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Children's Representations of Numbers

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The purpose of this paper was to examine early numerical representations between American and Korean children. Fifty-five first graders (35 Korean and 20 American) participated in the study. According to the findings of the current study, the author concluded that the Korean children had a stronger conception of base ten representations of numbers than that of the American children. The Korean children used various strategic reasoning such as decomposition and recomposition on the basis of base 10 structure to solve addition and subtraction problems effectively. However, the author cannot conclude that language differences would be the largest factor that would make Korean children salient in the representations of base ten structures.

1. INTRODUCTION

> Having studied all aspects of the development of intellectual operations, and having attempted to analyse some of the characteristics of perceptual development, it was necessary to go on to tackle the question of the evolution of mental images.
>
> — Piaget and Inhelder, "Mental Imagery in the Child"

The idea that language influences the nature of thought has a long history within psychology, being expressed in its most celebrated form via the so-called weak version of Sapir-Whorf hypothesis (Sapir 1921). Concerning the language and number tasks, some mathematics educators argue that linguistic differences play an important part in explaining the superiority of Korean, Chinese, and Japanese children on number tasks (Fuson & Kwon 1992; Miura et al. 1994; Song & Ginsburg 1988). However, Towse and Saxton (1997) argue that the influence of language upon cognitive development is minimal and criticize the methodologies that were previously used. That is, they found that the pattern of previous demonstration strongly cued the later tasks.

In the *Principles and Standards for School Mathematics* (NCTM 2000), representation is emphasized as one of the new standards in the process of school mathematics. The version states that "the way in which mathematical ideas are represented is fundamental to how people can understand and use those ideas (p. 69)" in problem solving and communication. The purpose of this paper was to examine early numerical representations between Korean and American children. In particular, the author investigated the children's base-ten representations.

2. The Korean and The English Number Naming Systems

Korean and other Asian languages have the regular number naming systems in which a number word is said and then the value of that number is named. For example, 27 is read as "ee ship chil" or "sumul ilgop" in Korean, "èr shí qī" in Chinese, and "ni juu nana" in Japanese, all of which means literally as "two *ten* seven (*one*)".

However, English and other Western number naming systems are irregular and lack the elements of tens and ones in their number words. For example, "thirteen" for 13 in English, "sechsundzwanzig" (six and twenty) for 26 in German, and "quatre-vingt-dix-neuf" (four-twenty-ten-nine) for 99 in French do not express the place value construction of the numbers.

Korean children actually learn two number naming systems, the informal and formal systems. The informal Korean system is used for counting a relatively small number of objects, and the formal system is for numerical calculation with relatively big numbers in schools. However, both systems are structured similarly with regard to how numerals are read. In both systems, the two digit number AB is read as "A *ten* B" in their two digit numerals. In the formal system the digit in the tens place is named directly followed by the number in the ones digit, such as "one ten one", "one ten two", $\cdots$, and "nine ten nine". The same rule is applied in the informal system; however the multiples of ten (e.g., 20, 30, $\cdots$, 90) words share some phonetic similarity to the basic words for two, three, $\cdots$, and nine as in English. For all other numbers, the regularity of the formal system applies. Table 1 shows the Korean number naming systems.

Song and Ginsburg (1988) argued that mastery of the dual system might contribute further to Korean children's understanding of numbers even though young Korean children have difficulty in learning the numerals for a period of time. In the study of Miura et al. (1994), the Korean first graders performed better than all other Western children on the place-value understanding tasks. In fact, Korean children mostly used the formal system in their numerical computation in school.

The English number naming system lacks the elements of tens and ones in two digit numbers, especially in the numbers between ten and twenty. The tens and ones are reversed only in the teen words so that the 'nine' is first in the teen words —"nine teen" instead of "teen nine" or "ten nine". In addition, the decades are reversed from their written order so that numbers such as 16 (sixteen) and 60 (sixty) are phonetically similar.

Table 1. Korean Formal and Informal Systems of Number Words
(Adapted from Fuson & Kwon 1992)

