau_j)-\mathrm{sCl}(\mu^c,r,s))^c$ is a (τ_i,τ_j)-fuzzy (r,s)-semiopen set of X. Thus

$$\begin{aligned}&((\tau_i,\tau_j)-\mathrm{sCl}(\mu^c,r,s))^c\\ &=(\tau_i,\tau_j)-\mathrm{sInt}(((\tau_i,\tau_j)-\mathrm{sCl}(\mu^c,r,s))^c,r,s)\\ &\le(\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s)\end{aligned}$$

and hence

$$((\tau_i,\tau_j)-\mathrm{sInt}(\mu,r,s))^c \le (\tau_i,\tau_j)-\mathrm{sCl}(\mu^c,r,s).$$

(2) Similar to (1).

4. Fuzzy pairwise (r,s)-semicontinuous mappings

Definition 4.1 [7] Let $f:(X,\tau_1,\tau_2)\to(Y,\omega_1,\omega_2)$ be a mapping from a smooth bitopological spaces X to another smooth bitopological space Y and $r,s\in I_0$. Then f is called a fuzzy pairwise (r,s)-continuous ((r,s)-open and (r,s)-closed, respectively) mapping if the induced mapping $f:(X,\tau_1)\to(Y,\omega_1)$ is a fuzzy r-continuous (r-open and r-closed, respectively) mapping and the induced mapping $f:(X,\tau_2)\to(Y,\omega_2)$ is a fuzzy s-continuous (s-open and s-closed, respectively) mapping.

Definition 4.2 [7] Let $f:(X,\tau_1,\tau_2)\to(Y,\omega_1,\omega_2)$ be a mapping from a smooth bitopological spaces X to another smooth bitopological space Y and $r,s\in I_0$. Then f is called

(1) a fuzzy pairwise (r,s)-semicontinuous mapping if $f^{-1}(\mu)$ is a (τ_1,τ_2)-fuzzy (r,s)-semiopen set of X for each ω_1-fuzzy r-open set μ of Y and $f^{-1}(\nu)$ is a (τ_2,τ_1)-fuzzy (s,r)-semiopen set of X for each ω_2-fuzzy s-open set ν of Y,

(2) a fuzzy pairwise (r,s)-semiopen mapping if $f(\rho)$ is a (ω_1,ω_2)-fuzzy (r,s)-semiopen set of Y for each τ_1-fuzzy r-open set ρ of X and $f(\lambda)$ is a (ω_2,ω_1)-fuzzy (s,r)-semiopen set of Y for each τ_2-fuzzy s-open set λ of X,

(3) a fuzzy pairwise (r,s)-semiclosed mapping if $f(\rho)$ is a (ω_1,ω_2)-fuzzy (r,s)-semiclosed set of Y for each τ_1-fuzzy r-closed set ρ of X and $f(\lambda)$ is a (ω_2,ω_1)-fuzzy (s,r)-semiclosed set of Y for each τ_2-fuzzy s-closed set λ of X.

Theorem 4.3 Let $f:(X,\tau_1,\tau_2)\to(Y,\omega_1,\omega_2)$ be a mapping and $r,s\in I_0$. Then the following statements are equivalent:

(1) f is a fuzzy pairwise (r,s)-semicontinuous mapping.

(2) For each fuzzy set ρ of X,

$$f((\tau_1,\tau_2)-\mathrm{sCl}(\rho,r,s))\le\omega_1-\mathrm{Cl}(f(\rho),r)$$

and

$$f((\tau_2,\tau_1)-\mathrm{sCl}(\rho,s,r))\le\omega_2-\mathrm{Cl}(f(\rho),s).$$

(3) For each fuzzy set μ of Y,

$$(\tau_1,\tau_2)-\mathrm{sCl}(f^{-1}(\mu),r,s)\le f^{-1}(\omega_1-\mathrm{Cl}(\mu,r))$$

and

$$(\tau_2,\tau_1)-\mathrm{sCl}(f^{-1}(\mu),s,r)\le f^{-1}(\omega_2-\mathrm{Cl}(\mu,s)).$$

