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Effect of Kurtosis on the Flow Factors Using Average Flow Model

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Abstract : The roughness effects are very important due to the presence of interacting asperities in mixed lubrication regime. An average Reynolds equation using flow factors is useful to determine the effects of surface roughness on mixed lubrication. In this study, the effect of kurtosis on flow factors is investigated using random rough surfaces generated numerically. The results show that flow factors are very sensitive to h/σ according to the value of kurtosis in the partial lubrication regime.

Keywords : Flow factor, mixed lubrication, kurtosis

Introduction

Lubricant is put in to reduce friction which arises when the surfaces of two solid bodies are brought into contact. However, when the film thickness is of the order of roughness heights, the roughness effects should be considered. The study of surface roughness effects has gained an increasing attention after the introduction of stochastic concepts for a slider bearing with transverse roughness by Tzeng and Saibel [1]. Many studies of mixed lubrication analyses have been performed. Patir and Cheng derived average Reynolds equation in terms of flow factors considered the effects of roughness [2,3]. Teal and Lebeck introduced flow factors using corner point method [4]. Lunde and Tonder studied about the influence of boundary condition in mixed lubrication analysis using average flow model [5]. Most of these results are obtained for having a Gaussian distribution of roughness height, but most of the engineering surfaces have non-Gaussian distribution of roughness height. Therefore mixed lubrication analysis of rough surface having non-Gaussian distribution of roughness height needs to be performed. The effect of skewness on flow factor in mixed lubrication was studied by Patir [6]. In this study, the effect of kurtosis on flow factors is investigated using 3 dimensional rough surfaces generated numerically.

Kurtosis and Surface Roughness

In the past studies, the surface parameters expressed as the Root Mean Square (RMS) or the Center Line Average Roughness (Ra) are mainly used. This method tells nothing about the actual surface profile. Though surface profiles have the same RMS or Ra, they could have different asperity distributions. So, in order to represent more exact surface profile, Skewness(Sk) and Kurtosis(Ku) which give an information about the shape of asperities distribution should be used. The skewness shows the degree of symmetry of the profile, and the kurtosis shows the degree of pointness or bluntness of waveform. The kurtosis is defined as

$$ku = \frac{1}{\sigma^4}\int_{-\infty}^{\infty} z^4 \overline{\psi} dz \qquad (1)$$

where, σ : rms roughness
z : Surface Roughness Height
$\overline{\psi}$: Power Spectral Density Function

The kurtosis always has a positive value and measures the sharpness of symmetric distribution. For a gaussian distribution the curve has a kurtosis of 3. When most of the profile heights are close to the reference line, kurtosis is quite large; a relatively flat height distribution has a kurtosis near zero. Curves with values of Ku less than 3 are called "platikurtic" and those with Ku greater than 3 as called "leptokurtic." [7]

Surface Generation Procedure

The contact simulation of the rough surface would require surface roughness data, which can either be measured from rough surface or generated numerically to be in digital form. Accurate measurement of a profile using a stylus profilometer seem to be relatively simple, but the measurement of a surface is much more complex. Surface measurements are done by taking a number of paralleled profile measurements which require an accurate relocation technique and an additional software requirement to align the profiles numerically. However, Randomly generating surface roughness by numerical means is simpler and offers certain advantages. In addition to eliminating all the hardware requirements for surface measurement, it also eliminates the need to filter out the unwonted wavelength from a measured surface.

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Furthermore, it is much convenient to generate random surface roughness with statistical properties such as the distribution and the correlation of roughness height.

