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A Review of Mathematics Education Reform in the United States: Ideal, Practice, and Implication

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The current reform recommendations in the United States have been widely recognized but the outcomes with regard to the teaching practice in the classroom were evaluated as ineffective to foster students' mathematical learning. Given this, this paper reviews the current mathematics education reform in terms of the typical and the recommended teaching practices. An analysis of influences of theoretical perspectives is then provided as an attempt to explore the underpinnings that implicitly motivate the current reform movement. This paper then identifies the difficulties in implementing reform ideals. Building on the review and the analysis, this paper finally provides implications of re-conceptualizing mathematics instruction in the current reform era.

I. OPENING

The call for reform in mathematics education is a reaction to the ample and increasing evidence that mathematics education is not adequately promoting students' mathematical development. Students learned even basic mathematical concepts and procedures only at a superficial level. The problems that have motivated the current reform movement include;

(a) learning without understanding of mathematical ideas (Hiebert, Carpenter, Fennema, Fuson, Human, Murray et al. 1997),
(b) increasingly negative mathematical disposition as students advance through school (Brown, Carpenter, Kouba, Lindquist, Silver & Swafford 1988),
(c) lack of creative mathematical thinking (Peterson 1988),
(d) lack of self-esteem in terms of mathematical ability (McLeod 1994),
(e) lack of problem-solving ability (Charles & Silver 1988; Hiebert, Carpenter, Fennema, Fuson, Human, Murray et al. 1996), and
(f) conceptions of mathematics as a set of rules (Swafford & Brown 1990).

The Third International Mathematics and Science Study (TIMSS) and TIMSS-Repeat provide multinational perspectives on the problems and issues encountered in the United States (Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, et al. 2000; National Center for Education Statistics [NCES] 1996; 1997; Schmidt, McKnight & Raizen 1997). These include;

(a) fragmented curricula covering far more topics in a year than is typical internationally,
(b) low level curricular expectations of what should be the basics in mathematics, in particular for the middle school years,
(c) inclusive but unfocused textbooks that cover many topics but do so in a comparatively shallow manner,
(d) teaching methods focusing on skills rather than thinking and understanding,
(e) a decentralized educational system in which agencies and organizations do not always work towards common goals or combined results, and
(f) an American ideology of mass production and mass education.

Many of these factors are rooted in legal and cultural aspects of the United States society, and hence present no immediate prospects for change. However, teaching methods have been critiqued in the United States, specifically in comparison with Japanese teaching practices (Stigler, Fernandez & Yoshida 1996; TIMSS 1996), and broad-scale efforts have been launched to influence the ways mathematics is taught.

Given this, this paper describes the current mathematics education reform in the United States in terms of the typical and the recommended teaching methods. This paper then reviews the theoretical factors that have motivated the reform and identifies the problems of implementing reform ideals. Building on the review, this paper finally opens for the discussion on re-conceptualizing mathematics instruction.

II. IDEALS OF MATHEMATICS EDUCATION REFORM

2.1. Typical Teaching Practices

The teaching methods perceived as modal in the United States are teacher-centered wherein teachers deliver a pre-given mathematical curriculum mainly through explanation and demonstration, asking students to practice the methods (NCES 1996, 1997; TIMSS 1996). The term *teacher-centered* refers to a teacher's explanations and ideas constituting the focus of classroom mathematical practice. Despite the call for emphasis on problem-solving in mathematics education since the early 1980s (National Council of Teachers of Mathematics [NCTM] 1980), there has been little indication of instructional changes in traditional teacher-centered practices (Cobb, Wood, Yackel & McNeal 1992; NCTM 1989; TIMSS 1996).

In a typical U. S. mathematics class, most of the time is devoted to the teacher's lecture or demonstration and then to students' individual seatwork. The teacher begins with a brief review of short-answer questions or homework problems. The teacher then demonstrates or explains how to solve the next category of mathematics problems. The teacher's foremost concern is to display a standard method in a clear and definite way rather than to encourage students to express their own thinking as a means to gaining a conceptual understanding of mathematical principles and processes. During the remainder of the class, students practice the demonstrated method on similar problems while the teacher moves around the room answering individual questions. Students are rarely actively involved in posing problems, offering alternative solution methods, or debating mathematical ideas. Thus, the teaching practice can be described as the delivery of information from a knowledgeable teacher to uninformed students.