(4) For each fuzzy set μ of Y,

$$f^{-1}(\omega_1-\mathrm{Int}(\mu,r))\le(\tau_1,\tau_2)-\mathrm{sInt}(f^{-1}(\mu),r,s)$$

and

$$f^{-1}(\omega_2-\mathrm{Int}(\mu,s))\le(\tau_2,\tau_1)-\mathrm{sInt}(f^{-1}(\mu),s,r).$$

Proof. (1) $\Rightarrow$ (2) Let ρ be any fuzzy set of X. Then $\omega_1-\mathrm{Cl}(f(\rho),r)$ is ω_1-fuzzy r-closed and $\omega_2-\mathrm{Cl}(f(\rho),s)$ is ω_2-fuzzy s-closed in Y. Since f is a fuzzy pairwise (r,s)-semicontinuous mapping, we have $f^{-1}(\omega_1-\mathrm{Cl}(f(\rho),r))$ is a (τ_1,τ_2)-fuzzy (r,s)-semiclosed set and $f^{-1}(\omega_2-\mathrm{Cl}(f(\rho),s))$ is a (τ_2,τ_1)-fuzzy (s,r)-semiclosed set of X. Thus

$$\begin{aligned}&(\tau_1,\tau_2)-\mathrm{sCl}(\rho,r,s)\\&\le(\tau_1,\tau_2)-\mathrm{sCl}(f^{-1}(\omega_1-\mathrm{Cl}(f(\rho),r)),r,s)\\&=f^{-1}(\omega_1-\mathrm{Cl}(f(\rho),r))\end{aligned}$$

and

$$\begin{aligned}&(\tau_2,\tau_1)-\mathrm{sCl}(\rho,s,r)\\&\le(\tau_2,\tau_1)-\mathrm{sCl}(f^{-1}(\omega_2-\mathrm{Cl}(f(\rho),s)),s,r)\\&=f^{-1}(\omega_2-\mathrm{Cl}(f(\rho),s)).\end{aligned}$$

Hence

$$\begin{aligned}f((\tau_1,\tau_2)-\mathrm{sCl}(\rho,r,s))&\le ff^{-1}(\omega_1-\mathrm{Cl}(f(\rho),r))\\&\le\omega_1-\mathrm{Cl}(f(\rho),r)\end{aligned}$$

and

$$\begin{aligned}f((\tau_2,\tau_1)-\mathrm{sCl}(\rho,s,r))&\le ff^{-1}(\omega_2-\mathrm{Cl}(f(\rho),s))\\&\le\omega_2-\mathrm{Cl}(f(\rho),s).\end{aligned}$$

(2) $\Rightarrow$ (3) Let μ be any fuzzy set of Y. Then

$$\begin{aligned}f((\tau_1,\tau_2)-\mathrm{sCl}(f^{-1}(\mu),r,s))&\le\omega_1-\mathrm{Cl}(ff^{-1}(\mu),r)\\&\le\omega_1-\mathrm{Cl}(\mu,r)\end{aligned}$$

and

$$\begin{aligned}f((\tau_2,\tau_1)-\mathrm{sCl}(f^{-1}(\mu),s,r))&\le\omega_2-\mathrm{Cl}(ff^{-1}(\mu),s)\\&\le\omega_2-\mathrm{Cl}(\mu,s).\end{aligned}$$

Thus

$$\begin{aligned}&(\tau_1,\tau_2)-\mathrm{sCl}(f^{-1}(\mu),r,s)\\&\le f^{-1}f((\tau_1,\tau_2)-\mathrm{sCl}(f^{-1}(\mu),r,s))\\&\le f^{-1}(\omega_1-\mathrm{Cl}(\mu,r))\end{aligned}$$

and

$$(\tau_2,\tau_1)-\mathrm{sCl}(f^{-1}(\mu),s,r)$$
$$\le f^{-1}f((\tau_2,\tau_1)-\mathrm{sCl}(f^{-1}(\mu),s,r))$$
$$\le f^{-1}(\omega_2-\mathrm{Cl}(\mu,s)).$$