Number in Arabian rumerals	Formal Korean system*	Informal Korean system*	Number word structure	English translation
1	Eel	Hahna	a	One
2	Ee	Dool	b	Two
3	Sahm	Set	c	Three
4	Sah	Net	d	Four
5	Oh	Tahsut	e	Five
6	Yook	Yersut	f	Six
7	Chil	Ilgop	g	Seven
8	Pal	Yerdul	h	Eight
9	Goo	Ahop	i	Nine
10	Ship	Yerl	j	Ten
11	Ship Eel	Yerl hahna	ja	Ten one
12	Ship ee	Yerl dool	jb	Ten two
13	Ship sahm	Yerl set	jc	Ten three
14	Ship sah	Yerl net	jd	Ten four
15	Ship oh	Yerl tahsut	je	Ten five
16	Ship yook	Yerl yersut	jf	Ten six
17	Ship chil	Yerl ilgop	jg	Ten seven
18	Ship pal	Yerl yerdul	jh	Ten eight
19	Ship goo	Yerl ahop	ji	Ten nine
20	Ee ship	Sumul**	bj	Two ten
21	Ee ship eel	Sumul hahna	bja	Two ten one
29	Ee ship goo	Sumul ahop	bji	Two ten nine
30	Sahm ship	Sulheun**	cj	Three ten
31	Sahm ship eel	Sulheun hahna	cja	Three ten one
40	Sah ship	Maheun**	dj	Four ten
50	Oh ship	Shiheun**	ej	Five ten
60	Yook ship	Yehsun**	fj	Six ten
70	Chil ship	Ilheun**	gj	Seven ten
80	Pal ship	Yerdeun**	hj	Eight ten
90	Goo ship	Aheun**	ij	Nine ten
100	Baek	Baek	ak	Hundred

* They do not represent the Romanization of Korean Language.
** In the informal system, 20, 30, ··· do not follow the exact rule as the formal system does

In Korean, 16 is read as "ten six", and 60 as "six ten". Thus, English-speaking children have to memorize them when learning the number sequence.

3. METHOD

Fifty-five first graders (35 Korean and 20 American) participated in the first and last tasks as a class. From each country, ten children were chosen randomly to participate in the other three tasks during individual interviews. For each child, the interviewer also examined the strategies that the children used for the fifth task. The author selected first grade children to avoid the institutional influence of number representation as much as possible. The classroom teachers conducted interviews with the children in each country. Ages of the children interviewed from Korea and America ranged from 6 years 7 months to 7 years 2 months (M = 7 years 1 month and SD = 2 months) and 6 years 6 months to 7 years 2 months (M = 7 years and SD = 3 months), respectively. There was no significant age difference between two groups, $t(53) = 1.48$. The five tasks were shown as in Table 2

The author let the teachers use the tasks to investigate the children's strategies and their numerical representations. The teachers kept in mind that they should be sensitive when the children's strategies might be dominantly affected by the previous task as Towse and Saxton (1997) confirmed. For example, in the second task, most children who used only unit blocks for 12 also used only unit blocks when they were asked to explain 27.

4. RESULTS

The children represented 11 or 12 objects variously. The drawings of Figures 1 and 2 show a Korean child's drawing and an American child's drawing, respectively, when they were asked to draw any 11 objects. Nine out of ten American children drew the 11 objects in a single row, whereas four out of ten Korean children drew 10 objects in one row and one object in another row.

However, in this task, the author could not see any strong evidence that Korean children grouped objects by 10. Also, most American children drew more abstract objects such as circles and triangles than the Korean children. This might result from a different interpretation of the term "objects" between English and Korean children. That is, the American children might interpret the word "objects" as more abstract than the Korean children.

In the second task, the teachers used a scenario with the toy Pooh. Most of the children in this study had never seen the base 10 blocks before the teachers introduced them. To avoid giving any clue for a specific use of blocks, the interviewers did not show any demonstrations and, instead, told the children that they could use both of the blocks and that the bigger block (ten) consists of ten small blocks (ones). Again, there was no difference in the use of blocks. That is, 7 Korean and 6 American children used one ten-block and two unit-blocks to represent 12. They consistently used ones-blocks only or tens-blocks and unit blocks together for representing 12 and 27.

Table 2. The five tasks

1. [Drawing pictures]

   Let's play a game. I will tell you some numbers and you imagine some things in your head. Now, close your eyes and imagine 3 objects in your head. [We may suggest ping-pong balls if the child is having difficulty imagining the objects.] Erase it. Now, imagine 11 (or 12) objects in your head. [pause] Are you done? Now, open your eyes and draw the picture of the 11 (or 12) objects you imagined in your head.

2. [Scenario using base 10 blocks]

   Pooh came from another planet, he doesn't know about numbers. You want to teach him the number 12 using these blocks. There are small and long things. Count how many small things are in the long one. Right, 10 small things make one long one. Use these blocks to teach Pooh 12. OK. You did a good job. Now, teach him the number 27.

3. [Decomposing numbers]

   Here are some sticks [25]. Count them. How many sticks are here? Here are some rubber bands. Divide the sticks into groups and wrap them up. Can you show me other ways of dividing them? [If the child did not divide into 3 groups, you may suggest to him/her to divide the sticks into 3 groups.]

4. [Concept of base 10]

   You did a great job. Think about 78. How many 10s are in the number 78? [If the child can't do this problem, propose 34] [When you do not know how the child got the answer, say, "Show me how you got the answer"]
   Great job! Then, how many 5s are in 78? [Or how many 5s are in 34?]