Therefore, the rough surfaces used in this study were computer generated based on a numerical method. Most of the statistical parameters of a rough surface can be derived from two statistical functions; the autocorrelation function (ACF) and the probability density function. The ACF of a surface is defined as

$$R(\lambda_x, \lambda_y) = E\{z(x, y), z(x+\lambda_x, y+\lambda_y)\} \tag{2}$$

where, E: Expendency Operator

λ_x, λ_y: Autocorrelation Length along x- and y-direction

Since a digital form of surface roughness is sought, it is more convenient to use an index notation. Let z_{ij} denote the roughness amplitude at $x = i\Delta x$ and $y = i\Delta y$ where Δx and Δy are the sampling intervals in the x and y directions. Similarly, R_{pq} is defined as

$$R_{pq} = R(p\Delta x, q\Delta y) = E(z_{ij}, z_{i+p, j+q}) \tag{3}$$

Probability density function of the roughness heights is divided into gaussian and nongaussian. Through the use of probability density function on the gaussian distribution, the roughness height with the desired frequency and correlation properties can be obtained.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\{-(z-\bar{z})^2/2\sigma^2\} \tag{4}$$

where, $\bar{z}$: Average value of roughness heights

Eq. (5) shows the probability density function on nongaussian distribution and the desired roughness height distribution having the property of gaussian or nongaussian can be obtained by converting the weighting factor a and b. If a value of a is greater than a value of b, the roughness heights concentrate on the valley, and if a value of a is less than a value of b, those concentrate on the peak.(skewness) If the values of a and b are the same and those are greater or less than 3, the roughness height has a symmetric but nongaussian distribution.(kurtosis) [8,9,10]

$$f(x) = \frac{1}{\beta(a, b)}z^{(a-1)}\cdot(1-z)^{(b-1)} \tag{5}$$

where, $\beta(a,b)$: Beta Function

a, b: Weighting Factor

Flow factors

Pressure flow factor ϕ_x

To obtain pressure flow factor ϕ_x, we assume $U_1 = U_2 = \bar{U}$ (pure rolling) and consider the following model problem for simulation (see Fig. 1)

$$\frac{\partial}{\partial x}\left(\frac{h_T^3}{12\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{h_T^3}{12\mu}\frac{\partial p}{\partial y}\right) = \bar{U}\frac{\partial h_T}{\partial x} + \frac{\partial h_T}{\partial t} \tag{6}$$

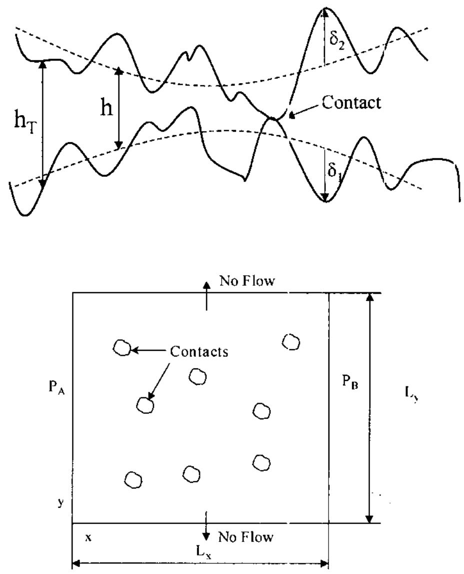

Fig. 1. The model problem for simulation.

$$h_T = h + \delta_1 + \delta_2 \ (\text{h} = \text{constant}) \tag{7}$$

h is constant and δ_1, δ_2 are function of position on each surfaces, but the movement of the surfaces makes them time independent relative to a stationary reference plane. Therefore the two terms on the RHS of Eq. (7) cancel, and Eq. (7) becomes

$$\frac{\partial}{\partial x}\left(\frac{h_T^3}{12\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{h_T^3}{12\mu}\frac{\partial p}{\partial y}\right) = 0 \tag{8}$$