(3) $\Rightarrow$ (4) Let μ be any fuzzy set of Y. Then

$$(\tau_1,\tau_2)-\mathrm{sCl}(f^{-1}(\mu)^c,r,s)\le f^{-1}(\omega_1-\mathrm{Cl}(\mu^c,r))$$

and

$$(\tau_2,\tau_1)-\mathrm{sCl}(f^{-1}(\mu)^c,s,r)\le f^{-1}(\omega_2-\mathrm{Cl}(\mu^c,s)).$$

By Theorem 3.2, we have

$$\begin{aligned}f^{-1}(\omega_1-\mathrm{Int}(\mu,r)) &=(f^{-1}(\omega_1-\mathrm{Cl}(\mu^c,r)))^c\\ &\le((\tau_1,\tau_2)-\mathrm{sCl}(f^{-1}(\mu)^c,r,s))^c\\ &=(\tau_1,\tau_2)-\mathrm{sInt}(f^{-1}(\mu),r,s)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(\omega_2-\mathrm{Int}(\mu,s)) &=(f^{-1}(\omega_2-\mathrm{Cl}(\mu^c,s)))^c\\ &\le((\tau_2,\tau_1)-\mathrm{sCl}(f^{-1}(\mu)^c,s,r))^c\\ &=(\tau_2,\tau_1)-\mathrm{sInt}(f^{-1}(\mu),s,r).\end{aligned}$$

(4) $\Rightarrow$ (1) Let μ be any ω_1-fuzzy r-open set and ν any ω_2-fuzzy s-open set of Y. Then $\omega_1-\mathrm{Int}(\mu,r)=\mu$ and $\omega_2-\mathrm{Int}(\nu,s)=\nu$. Thus

$$\begin{aligned}f^{-1}(\mu) &=f^{-1}(\omega_1-\mathrm{Int}(\mu,r))\\ &\le(\tau_1,\tau_2)-\mathrm{sInt}(f^{-1}(\mu),r,s)\le f^{-1}(\mu)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(\nu) &=f^{-1}(\omega_2-\mathrm{Int}(\nu,s))\\ &\le(\tau_2,\tau_1)-\mathrm{sInt}(f^{-1}(\nu),s,r)\le f^{-1}(\nu).\end{aligned}$$

So $f^{-1}(\mu)=(\tau_1,\tau_2)-\mathrm{sInt}(f^{-1}(\mu),r,s)$ and $f^{-1}(\nu)=(\tau_2,\tau_1)-\mathrm{sInt}(f^{-1}(\nu),s,r)$. Hence $f^{-1}(\mu)$ is a (τ_1,τ_2)-fuzzy (r,s)-semiopen set and $f^{-1}(\nu)$ is a (τ_2,τ_1)-fuzzy (s,r)-semiopen set of X. Therefore f is a fuzzy pairwise (r,s)-semicontinuous mapping.

Theorem 4.4 Let $f:(X,\tau_1,\tau_2)\to(Y,\omega_1,\omega_2)$ be a bijection and $r,s\in I_0$. Then f is a fuzzy pairwise (r,s)-semicontinuous mapping if and only if

$$\omega_1-\mathrm{Int}(f(\rho),r)\le f((\tau_1,\tau_2)-\mathrm{sInt}(\rho,r,s))$$

and

$$\omega_2-\mathrm{Int}(f(\rho),s)\le f((\tau_2,\tau_1)-\mathrm{sInt}(\rho,s,r))$$

for each fuzzy set ρ of X.

