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Tools for the Acquisition of Graphing Ability: Real-Time Graphing Technology[1,2]

KWON, OH NAM
Department of Mathematics Education, College of Education Ewha Womans University,
11-1 Daehyun-dong, Seodaemun-gu, Seoul 120-750, Korea;
E-mail: onkwon@mm.ewha.ac.kr

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This study investigates the impact of Calculator-Based Ranger (CBR) activities in the performance of middle school students' graphing abilities of physical phenomena. Two issues about CBR activities on graphing abilities were addressed in this study;
(1) the effect of CBR activities on graphing abilities, and
(2) the influence of instructional styles on students' graphing abilities.
Following the use of CBR activities, students' graphing abilities were significantly more developed in three components-interpreting, modeling, and transforming. Significant differences were found in students' achievement depending on instructional styles related to differentiation, which is closely connected to transforming distance-time graphs to velocity-time graphs. The findings of this study indicate that CBR activities may enhance students in constructing appropriate webs of related concepts and ability to qualitatively interpret graphs. Using collaborative CBR activities to introduce and explore graphing of physical phenomena is, therefore, recommended for inclusion in the secondary mathematics curriculum.

1. INTRODUCTION

Graphing represents a key symbol system of mathematical and scientific communications. Graphs play many roles including the construction of mathematical and scientific facts, coordination of activities across different domains of knowledge, and making visible phenomena otherwise impossible to see (Latour 1987).

Despite their widespread use in science, and despite the place of graphing activities in

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science and mathematics curricula, graphs are notorious for their difficulties. An extensive review of the literature in mathematics and science showed that many students experience difficulties in producing and interpreting graphs (Leinhart, Zalslavsky & Stein 1990).

Real-time graphing technology activities can be valuable tools in the investigation of students' use of graphs. With the presence of real-time technology such as Calculator-Based Ranger (CBR), these difficulties can interact to enhance graphical understanding. This study explores the impact of CBR activities on middle-school students' abilities to interpret, model, and transform graphical information of physical phenomena.

1.1. The use of real-time graphing technology in the learning of kinematics

The microcomputer-based laboratory (MBL), calculator-based laboratory (CBL), and calculator-based ranger (CBR) are real-time graphing technologies that collect data with various probes and then store the data into a computer or a calculator (Figure 1). The data can be analyzed and displayed in different formats, and the students can make graphs as data are collected or at a later time.

Figure 1. CBR and its activity

Researchers claim that MBL activities are effective in improving students' understanding of graphs of physical events (Barclay 1986; MoKoros 1986; Thorton 1987). MBL activities may be particularly useful in kinematics where students have serious difficulties with graphing. Real-time graphing of data on the computer screen is fast and dynamic with the graph forming on the screen as the event progresses; thus, both the speed and the dynamism may have a considerable impact on information processing. Real-time graphing allows learners to process information about the event and the graph simultaneously rather than sequentially. Research indicates that activities involving real-time graphing technology support empirically the idea that intuitions based on students' knowledge of real-world situations operate successfully when reasoning in the graphing domain. While MBL, with its capability to appears to be a versatile vehicle for developing students' intuitions and providing rich experiences in visualization, two drawbacks with this learning environment are the cost and space requirements. However CBR devices, while interface with graphing calculators, perform essentially the same functions as the MBL but are less expensive, more portable, and do not require special facilities. Few mathematics education studies to data, however, have examined the effects of CBR activities. Therefore there is a genuine need to investigate the impact of CBR activities in the mathematics classroom (Cates 2000).

1.2. Graph and graphing ability

Based on a review of research, in this study, graphing ability is defined as an ability to use a graph as a qualitative analysis of a whole picture. It facilitates students to connect graphs with physical concepts and real-world situations, while also allowing them to translate between graphs and physical events. Graphing ability consists of three components: interpreting, modeling, and transforming. These components are based on Leinhardt's classification of functions framework and O'Callaghan's framework (cf. Leinhardt et al. 1990; O'Callaghan 1998). All three terms are described below.

Interpreting. Interpreting means that students are able to get meaning in a concrete context as well as convert multiple representations into verbal expressions (O'Callaghan 1998). In this study, it is defined as an ability to translate from graphs to verbal expressions. Students can extract information from graphs that they need to solve problems and make different types of interpretations or focus on different aspects of a graph. Interpreting can be global, referring to properties of the entire graph or major portions of the graph, or local, referring to properties of a point on the graph (Leinhardt et al. 1990). This component evolves as students develop and integrate structures as specific values and patterns of behavior (O'Callaghan 1998).