Boundary conditions for obtaining ϕ. are

1) $p = p_A$ at $x = 0$

2) $p = p_B$ at $x = L_x$

3) at $\frac{\partial p}{\partial y} = 0$, $y = L_y$

4) No flow at contact points

The model problem is solved for pressure using finite different methods. ϕ_x becomes

$$\phi_x = \frac{\frac{1}{L_y}\int_0^{L_y}\frac{h_T^3}{12\mu}\frac{\partial p}{\partial x}dy}{\frac{h^3}{12\mu}\frac{\partial \bar{p}}{\partial x}} \tag{9}$$

$$\text{where, } \frac{\partial \bar{p}}{\partial x} = \frac{p_B - p_A}{L_x} \tag{10}$$

Shear flow factor ϕ_s

Shear flow factor ϕ_s is the flow factor about an additional flow transport resulting from the combined effect of sliding and roughness. The shear flow factor ϕ_s is obtained by numerical flow simulation on a model bearing similar to the one used for ϕ_x. The rolling velocity is taken as zero. The model problem for

ϕ_s then becomes pure sliding of two nominally parallel surfaces.

$$\frac{\partial}{\partial x}\left(\frac{h_T^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{h_T^3}{12\mu}\frac{\partial p}{\partial y}\right) = \frac{\partial h_T}{\partial t} \quad (11)$$

The boundary conditions are chosen as:

1) $p = p_A$ at $x = 0, x = L_x$

2) at $y = 0, y = L_y$

3) No flow at contact points

Choose a time interval Δt such that each surface moves a distance Δx during that time. Since each surface moves with a velocity $U_s/2$, Δt and Δx are related by

$$\frac{1}{2}U_s\Delta t = \Delta x \quad (12)$$

The Reynolds equation is approximated by the finite difference formulas. After numerically solving the pressure in the model bearing, the expected flow is obtained by averaging the local flow over the entire bearing.

$$\bar{q}_x = E\left(-\frac{h_T^3}{12\mu}\frac{\partial p}{\partial x}\right) = \frac{1}{L_xL_y}\int_0^{L_y}\int_0^{L_x}\left(-\frac{h_T^3}{12\mu}\frac{\partial p}{\partial y}\right) \quad (13)$$

Since the mean pressure gradient is zero, and there is no rolling velocity, this expression is equal to the additional flow transport due to sliding:

$$\bar{q}_x = \frac{U_s}{2}\sigma\phi_s \quad (14)$$

Therefore, ϕ_s is obtained by

$$\phi_s = \frac{2}{U_s\sigma}E\left(-\frac{h_T^3}{12\mu}\frac{\partial p}{\partial x}\right) \quad (15)$$

Average shear stress factor ϕ_{fp}, ϕ_{fs}

In the partial lubrication regime, the total friction force consists of hydrodynamic and asperity friction forces. The local shear stress at $z = 0$ or $z = h_T$ is given by

$$\tau = \mu\frac{U_2 - U_1}{h_T} \pm \frac{h_T}{2}\frac{\partial p}{\partial x} \quad (16)$$

Where, (+) is surface $2(z = h_T)$, (-) is surface $1(z = 0)$. The local shear stress is a randomly varying quantity. The new stress can be obtained by taking expectations along the control volume.

$$\bar{\tau} = \mu(U_2 - U_1)E\left(\frac{1}{h_T}\right) \pm E\left(\frac{h_T}{2}\frac{\partial p}{\partial x}\right) \quad (17)$$

Similar to the definitions for mean oil flow, empirical shear stress factors ϕ_{fp} and ϕ_{fs} are defined, such that the mean shear stress is given in terms of mean quantities:

$$\bar{\tau} = \frac{\mu(U_2 - U_1)}{h}\left[hE\left[\frac{1}{h_T}\right] \pm \phi_{fs}\right] \pm \phi_{fp}\frac{h}{2}\frac{\partial \bar{p}}{\partial x} \quad (18)$$