This teacher-centered method has its roots in related views of what is mathematics, how the teacher should teach mathematics, and how students can best learn it (Ball 1988; Smith 1996; Thompson 1992). Teachers often regard mathematics as a fixed and static body of facts and procedures, mainly for symbol manipulations. School mathematics is arranged in textbooks as specific curricular contents which students are to master. The teacher's major role is to provide a direct and clear demonstration of particular solution procedures to given mathematics problems, and then to assist students to acquire and consolidate problem-solving skills by giving them a chance to practice, administrating periodic tests to check their competence, and repeating the demonstration or step-by-step instruction whenever needed.

In fact, the teacher focuses more on whether students are able to *perform* (standard algorithms) than on whether they *understand* (mathematical principles and processes). Students' central role then is to pay their full attention to the teacher's demonstration and explanation, to memorize facts, and to practice routine procedures with many problems until they master them. This interlocking set of conceptions of mathematics and its teaching and learning has become so entrenched as to be the school mathematics tradition (Cobb, Wood, Yackel & McNeal 1992).

2.2. Recommended Teaching Practices

Against the typical instructional practice described above, educational leaders are seeking to change teacher-centered pedagogy to a student-centered approach. NCTM has initiated and propelled a reform movement by the publication of *Standards* on curriculum, teaching and assessment (NCTM 1989; 1991; 1995; 2000). This reform requires substantial changes in the teaching and learning of mathematics. In particular, five major shifts from the current practice have been recommended;

(a) toward classrooms as mathematical communities, away from classrooms as simply a collection of individuals,
(b) toward logic and mathematical evidence as verification, away from the teacher as the sole authority for right answers,
(c) toward mathematical reasoning, away from merely memorizing procedures,
(d) toward conjecturing, inventing and problem solving, away from an emphasis on mechanistic answer finding, and
(e) toward connecting mathematics, its ideas and its applications, away from treating mathematics as a body of isolated concepts and procedures (NCTM 1991, p. 3).

The term *student-centered* refers to students' contributions and responses constituting the center of mathematics activity. Instead of listening and following a teacher's instruction, the students in a student-centered classroom are expected to have the opportunity to be enculturated into a mathematical discourse in which they invent, explain, and justify their own mathematical ideas and critique others' ideas.

The teacher in a reform classroom is expected to provide worthwhile mathematical tasks on the basis of knowledge of mathematics and students' understandings in order to engage students' interests and intellect. The teacher also manages classroom discourse in ways that probe various mathematical ideas and deepen students' conceptual understanding. In order to promote such discourse, the teacher should be sensitive to students' engagement in discussions. The teacher specifically needs to listen carefully to their ideas, to ask for clarification or justification, and to decide which responses need to be deeply explored in discussions.

Creating a learning environment with mathematically rich tasks and discourse supports the new curricular emphasis of mathematics as problem solving, reasoning and proof, communication, connections, and representation (NCTM 1989; 2000). The reform-oriented teacher also is expected to continually analyze the effects of the learning environment on students' knowledge, skills, and disposition. In these respects, the teacher's role in a reform mathematics classroom is to implement new social norms that foster all students' mathematical learning.

III. THEORETICAL INFLUENCES TO THE REFORM

The current mathematics education reform movement has been motivated by diverse factors including the depressing outcomes of typical mathematical instruction described at the beginning of this paper, and the increasing research knowledge that has offered different perspectives of mathematics learning and teaching. The call for reform also is situated in the historical contexts in which various attempts for better mathematical instruction and their concomitant results have been interwoven.

Among others, this section focuses on a review of influences of theoretical perspectives in order to explore the underpinnings that implicitly motivate the current reform movement. Accumulating research on students' learning of mathematics has provided important foundations for the reform movement, because such studies lead to reflection on the nature of mathematical learning and teaching that has been typically assumed.

The reform recommendations are generally geared at a combination of learning as students' construction and their mathematical enculturation (Silver 1990; Steffe, Nesher, Cobb, Goldin & Greer 1996). The former reflects the influences of constructivist perspectives, whereas the latter reflects those of sociocultural perspectives.

Whereas the previous reform documents (NCTM 1989; 1991; 1995) contained little explicit discussion of the theoretical perspectives they reflect, the *Principles and Standards for School Mathematics* (NCTM 2000) acknowledges the influence of these two perspectives of learning mathematics. On the one hand, the reform emphasizes the importance of building on individual students' prior knowledge and making connections for their conceptual organization. On the other hand, the reform stresses the processes by which students become active members of a mathematical community in their classroom.