Modeling. Mathematical modeling involves recovering a mathematics process from the experience of describing and interpreting physical and social phenomena (O'Callaghan 1998). In this study, it is defined as an ability to translate from real-world situations to graphs. This component can clarify understanding of the complex phenomenon in real-world situations and select appropriate graphs corresponding to the real context. Modeling entails the use of a graph to form an abstract representation of the quantitative relationships in that situation (Fey 1984).

Transforming. Transforming involves interpreting multiple graphs that describe different aspects of the same sequence of events and modeling various real world events. In this study, it is defined as an ability to see and draw a variety of graphs depicting events. This component integrates both interpreting (connecting given graphs with the real world) and modeling (connecting graphs with physical concepts) (Leinhardt et al. 1990).

2. RESEARCH QUESTIONS

The general hypothesis to be tested was that CBR activities would enhance students' graphing ability. This broad statement generated the two specific research questions listed below.

1. Does the use of CBR activities improve students' graphing ability in interpreting, modeling, and transforming graphs of physical phenomena?
2. Is students' graphing ability affected by different ways of learning (traditional-lecture style or CBR-based laboratory learning)?

3. METHODS

3.1. Subjects

The participants in this study were 590 students in 18 intact classes at five middle schools and one high school, all of which are located in Seoul, Korea. These classes were randomly selected at each school. They had not used technology such as CBRs and graphing calculators in their mathematics classroom prior to this study. For this research, 428 students of average age 13.5 years participated using the CBR (CBR group), and these students were tested with both pretests and posttests. Among these 428 students, 300 were in the seventh grade, and 128 were in the eighth grade.

One hundred sixty-two eleventh-grade students of average age 17 years comprised the test-only group (TOG). In a traditional classroom with lecture-based learning and without any technology such as computers or graphing calculators and hands-on activities TOG students experienced instruction on the concept of differentiation.

Korea is a high-performing country on most international comparisons of mathematics achievement. Lew (1999) argues, however, that most Korean students seem quite unable to relate their well-developed manipulative skills to the real world, as secondary mathematics lessons in Korea put much emphasis on computation and algorithmic skills. Since TOG students were taught the concept of differentiation and its application only in the context of velocity and acceleration by a traditional lecture style, they had little emphasis on the graphical representations of differentiation in relation to real-life phenomena. On the other hand, the CBR students experienced the notion of differentiation based on physical phenomena in realistic contexts.

3.2. CBR Environment

The study took place in July, 2000 and took six class periods total for pretest, treatment, and posttest. One class period lasted for 45 minutes. Four in-class CBR labs developed for this study were designed to actively engage students in the learning process and to promote conceptual understanding of the graphs used to interpret and model the observed physical phenomena. Activities were conceptually structured via worksheets to guide students in investigating the effect of speed of movement on distance-time and velocity-time graphs using the CBR.

During Day 1 of the treatment, the lesson focused on students' understanding of position graphs. During Day 2, the lesson focused on distinguishing between velocity and distance graphs. Speed-controlled miniature cars and balls were used for this activity. During Day 3, activities focused on students' reproduction of graphs using their body motion and CBR's "Match it" program (e.g., "Walk to produce the following distance graph"). As students moved steadily and slowly away from the CBR, the distance-time graph was displayed on the screen, and they were asked to predict the velocity-time graph of their movements. On Day 4, the focus was prediction and explanation activities of real-life phenomena and their relationship to the graphs.

3.3. Performance Measures

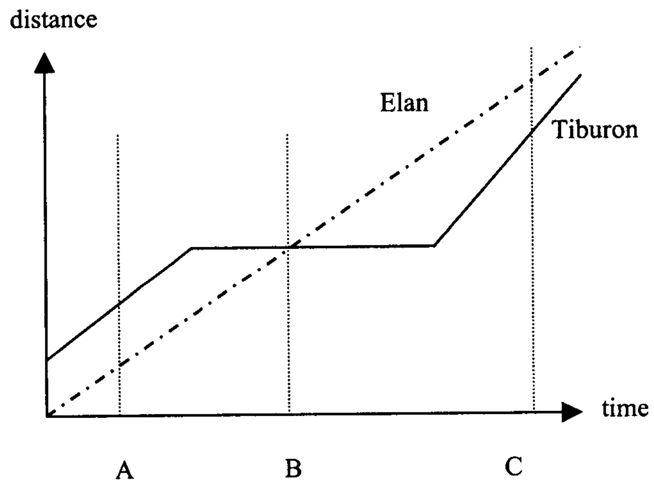
The Graphing Interpretation Skill Test (GIST) based on Test of Graphing in Science by McKenzie and Padilla (1986), as well as the Motion Content Test (MCT), were both constructed and validated by Michael (1995). The pretest and posttest, adapted from Michael, were designed to assess students' graphing ability as described in research questions 1–2. The posttest was conceptually the same and essentially an alternative version of the pretest. The tests consisted of 27 items, including 20 multiple-choice items and 7 free-response items. The reliability (Cronbach α) for the pretest was 0.8753. The reliability (Cronbach α) for the posttest was 0.8608. The two tests were consulted and validated by two professors in mathematics, 12 in-service mathematics teachers (8 from middle schools, 4 from high schools), and 5 graduate students in mathematics education. Each correct multiple-choice item was worth 3 points, and each incorrect answer was 0 points. Each free-response item was scored as 3, 2, 1, 0 according to the quality of the answer.