Proof. Let ρ be any fuzzy set of X. Since f is one-to-one,

$$\begin{aligned}f^{-1}(\omega_1-\mathrm{Int}(f(\rho),r)) &\le(\tau_1,\tau_2)-\mathrm{sInt}(f^{-1}f(\rho),r,s)\\ &=(\tau_1,\tau_2)-\mathrm{sInt}(\rho,r,s)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(\omega_2-\mathrm{Int}(f(\rho),s)) &\le(\tau_2,\tau_1)-\mathrm{sInt}(f^{-1}f(\rho),s,r)\\ &=(\tau_2,\tau_1)-\mathrm{sInt}(\rho,s,r).\end{aligned}$$

Since f is onto,

$$\begin{aligned}\omega_1-\mathrm{Int}(f(\rho),r) &=ff^{-1}(\omega_1-\mathrm{Int}(f(\rho),r))\\ &\le f((\tau_1,\tau_2)-\mathrm{sInt}(\rho,r,s))\end{aligned}$$

and

$$\begin{aligned}\omega_2-\mathrm{Int}(f(\rho),s) &=ff^{-1}(\omega_2-\mathrm{Int}(f(\rho),s))\\ &\le f((\tau_2,\tau_1)-\mathrm{sInt}(\rho,s,r)).\end{aligned}$$

Conversely, let μ be any fuzzy set of Y. Since f is onto,

$$\begin{aligned}\omega_1-\mathrm{Int}(\mu,r) &=\omega_1-\mathrm{Int}(ff^{-1}(\mu),r)\\ &\le f((\tau_1,\tau_2)-\mathrm{sInt}(f^{-1}(\mu),r,s))\end{aligned}$$

and

$$\begin{aligned}\omega_2-\mathrm{Int}(\mu,s) &=\omega_2-\mathrm{Int}(ff^{-1}(\mu),s)\\ &\le f((\tau_2,\tau_1)-\mathrm{sInt}(f^{-1}(\mu),s,r)).\end{aligned}$$

Since f is one-to-one,

$$\begin{aligned}f^{-1}(\omega_1-\mathrm{Int}(\mu,r)) &\le f^{-1}f((\tau_1,\tau_2)-\mathrm{sInt}(f^{-1}(\mu),r,s))\\ &=(\tau_1,\tau_2)-\mathrm{sInt}(f^{-1}(\mu),r,s)\end{aligned}$$

and

$$\begin{aligned}f^{-1}(\omega_2-\mathrm{Int}(\mu,s)) &\le f^{-1}f((\tau_2,\tau_1)-\mathrm{sInt}(f^{-1}(\mu),s,r))\\ &=(\tau_2,\tau_1)-\mathrm{sInt}(f^{-1}(\mu),s,r).\end{aligned}$$

Hence the theorem follows.

Theorem 4.5 Let $f:(X,\tau_1,\tau_2)\to(Y,\omega_1,\omega_2)$ be a mapping and $r,s\in I_0$. Then the following statements are equivalent:

(1) f is a fuzzy pairwise (r,s)-semiopen mapping.

(2) For each fuzzy set ρ of X,

$$f(\tau_1-\mathrm{Int}(\rho,r))\le(\omega_1,\omega_2)-\mathrm{sInt}(f(\rho),r,s)$$

and

$$f(\tau_2-\mathrm{Int}(\rho,s))\le(\omega_2,\omega_1)-\mathrm{sInt}(f(\rho),s,r).$$

(3) For each fuzzy set μ of Y,

$$\tau_1-\mathrm{Int}(f^{-1}(\mu),r)\le f^{-1}((\omega_1,\omega_2)-\mathrm{sInt}(\mu,r,s))$$

and

$$\tau_2-\mathrm{Int}(f^{-1}(\mu),s)\le f^{-1}((\omega_2,\omega_1)-\mathrm{sInt}(\mu,s,r)).$$