ϕ_{fp} is a correction factor for the mean pressure flow component of the shear stress. It is obtained through simulation using the model problem for ϕ_x. Since the sliding velocity is zero for this problem, comparison of Eq. (17), (18) reveals

$$\phi_{fp} = \frac{E\left[\frac{h_T}{2}\frac{\partial p}{\partial x}\right]}{\frac{h}{2}\frac{\partial \bar{p}}{\partial x}} = \frac{\frac{1}{L_xL_y}\int_0^{L_y}\int_0^{L_x} h_T\frac{\partial p}{\partial x}dxdy}{h\frac{\partial \bar{p}}{\partial x}} \quad (19)$$

ϕ_{fs} is another correction term which arises from the combined effect of sliding and roughness, similar to the ϕ_s term in mean flow. ϕ_{fs} is obtained through simulation using the model problem for ϕ_s. Equating Eq. (17), (18) canceling common terms and noting $\partial\bar{p}/\partial x$ for this model problem, ϕ_{fs} becomes:

$$\phi_{fs} = \frac{h}{\mu(U_2 - U_1)}E\left[\frac{h_T}{2}\frac{\partial p}{\partial x}\right] = -\frac{h}{\mu U_s}\frac{1}{L_xL_y}\int_0^{L_y}\int_0^{L_x}\left[\frac{h_T}{2}\frac{\partial p}{\partial x}\right]dxdy \quad (20)$$

Results and Discussion

Fig. 2 show results of the generated surfaces having values of $Ku = 2.0, 3.0$ and 4.0, respectively. The platykurtic surface of $Ku = 2.0$ in Fig. 2 appears more blunted than the Gaussian surface in Fig. 2, while the leptokurtic surface of $Ku = 4.0$ in Fig. 2 appears more pointed than it. As kurtosis value increases contact area gets smaller. Fig. 3-6 shows the result calculated numerically for surfaces as a function of h/σ and kurtosis of the combined roughness. Each point of curves in these Figs is calculated by averaging results from 6 different surfaces all having the same statistical roughness properties.

As h/σ increases, regardless of kurtosis, the pressure flow factor ϕ_x approaches to 1 which is the expected value for smooth surface. The ϕ_x becomes very sensitive to h/σ in the partial lubrication regime. The contact area of surface having $Ku = 4.0$ is larger than that of $Ku = 2.0$ and $Ku = 3.0$ due to a blunted shape of asperity peak. Therefore ϕ_x of surfaces having $Ku = 4.0$ is larger than that of Ku = 3.0 and ϕ_x of surfaces having $Ku = 2.0$ is smaller than that of $Ku = 3.0$ by the no flow conditions at contact points.

The shear flow factor ϕ_s is a function of h/σ and the standard deviations σ_1, σ_2 of the two opposing surfaces. In this study, as we assume that a rough surface moves on a flat surface, ϕ_s's have the values of positive. The ϕ_s values approach rather slowly to zero as h/σ increases. But as h/σ decreases, they increase up to a certain point, and then decrease rapidly toward zero. This strange behavior can be attributed to the increasing number of contacts in the partial lubrication regime. The surface having $Ku = 2.0$ has the largest ϕ_s value because the radius of asperities of the rough surface having $Ku = 2.0$ is largest.