5. [Addition and subtraction]

   c. $5 + 8$
   c. $13 - 7$
   c. $15+13$
   d. $23-17$

In the third task, 7 Korean children wrapped the sticks up based on the base ten or five structure, whereas most American children wrapped them up arbitrarily. The difference may be partially affected by the textbooks that Korean children use. In the textbook, they regularly practice wrapping up by 10s and counting by 2s, 3s, 4s, and 5s using small rods as manipulative materials. However, such activities are rarely found in the textbook that these American children used. This may have also affected the Korean children's numerical representations.

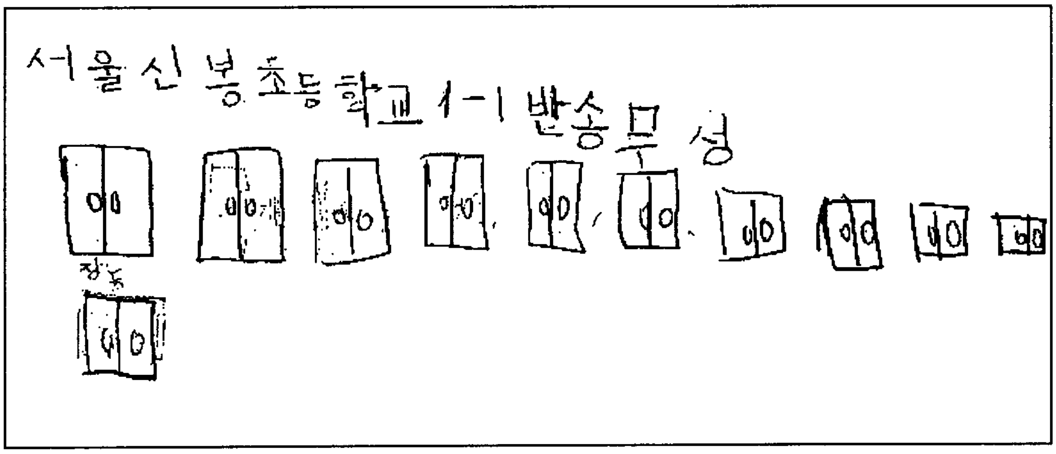

Figure 1. Korean child's drawing

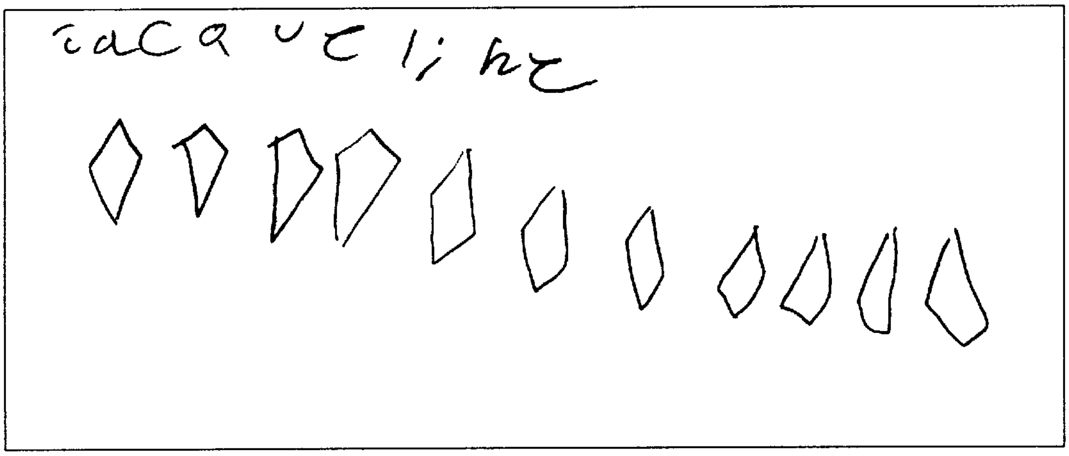

Figure 2. American child's drawing

When the children who wrapped the sticks arbitrarily were asked to divide 25 sticks into 3 groups, 4 American children tried to divide the 25 sticks into 3 equal groups. They did this by placing one stick at a time into one of the 3 groups. Once they had placed 8 sticks in each group, they had 1 stick left over and usually placed it in the first group. When the teacher said that the 3 groups needed not to be equal, the American students then randomly grabbed sticks and wrapped them up into three groups. However, from the beginning, 8 Korean students wrapped by 10, 10, and 5, sequentially.

In the fourth task, 8 Korean children responded correctly to questions about how many 10s are in 78, but it seemed difficult for the children to determine how many 5s were in 78. Only one Korean child got the correct answer for the second question by counting by 5s. Six American children did not understand what the question was asking. The interviewers also had to elaborate the instructions for this task as the children did not immediately understand the questions.