Proof. (1) $\Rightarrow$ (2) Let ρ be any fuzzy set of X. Clearly, $\tau_1-\mathrm{Int}(\rho,r)$ is τ_1-fuzzy r-open and $\tau_2-\mathrm{Int}(\rho,s)$ is τ_2-fuzzy s-open in X. Since f is a fuzzy pairwise (r,s)-semiopen mapping, we have $f(\tau_1-\mathrm{Int}(\rho,r))$ is a (ω_1,ω_2)-fuzzy (r,s)-semiopen set and $f(\tau_2-\mathrm{Int}(\rho,s))$ is a (ω_2,ω_1)-fuzzy (s,r)-semiopen set of Y. Thus

$$\begin{aligned} f(\tau_1-\mathrm{Int}(\rho,r)) &= (\omega_1,\omega_2)-\mathrm{sInt}(f(\tau_1-\mathrm{Int}(\rho,r)),r,s) \\ &\le (\omega_1,\omega_2)-\mathrm{sInt}(f(\rho),r,s) \end{aligned}$$

and

$$\begin{aligned} f(\tau_2-\mathrm{Int}(\rho,s)) &= (\omega_2,\omega_1)-\mathrm{sInt}(f(\tau_2-\mathrm{Int}(\rho,s)),s,r) \\ &\le (\omega_2,\omega_1)-\mathrm{sInt}(f(\rho),s,r). \end{aligned}$$

(2) $\Rightarrow$ (3) Let μ be any fuzzy set of Y. Then $f^{-1}(\mu)$ is a fuzzy set of X. By (2),

$$\begin{aligned} f(\tau_1-\mathrm{Int}(f^{-1}(\mu),r)) &\le (\omega_1,\omega_2)-\mathrm{sInt}(ff^{-1}(\mu),r,s) \\ &\le (\omega_1,\omega_2)-\mathrm{sInt}(\mu,r,s) \end{aligned}$$

and

$$\begin{aligned} f(\tau_2-\mathrm{Int}(f^{-1}(\mu),s)) &\le (\omega_2,\omega_1)-\mathrm{sInt}(ff^{-1}(\mu),s,r) \\ &\le (\omega_2,\omega_1)-\mathrm{sInt}(\mu,s,r). \end{aligned}$$

Thus we have

$$\begin{aligned} \tau_1-\mathrm{Int}(f^{-1}(\mu),r) &\le f^{-1}f(\tau_1-\mathrm{Int}(f^{-1}(\mu),r)) \\ &\le f^{-1}((\omega_1,\omega_2)-\mathrm{sInt}(\mu,r,s)) \end{aligned}$$

and

$$\begin{aligned} \tau_2-\mathrm{Int}(f^{-1}(\mu),s) &\le f^{-1}f(\tau_2-\mathrm{Int}(f^{-1}(\mu),s)) \\ &\le f^{-1}((\omega_2,\omega_1)-\mathrm{sInt}(\mu,s,r)). \end{aligned}$$

(3) $\Rightarrow$ (1) Let ρ be any τ_1-fuzzy r-open set and λ any τ_2-fuzzy s-open set of X. Then $\tau_1-\mathrm{Int}(\rho,r)=\rho$ and $\tau_2-\mathrm{Int}(\lambda,s)=\lambda$. By (3),

$$\begin{aligned} \rho=\tau_1-\mathrm{Int}(\rho,r) &\le \tau_1-\mathrm{Int}(f^{-1}f(\rho),r) \\ &\le f^{-1}((\omega_1,\omega_2)-\mathrm{sInt}(f(\rho),r,s)) \end{aligned}$$

and

$$\begin{aligned} \lambda=\tau_2-\mathrm{Int}(\lambda,s) &\le \tau_2-\mathrm{Int}(f^{-1}f(\lambda),s) \\ &\le f^{-1}((\omega_2,\omega_1)-\mathrm{sInt}(f(\lambda),s,r)). \end{aligned}$$

Hence we have

$$\begin{aligned} f(\rho) &\le ff^{-1}((\omega_1,\omega_2)-\mathrm{sInt}(f(\rho),r,s)) \\ &\le (\omega_1,\omega_2)-\mathrm{sInt}(f(\rho),r,s) \le f(\rho) \end{aligned}$$

and

$$\begin{aligned} f(\lambda) &\le ff^{-1}((\omega_2,\omega_1)-\mathrm{sInt}(f(\lambda),s,r)) \\ &\le (\omega_2,\omega_1)-\mathrm{sInt}(f(\lambda),s,r) \le f(\lambda). \end{aligned}$$