3.1. Constructivist Perspectives

A constructivist approach evolving from Piaget's genetic epistemology assumes that children construct their own knowledge through reflection on their actions in the world, through assimilation and accommodation (von Glasersfeld 1995). The constructivist view of mathematics learning has been influential since the early of 1980s: Mathematical learning consists of students' own construction of mathematical concepts and procedures (cf. Kamii 1990; Steffe & Blake 1983; von Glasersfeld 1991).

From the constructivist point of view, students do not simply add new information to their own cognitive structures that have been established. Instead, they connect or construct new relationships among the interpretive structures. Thus a constructivist teacher is very concerned about the possibility that an individual's knowledge structures may either be isolated from each other, rather than integrated together. This perspective has helped to overturn the view of mathematical learning as passive reception and mathematical teaching as the transmission of teachers' knowledge.

Although there have been several versions of constructivism (Confrey 1995; Ernest, 1996; O'Connor 1998), the discussion here attempts to capture its fundamental aspects.

The constructivist perspective assumes that learning occurs through cognitive conflicts by which the individual's mental structure evolves into more viable structure (von Glasersfeld 1995). Thus, the main concern of constructivist teaching in mathematics education is to help students enhance their cognitive structures with respect to specific mathematical content (Cobb & Steffe 1983). Social interaction contributes to this to the extent it raises cognitive conflict and perturbation leading to cognitive reorganization in the process of individual' sense making (Steffe & Kieren 1994).

Consequently, the crucial role of a constructivist teacher is to provide a learning environment wherein students can confront the limitations of their current understanding of a specific mathematical concept, which in turn leads to conceptual changes (Confrey 1990). For this reason, it is important for a teacher to conjecture about a student's previous construction of a mathematical topic and to develop extremely detailed teaching strategies in order to modify the student's thinking (Cobb & Steffe 1983; Simon 1995). The teacher continually re-assesses his or her conceptual portrait of the student and the corresponding teaching model based on the effectiveness of the interactions with the student.

Constructivist perspectives inform the recommended teaching practice, student-centered pedagogy, of the current reform in mathematics education. One way is that constructivist-based research provides models of students' conceptual understandings that can inform teachers' attempts to create cognitive conflict for students resulting in the evolution of more mathematically powerful knowledge structures. Another way is that constructivist perspectives strongly portray students as active learners and thus encourage a teacher to probe carefully their meaning-making.

3.2. Sociocultural Perspectives

Although some mathematics education researchers espoused Vygotsky's *zone of proximal development* as a useful theoretical and pedagogical construct in the early 1980s (cf. Carpenter 1980; Fuson 1980), the influence of sociocultural perspectives on mathematics education is relatively recent (Schmittau & Taylor 1993). The influence has been propelled by anthropological studies which have explored the relations between cultural activities and cognitive development, specifically the comparisons of children's mathematical thinking in and out of school culture (Carraher, Carraher & Schliemann 1985; Lave 1988; Schliemann & Carraher 1996). Such studies have often demonstrated that school mathematical knowledge is noticeably inaccessible in out-of-school settings, suggesting that individuals' arithmetical activities are profoundly influenced by their participation in encompassing cultural practices. These studies urged mathematics educators to broaden their foci of attention so that they become sensitive to incorporating cultural and social dimensions in instruction.

Sociocultural perspectives, inspired by Vygotsky's work, claim that individuals' cognitive processes are subsumed by social and cultural processes, locating learning not in the individual's mind but in the participation of social, cultural, and historical practices (Cobb 1994). This claim reflects a move away from explaining cognition as an individual mental process to understanding the interpersonal context of cognitive growth (Forman, Minick & Stone 1993). Sociocultural perspectives conceptualize learning as a situated process, which arises from engagement in socioculturally shared endeavors through the *zone of proximal development* (Vygotsky 1978), *activity system* (Engeström 1987), *cognitive apprenticeship* (Collins, Brown & Newman 1989), *construction zone* (Newman, Griffin & Cole 1989), or *legitimate peripheral participation* (Lave & Wenger 1991). Learning is characterized as mutual appropriation by which the teacher (or master) and the students (or apprentices) continually coopt each other's contributions until the students are engaged in expected practices (Leont'ev 1981). In short, learning in the sociocultural perspectives is seen as a process of enculturation into a community of practice.