In the last task, the Korean children scored higher than the American children. When solving the problems, the American children predominantly used their fingers and count-on or count-down strategies, whereas most Korean children used strategic methods such as mental decomposing or recomposing the numbers into 5s or 10s. For example, for 5 + 8, Korean children decomposed the 8 into 3 and 5, which was added to 5 (i.e., 5 + 5 = 10; 10 + 3 = 13). For 13 – 7, more Korean children than American children decomposed the 7 into 3 and 4, and then 3 was subtracted from 3 (3 – 3 = 0) and 4 from 10 (10 – 4 = 6), which resulted in 0 + 6 = 6. I observed Korean children distinctively using the base ten concepts, unlike their American counterparts. Due to the nature of English number words, American children may not explicitly see the structure of 10 + 3 in 13. By comparison, the structure of the Korean number words may induce composition strategies. Actually, base ten decomposition and recomposition methods are presented in the Korean first grade textbook.

The two groups performed well on the third question in the fifth task, 15 + 13. During the interview, the American teacher had the children recall the separation of the tens and ones columns. For some students, she reminded them of the place value chart by vertically gesturing with her hand. During whole class instruction, the teacher had emphasized the separation of the two columns when calculating with two digit numbers. In this question, children from both countries predominantly solved the problem by separating the two columns. However, few of the American children could reason why they separated that way.

On the last question, which involved trading and was new for the children, eleven Korean and two American children obtained correct answers. Fifteen Korean and eight American children answered 14, which they obtained by subtracting 1 from 2 in the 10s column and 3 from 7 in the one column. Neither American nor Korean children used manipulative such as sticks and cubes, although they were available on the table, for counting. Instead, more American children used their fingers when they counted, whereas Korean children tended to use mental computation instead of their fingers.

5. CONCLUSION

Korean children routinely score higher on assessments of their numerical knowledge as well as other fields of knowledge in mathematics than students of the same ages in Western countries (IAEEA 1997). In the present study, the Korean children also scored higher than the American children in the tasks. Many researchers argue that Korean children and other Asian children construct base ten mental representations for numbers because the Korean language has a regular number naming system. They also argue that this leads Korean children to use various strategies and as a result they tend to be more skillful in number calculations than American children.

According to the findings of the current study, I also conclude that the Korean children had a stronger conception of base ten representations of numbers than that of the American children. They scored higher in the addition and subtraction problems than the American children. The Korean children used various strategic reasoning such as decomposition and recomposition on the basis of base 10 structure to solve addition and subtraction problems effectively. Especially, the Korean children used compensation strategies, which are based on "add to ten" method. I observed that the Korean children had strong base ten numerical structure and, thus, infer that the Korean regular number naming system may facilitate Korean children's base ten numerical representation structure.

However, I cannot conclude that language differences would be the largest factor that would make Korean children salient in the representations of base ten structure because the children are also strongly affected by their previous experiences in the culture. We need to examine other cultural factors such as the effects of pre-schooling, textbooks, teaching methods in the classroom, parents' assistance at home, and so forth. Future studies are necessary for full explanation of the difference of the numerical representations of the children between the two countries. In particular, data is needed from a large sample, and other cultural factors need to be considered. This can be achieved by a careful analysis of the complex factors that contribute to the children's numerical representations.

Finally, I propose the following several recommendations. First, curriculum developers, classroom teachers, and parents should consider that children need opportunities to facilitate base-ten representational structure of numbers because it can bolster children to use strategic reasoning. Especially, English-speaking children need to experience various numerical representations mentally like using decomposition and recomposition methods as well as physically using base 10 blocks as a manipulative. Furthermore, as Neuman (1998) and Steffe et al. (1983) classified, we need a more elaborated model of children's ways of experiencing of numbers with Korean children. Second, the manipulative in the mathematics classrooms should be used with care. According to the present study, most children used fingers or mental calculation instead of using manipulative, which were available on the table. Many parents worry about their children's use of fingers when computing and force them not to use fingers. However, the use of fingers is natural for most children whose numerical developments do not reach an abstract level. Third, research relating the effect of language should be elaborated in settings and procedures of experiments and extended in terms of duration and size of participants. American children in the study considered "subjects" in a more abstract manner than Korean children do. Any two words within one language as well as between two languages do not have same meaning and nuance. As McGarrigle and Donaldson (1974) criticized Piaget experiments, a word meaning among adults often do not mean the same thing among children. Researchers should know the subtle differences of meaning and nuance between two languages. At last, bilingualism should be investigated in mathematics education. With the intense use of English from the early ages in Korea, many children can use two languages, Korean and English. Even though many researchers argue that bilingualism is advantageous in reasoning (Náñez, Padilla & Lopez 1992), more research is in demand in mathematics education concerning bilingualism. We should develop more elegant models by using elaborated research methods concerning languages and numerical capacities among countries as well as within a country.

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