Thus $f(\rho)=(\omega_1,\omega_2)-\mathrm{sInt}(f(\rho),r,s)$ and $f(\lambda)=(\omega_2,\omega_1)-\mathrm{sInt}(f(\lambda),s,r)$. Hence $f(\rho)$ is (ω_1,ω_2)-fuzzy (r,s)-semiopen and $f(\lambda)$ is (ω_2,ω_1)-fuzzy (s,r)-semiopen in Y. Therefore f is a fuzzy pairwise (r,s)-semiopen mapping.

Theorem 4.6 Let $f:(X,\tau_1,\tau_2)\to(Y,\omega_1,\omega_2)$ be a mapping and $r,s\in I_0$. Then the following statements are equivalent:

(1) f is a fuzzy pairwise (r,s)-semiclosed mapping.

(2) For each fuzzy set ρ of X,

$$(\omega_1,\omega_2)-\mathrm{sCl}(f(\rho),r,s)\le f(\tau_1-\mathrm{Cl}(\rho,r))$$

and

$$(\omega_2,\omega_1)-\mathrm{sCl}(f(\rho),s,r)\le f(\tau_2-\mathrm{Cl}(\rho,s)).$$

Proof. (1) $\Rightarrow$ (2) Let ρ be any fuzzy set of X. Clearly, $\tau_1-\mathrm{Cl}(\rho,r)$ is τ_1-fuzzy r-closed and $\tau_2-\mathrm{Cl}(\rho,s)$ is τ_2-fuzzy s-closed in X. Since f is a fuzzy pairwise (r,s)-semiclosed mapping, we have $f(\tau_1-\mathrm{Cl}(\rho,r))$ is a (ω_1,ω_2)-fuzzy (r,s)-semiclosed set and $f(\tau_2-\mathrm{Cl}(\rho,s))$ is a (ω_2,ω_1)-fuzzy (s,r)-semiopen set of Y. Thus

$$\begin{aligned} &(\omega_1,\omega_2)-\mathrm{sCl}(f(\rho),r,s) \\ &\le (\omega_1,\omega_2)-\mathrm{sCl}(f(\tau_1-\mathrm{Cl}(\rho,r)),r,s) \\ &= f(\tau_1-\mathrm{Cl}(\rho,r)) \end{aligned}$$

and

$$\begin{aligned} &(\omega_2,\omega_1)-\mathrm{sCl}(f(\rho),s,r) \\ &\le (\omega_2,\omega_1)-\mathrm{sCl}(f(\tau_2-\mathrm{Cl}(\rho,s)),s,r) \\ &= f(\tau_2-\mathrm{Cl}(\rho,s)). \end{aligned}$$

(2) $\Rightarrow$ (1) Let ρ be any τ_1-fuzzy r-closed set and λ any τ_2-fuzzy s-closed set of X. Then $\tau_1-\mathrm{Cl}(\rho,r)=\rho$ and $\tau_2-\mathrm{Cl}(\lambda,s)=\lambda$. By (2),

$$\begin{aligned} (\omega_1,\omega_2)-\mathrm{sCl}(f(\rho),r,s) &\le f(\tau_1-\mathrm{Cl}(\rho,r)) \\ &= f(\rho) \\ &\le (\omega_1,\omega_2)-\mathrm{sCl}(f(\rho),r,s) \end{aligned}$$

and

$$\begin{aligned} (\omega_2,\omega_1)-\mathrm{sCl}(f(\lambda),s,r) &\le f(\tau_2-\mathrm{Cl}(\lambda,s)) \\ &= f(\lambda) \\ &\le (\omega_2,\omega_1)-\mathrm{sCl}(f(\lambda),s,r). \end{aligned}$$

Thus $f(\rho)=(\omega_1,\omega_2)-\mathrm{sCl}(f(\rho),r,s)$ and $f(\lambda)=(\omega_2,\omega_1)-\mathrm{sCl}(f(\lambda),s,r)$. Hence $f(\rho)$ is (ω_1,ω_2)-fuzzy (r,s)-semiclosed and $f(\lambda)$ is (ω_2,ω_1)-fuzzy (s,r)-semiclosed in Y. Therefore f is fuzzy pairwise (r,s)-semiclosed.