The shear stress factor ϕ_{fp} and ϕ_{fs} are obtained using the

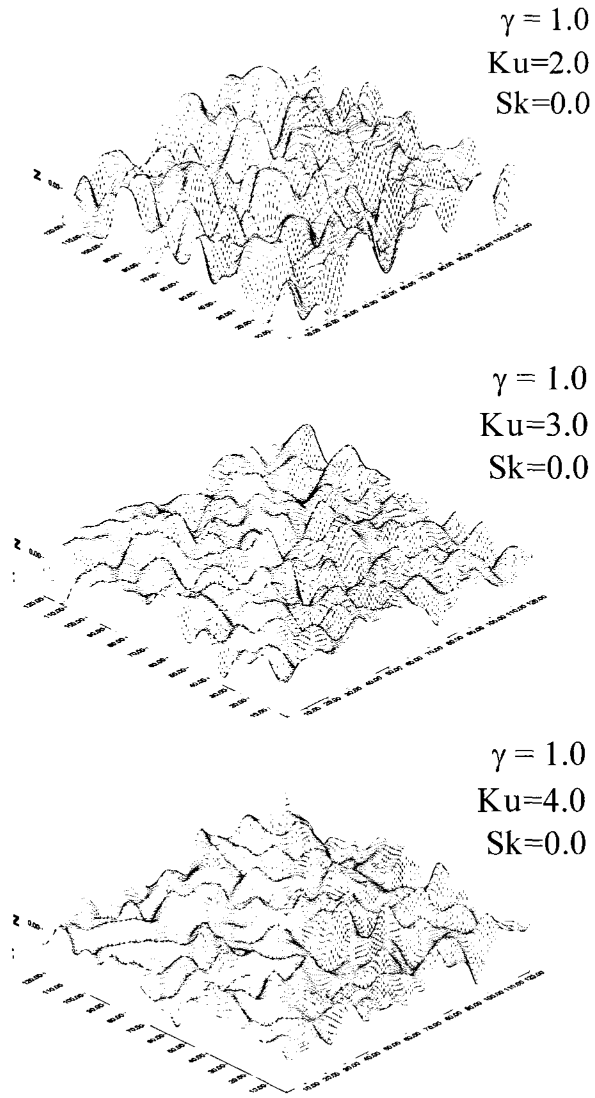

Fig. 2. 3-Dimensional rough surfaces generated numerically [*Ku* = 2,3,4].

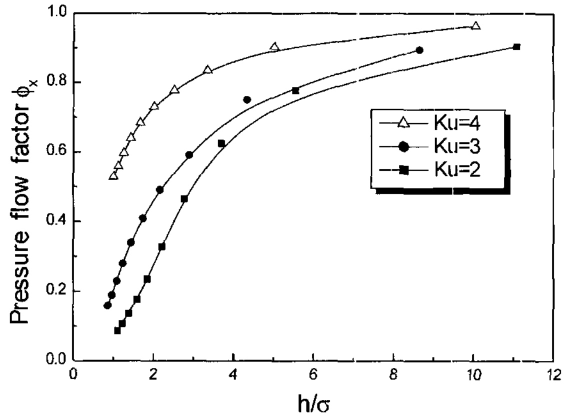

Fig. 3. Pressure flow factors ϕ_x for different kurtosis values.

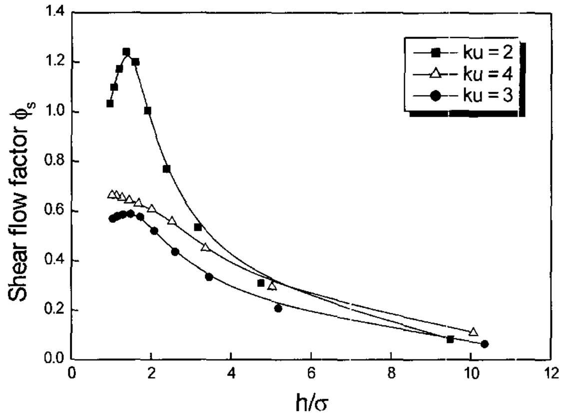

Fig. 4. Shear flow factors ϕ_s for different kurtosis values.

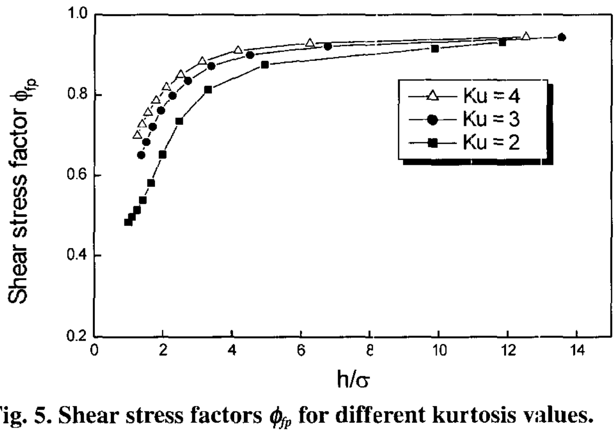

Fig. 5. Shear stress factors ϕ_{fp} for different kurtosis values.