For learning, sociocultural perspectives are concerned with access to the authentic practice in a community in contrast with the availability of rich instructional resources promoted by constructivism (Forman 1996). In this respect, mathematical activities in a classroom should reflect what mathematicians do. The teacher, serving as a representative of a mathematical community, organizes classroom activity settings in such a way that students experience the authentic nature of mathematical activities including mathematical ways of knowing, communicating, valuing, justifying, agreeing, arguing, etc (Collins, Brown & Newman 1989; Lampert, Rittenhouse & Crumbaugh 1996). The main concern of the teacher is whether or not his or her students' classroom practices progress toward those of a socioculturally established mathematics community.

Like constructivist perspectives, sociocultural perspectives inform the recommended teaching practices in the current reform era, but from different points of view. Whereas constructivist perspectives account for students' conceptual development, sociocultural perspectives illuminate the nature and effects of their participation in socially situated activities. Sociocultural perspectives strongly support for, among others, the establishment of a classroom as a mathematical community in which students are engaged in specifically mathematical ways of thinking and interacting (NCTM 2000).

IV. Problems in Implementing Reform

The reform movement has been successful in marshaling large-scale support for instructional innovation and in enlisting the participation, cooperation, and allegiance of large numbers of mathematics teachers (Knapp 1997). Forty-six states developed their own standards that were aligned with the *Standards* (Council of Chief State Shool Officers 1997). Many texts and programs advocate implementing a "standards-based" approach. In the TIMSS report, 95% of the eighth grade teachers in the study said that they are aware of the current reform ideas on mathematics teaching (NCES 1996). Moreover, when asked to evaluate their videotaped lessons in terms of current reform ideas, 70% of the teachers rated the lessons as reasonably in accord with the reform (Stigler & Hiebert 1998).

In contrast to the widespread awareness of the standards and the teachers' self-evaluation of their teaching practice, there has been a growing concern that many U. S. teachers do not quite grasp the vision of the current reform ideas (Hiebert et al. 1996; NCES 1996; 1997; RAC 1997). Teachers often interpret standards-based reform as a new list of teaching strategies and materials. For instance, when asked to justify why they think that the videotaped lessons in the TIMSS are consistent with the reform ideals, the majority of U. S. teachers pointed to "surface features, such as the use of real-world problems, manipulatives, or cooperative learning, rather than to the deeper characteristics of instruction such as the depth of understanding developed by their students" (Stigler & Hiebert 1998, p. 45). Similarly, Burrill (1997) identified widespread misinterpretation about standards-based teaching;

(a) the teacher is just a facilitator or "guide on the side",
(b) students should never practice,
(c) all work should be done in cooperative groups,
(d) manipulatives are the basis for all learning,
(e) students should write an explanation for every problem they solve, and
(f) mathematics should be graded to make students "feel good" (pp. 337–338).

The reform documents have emphasized problem solving as essentially doing mathematics in ways that problem-solving approaches are used for students to construct, investigate, and learn mathematics. However, the interpretation of problem solving too often is limited primarily to problem-solving strategies and heuristics to be mastered, omitting the development of students' understanding of mathematics (Bybee, Ferrini-Mundy & Loucks-Horsley 1997).

Other studies have shown how difficult it is to see basic instructional changes in mathematics classrooms, even with teachers who are committed to implementing reform recommendations (Carpenter, Franke & Levi 1998; Schifter & Fosnot 1993). Teachers tended to wait to be told the "right way" to teach mathematics and were eager to change their old teaching strategies in order to implement new ones that have been advocated in the current reform era. However, they didn't think that they have to fundamentally rethink their views about mathematics and how students should learn mathematics.

Just focusing on new teaching strategies is not sufficient to implement reform ideas. Providing manipulative materials or organizing a classroom into small groups does not guarantee that students are engaging in creative and reflective mathematical activities (Good, Clark & Clark 1997; Steffe & Kieren 1994). Whereas the current reform supports small-group instruction and cooperative learning, there is still a question as to whether small-group instruction promotes the development of students' abilities and understandings better than whole-group or individualized format (King 1993; Mulryan 1995). The real issue is then to understand not the form but the quality of an instructional method — what kinds of mathematical and social exchanges occur and in what ways such exchanges promote students' understanding of mathematics?