Theorem 4.7 Let $f:(X,\tau_1,\tau_2)\to(Y,\omega_1,\omega_2)$ be a bijection and $r,s\in I_0$. Then f is a fuzzy pairwise (r,s)-semiclosed mapping if and only if

$$f^{-1}((\omega_1,\omega_2)-\mathrm{sCl}(\mu,r,s))\le\tau_1-\mathrm{Cl}(f^{-1}(\mu),r)$$

and

$$f^{-1}((\omega_2,\omega_1)-\mathrm{sCl}(\mu,s,r))\le\tau_2-\mathrm{Cl}(f^{-1}(\mu),s)$$

for each fuzzy set μ of Y.

Proof. Let μ be any fuzzy set of Y. Since f is onto,

$$\begin{aligned}(\omega_1,\omega_2)-\mathrm{sCl}(\mu,r,s)&=(\omega_1,\omega_2)-\mathrm{sCl}(ff^{-1}(\mu),r,s)\\&\le f(\tau_1-\mathrm{Cl}(f^{-1}(\mu),r))\end{aligned}$$

and

$$\begin{aligned}(\omega_2,\omega_1)-\mathrm{sCl}(\mu,s,r)&=(\omega_2,\omega_1)-\mathrm{sCl}(ff^{-1}(\mu),s,r)\\&\le f(\tau_2-\mathrm{Cl}(f^{-1}(\mu),s)).\end{aligned}$$

Since f is one-to-one,

$$\begin{aligned}f^{-1}((\omega_1,\omega_2)-\mathrm{sCl}(\mu,r,s))&\le f^{-1}f(\tau_1-\mathrm{Cl}(f^{-1}(\mu),r))\\&=\tau_1-\mathrm{Cl}(f^{-1}(\mu),r)\end{aligned}$$

and

$$\begin{aligned}f^{-1}((\omega_2,\omega_1)-\mathrm{sCl}(\mu,s,r))&\le f^{-1}f(\tau_2-\mathrm{Cl}(f^{-1}(\mu),s))\\&=\tau_2-\mathrm{Cl}(f^{-1}(\mu),s).\end{aligned}$$

Conversely, let ρ be any fuzzy set of X. Since f is one-to-one,

$$\begin{aligned}f^{-1}((\omega_1,\omega_2)-\mathrm{sCl}(f(\rho),r,s))&\le\tau_1-\mathrm{Cl}(f^{-1}f(\rho),r)\\&=\tau_1-\mathrm{Cl}(\rho,r)\end{aligned}$$

and

$$\begin{aligned}f^{-1}((\omega_2,\omega_1)-\mathrm{sCl}(f(\rho),s,r))&\le\tau_2-\mathrm{Cl}(f^{-1}f(\rho),s)\\&=\tau_2-\mathrm{Cl}(\rho,s).\end{aligned}$$

Since f is onto,

$$\begin{aligned}(\omega_1,\omega_2)-\mathrm{sCl}(f(\rho),r,s)&=ff^{-1}((\omega_1,\omega_2)-\mathrm{sCl}(f(\rho),r,s))\\&\le f(\tau_1-\mathrm{Cl}(\rho,r))\end{aligned}$$

and

$$\begin{aligned}(\omega_2,\omega_1)-\mathrm{sCl}(f(\rho),s,r)&=ff^{-1}((\omega_2,\omega_1)-\mathrm{sCl}(f(\rho),s,r))\\&\le f(\tau_2-\mathrm{Cl}(\rho,s)).\end{aligned}$$

Hence the theorem follows.

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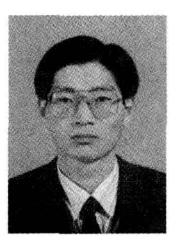

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