Before grading the pretests and posttests, the investigator consulted 4 in-service mathematics teachers to establish scoring rubrics. To obtain reliability of free-response items and expediate the grading, the same 4 teachers were divided into groups of 2. The tests were divided in half, and each group graded their portion respectively. In most cases, the two scores for each student were the same, but if the scores differed, all 4 teachers were consulted, and a compromise was reached with the approval of the investigator. Then the groups exchanged the marked tests, and each group reviewed the scoring marked by the previous group. The interpreting component has 8 items and a possible total score of 24. The modeling component has 13 items and a possible total score of 39. The transforming component has 6 items and a possible total score of 18. Sample test items for each of these component competencies are provided in Table 1.

4. DATA ANALYSIS

Graphing ability was evaluated in the form of pretest and the posttest scores. Table 2 shows means and standard deviations of three component scores: interpreting, modeling, and transforming, on the pretest and posttest for the CBR group.

Table 1. Sample Questions from the pretest and post-test

<table>
<tr><th>Category</th><th>Question</th></tr>
<tr><td>Interpreting</td><td>The distance-time graph shows the motion of two different cars, Elan and Tiburon, traveling at the same time in the same direction. Choose the time (A, B, C) when each of the statements is true.


(1) Find the time when Tiburon is located further than Elan from the origin.
(2) Find the time when Tiburon is moving faster than Elan.
(3) Find the time when the two cars are moving at the same speed.</td></tr>
</table>

Table 1 (continued).

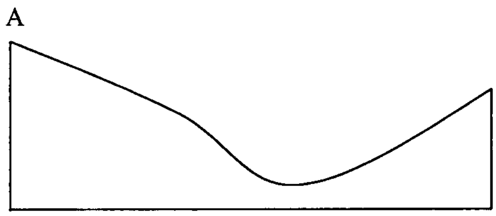
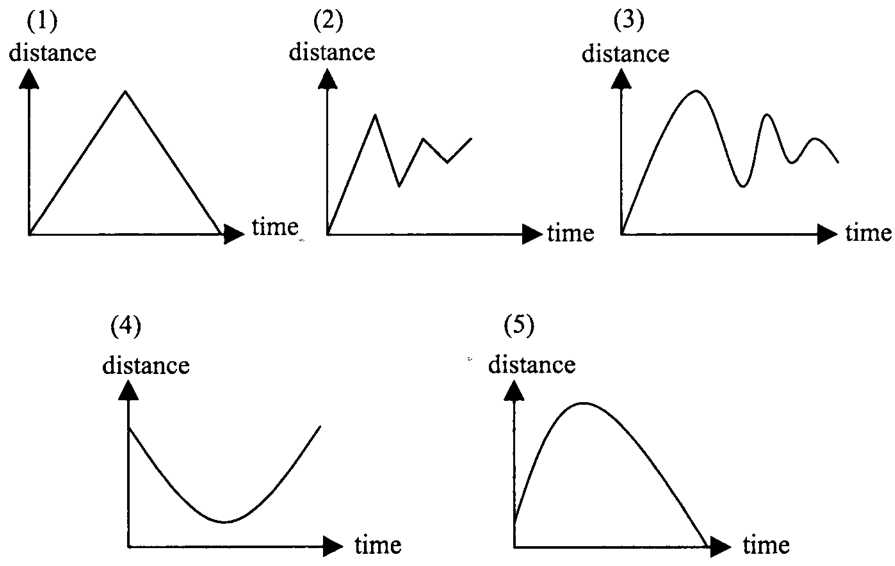
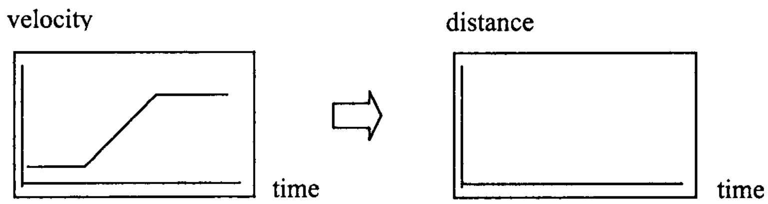
Category	Question
Modeling	Choose the distance-time graph of a ball when the ball rolls from point A.  
Transforming	Draw the distance-time graph for the given velocity-time graph. 