bearing model constructed for ϕ_x and ϕ_s. So, one expected similar characteristics for ϕ_x and ϕ_{fp}, and similar characteristics for ϕ_s and ϕ_{fs}. ϕ_x approaches to 1 regardless of kurtosis as h/σ increases. Comparing (9) with (19), it can be seen that ϕ_{fp} is a measure of the correlation between h_T and $\partial p/\partial x$ while ϕ_x is a measure of the correlation between h_T^3 and $\partial p/\partial x$. Therefore, we would expect similar qualitive behavior for these two factors, but ϕ_{fp} would show a weaker correlation in Fig. 5. Fig. 6 shows ϕ_{fs} values approach rather slowly to zero as *h*/σ increases similar to ϕ_s. But as *h*/σ decreases, they increase up to a certain point, and then decrease rapidly toward zero similar to ϕ_s.

Conclusions

In this study, flow factors for Gaussian and non-Gaussian surfaces generated numerically are evaluated in terms of kurtosis. From this study, the following results were derived.

The pressure flow factor ϕ_x increases with increasing kurtosis in mixed lubrication. As *h*/σ increases, ϕ_x approaches to 1 asymptotically regardless kurtosis values.

2. Shear flow factor ϕ_s is associated with an additional flow transport due to the combined effect of sliding and roughness. ϕ_s has maximum value at $Ku = 2$. As *h*/σ decreases, ϕ_s increases up to a certain point, and then decreases toward zero regardless kurtosis values. This behavior can be attributed to the increasing number of contacts in the mixed lubrication regime.

3. The shear stress factor ϕ_{fp} and ϕ_{fs} are obtained using the

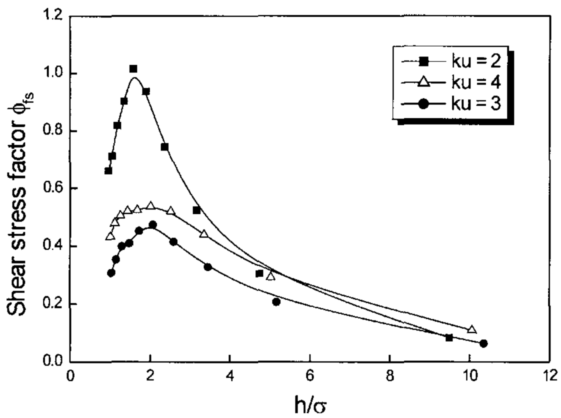

Fig. 6. Shear stress factors ϕ_{fs} for different kurtosis values.

bearing model constructed for ϕ_x and ϕ_s. So, one can expect similar characteristics for ϕ_x and ϕ_{fp}, and similar characteristics for ϕ_s and ϕ_{fs}.

Nomenclature

h	: nominal film thickness (compliance)
h_T	: local film thickness
$\overline{h_T}$	: average film thickness (seperation)
Ku	: kurtosis
p	: hydrodynamic pressure
$\bar{p}$	: mean hydrodynamic pressure
σ	: root mean square roughness
U_1, U_2	: velocity(x direction) of surfaces 1, 2
U	: rolling velocity
U_s	: sliding velocity
μ	: absolute viscosity
γ	: λ_x / λ_y
λ_x, λ_y	: autocorrelation length along x- and y- directions
$\bar{\tau}$	: average hydrodynamic shear stress
ϕ_x	: pressure flow factors
ϕ_s	: shear flow factor
ϕ_{fp}, ϕ_{fs}	: shear stress factors

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