The challenge for teachers is to provide the opportunity for students to develop mathematical power by being engaged in the classroom mathematical activities (NCTM 1989; 2000). In other words, teachers are expected to use the social structure of the classrooms to nurture students' development toward mathematical ways of thinking as well as their understanding of specific mathematical concepts and processes. This coordination requires new ways of thinking about the teaching/learning dynamic.

V. Implication and Discussion

The current reform urges the teacher to move from focusing exclusively on students' habituation and conceptual development, which have been traditionally emphasized, to incorporating it into the process of their engagement with mathematics in the classroom community. In this respect, the important aspects of mathematical process such as problem solving, representing, reasoning, proving, communicating, and making connections are emphasized (NCTM 1989; 2000). This emphasis on mathematical process as well as contents blurs a traditional dichotomy between "knowing that" and "knowing how". As Bauersfeld (1993) put it, "Participating in the processes of a mathe-matics classroom is participating in a culture of using mathematics, or better: a culture of mathematizing as a practice" (Bauersfeld 1993, p. 4; quoted in Yackel & Cobb 1996, p. 459). This view has been increasingly accepted by the mathematics education community (cf. Cobb & Bauersfeld 1995; Lampert 1990; Seeger, Voigt & Waschescio 1998).

As reviewed above, the current reform recommendations have been widely recognized but the outcomes with regard to the real transformation of teaching practices were evaluated as weak or ineffective to foster students' mathematical learning (Hiebert et al. 1996; Knapp 1997; RAC 1997). Teachers often change only the surface routines of their classroom practices, which makes the actual learning process on the part of students remain unchanged. Whereas teachers' misinterpretation of the visions and goals of current reform is manifest, their misunderstanding is not trivial. Lindquist, Ferrini-Mundy and Kilpatrick (1997) contend that a main impediment to effective reform is a perceived unitary reform pedagogy, which in fact has been eclectically chosen from different psychological theories with their own strengths and weaknesses.

Constructivism and socioculturalism are the central theoretical influences on the development of the current reform agenda, as reviewed earlier in this paper, but the issue of their integration has been theoretically challenging. For instance, Cobb (1994) calls for a pragmatic approach to counteract the acrimony that often arises between promoters of psychological and social dimensions of learning. In examining two metaphors for learning, acquisition and participation, Sfard (1998) questions the possibility of theoretic-cal unification because of their incommensurability, while agreeing with the necessity of both metaphors. Lerman (1996) also claims that adding the social to the individual leads only to an incoherent theory. O'Connor (1998) cautions:

It is naive to think that it will be easy to graft together a truly critical theory that will simultaneously illuminate the global and collective concerns of society, the social nature of knowledge construction in every content area, the nature of individual learning within a local collective, and the complex relation between an individual and the content itself (O'Connor 1998, p. 63).

In fact, the current theoretical limitations for integration are reflected in the difficulties encountered by dedicated researchers and teachers. In interacting with 51 teachers committed to implement standards-based curriculum programs, Manouchehri (1998) found that all the teachers had a difficulty in placing mastery of basic skills or algorithmic knowledge within their reform-oriented teaching. Ball (1993) also experienced a dilemma over the challenge of dual emphases on students' learning of mathematical concepts of the curriculum and their thinking or participation in mathematical discourse. As she put it, "With my ears to the ground, listening to my students, my eyes are focused on the mathematical horizon" (Ball 1993, p. 376). Lampert (1990) found, while attempt-ing to create discourse of school mathematics as closer to that of discipline, "Like teaching someone to dance, it required some telling, some showing, and some doing it with them along with regular rehearsals" (Lampert 1990, p. 58).

These challenges must be met by teachers and teacher educators if the reform intentions in mathematics education are ever to be realized. Teachers confront the complexities associated with the reform. On the one hand, they have to make sure students' understanding of specific mathematical content. On the other hand, they have to promote students' enculturation toward characteristically mathematical ways of thinking, reasoning, justifying, proving, and communicating through their classroom participation. In traditional mathematics instruction, teachers directly explain mathematical content. Teachers working within the reform visions also agree with a certain degree of direct teaching or modeling (cf. Lampert 1990; Wood, Cobb & Yackel 1995).

What is unclear is when and for what purposes a certain pedagogical strategy is appropriate within the teacher's objectives and minute-to-minute practices. How can a reform-minded teacher be sure that students will learn the specific mathematical concept, while he or she facilitates their general experience of mathematical ways of thinking in the classroom community? How can the teacher negotiate two divergent teaching objectives, conceptual and social development on the part of students?

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