Table 2. Means and standard deviations for pretest and posttest (CBR group)

	Tests				t	Sig.
	Pretest (n = 428)		Posttest (n = 428)			
	M	SD	M	SD		
Interpreting	9.86	7.65	15.63	6.28	13.14	.000
Modeling	14.40	7.82	21.94	8.10	14.54	.000
Transforming	4.56	4.10	5.25	4.28	2.47	.014
Total	28.83	16.51	42.82	15.83	13.57	.000

As predicted, the means on the posttest were significantly higher than on the pretest. Scores on the three components showed a significant change in students' graphing ability between the pretest and the posttest; $t(428) = 13.57$, $p < .001$, indicating that students gained significantly higher scores. Students improved significantly in interpreting, modeling, and transforming; $t(428) = 13.14$, $p < .001$, $t(428) = 14.54$, $p < .001$, and $t(428) = 2.47$, $p < .05$, respectively.

To investigate the impact of traditional lecture style and CBR-based laboratory learning, test scores of the CBR group were compared to the TOG (see Table 3).

Table 3. Means and standard deviations of posttest (the CBR and TOG groups)

	Groups				t	Sig.
	CBR		TOG			
	M	SD	M	SD		
Interpreting	15.63	6.28	11.37	7.80	6.85	.000
Modeling	21.94	8.10	17.22	9.84	5.93	.000
Transforming	5.25	4.28	4.17	3.98	2.77	.006
Total	42.82	15.83	32.76	19.45	16.45	.000

Mean scores of the TOG were significantly lower in all components than the mean score of the CBR group. This result indicates that although the TOG had been taught the concept of differentiation, they had more difficulties in linking realistic contexts of physical phenomena with graphs (interpreting and modeling), and in conceptually connecting a velocity-time graph and a distance-time graph (transforming).

5. Conclusion

Graphing has been considered a fundamental part of mathematics and science curricula, yet recent studies have indicated that students' understanding of graphs is rather limited. Hale (1996) noted that the qualitative interpretation of graphs is currently underrepresented in the mathematics curriculum. The findings of the present study provide evidence that CBR activities, which require students to explore the global feature of graphs and extract meanings about the relationship between two variables, may promote this aspect of graphical understanding.

Why does the CBR appear to be an effective tool for enhancing students' graphing abilities? The use of CBR activities may be effective because they provide students a physical experience: students use their own physical movements and manipulate physical lab materials such as speed-controllable miniature cars and balls. These physical experiences may be reinforced with the visual experiences of seeing the physical phenomena change. Even though there was an average 3.5 years of age gap between the CBR group and the TOG, students in the CBR group were able to interpret and model real-world phenomena and to transform among graphs in the context of physical events. The CBR group, who had physical experience in realistic situations, had developed a deeper understanding of the relationship between distance and velocity when compared to TOG who had developed this concept through paper-and-pencil based learning activities. This result lends support to the notion that the laboratory learning environments are more effective than the paper-and-pencil environment in developing understanding of graphs in the context of real-world situations.

The CBR's provision of real-time graphing has a considerable impact on students' learning of graphing. Students in the CBR environment receive immediate feedback by presenting data graphically. Real-time graphing, with its speed and dynamics, facilitates linking in time a physical event with a simultaneous graphic representation. Brasell (1987) has shown that real-time immediacy was crucial since even a short delay in displaying the graphed data impaired learning.

The use of CBR activities allowed for frequent repetition and numerous opportunities in experiencing graphing physical phenomena. These activities might reinforce students' concepts of different shapes of graphs representing different classes of motion events. Students in the CBR group were significantly better at transforming between distance graphs and velocity graphs than TOG students. This fact suggested that the CBR students might develop a set of mental connections for distance-time graphs and velocity-time graphs.

In summary, the present study demonstrates that CBR activities are pedagogically promising for enhancing students' graphing ability — interpreting, modeling, and transforming. This study suggests that CBR is a valuable tool for teaching global and qualitative interpretations of graphs. Korean students might more readily adapt their well-developed manipulative skills to real-world situations with the incorporation of CBR activities. Subsequently, real-time technologies such as CBR should be utilized and integrated into the mathematics curriculum of secondary schools.

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