

Relationships Between Teachers' Knowledge of School Mathematics and their Views of Mathematics Learning and Instructional Practice: A Case Study of Taiwan^{1, 2}

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This study explored teachers ($n = 219$) from northern, central, southern and eastern Taiwan concerning their views about children's learning difficulties, mathematical instruction and school mathematics curricular. Results showed that teachers' mathematics knowledge or their instruction methods had no significant influence on their views of children's learning difficulties. Even though teachers indicated that understanding of abstract mathematical concepts was the most prominent difficulty for children, they tended to employ direct instruction rather than constructive and cooperative problem solving in their teaching. However, teachers' views of children's learning difficulties did influence their instructional practice. Results from in-dept interviews revealed that there were some obstacles that prevented teachers from putting constructive perspectives of instruction into teaching practice. Further investigation is needed to develop a better understanding of epistemology and learning psychology as well as to help teachers create constructive learning situations.

Keyword: School mathematics knowledge, views of children's learning difficulty, views of mathematics instructional practice.

I. INTRODUCTION

Over the past few years, the reform movement in mathematics education in Taiwan has made teachers aware of the critical role they play in changing the traditional ways in

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which mathematics has been taught and learned in schools (Huang 1996; Leung & Wu 2000). Mathematics instruction has moved toward greater emphasis on mathematical processes, engaging students to learn mathematics with better understanding rather than mere practice and drill. In doing so, many teachers conduct constructive and cooperative small group discussion and problem-solving in their classrooms to help students learn better, and to involve students in the process of active knowledge construction (Huang 2000a; 2000b).

Thus, to improve students' learning efficiency, teachers should have in-depth knowledge not only of the specific mathematics they teach, but also of the mathematics that their students are to learn (Carpenter, Fennema & Fennema 1996; Ernest 1999; Fennema & Franke 1992).

There is common consensus that teachers' knowledge and views about learning and instruction influence their instructional practices (cf. Fennema & Franke 1992; Peterson, Fennema, Carpenter & Loef 1989; Fennema, Carpenter, Franke, Levi, Jacobs & Empson 1996). Furthermore, teachers' views of the mathematical thinking and learning difficulty of their students has great influence on the nature of teachers' instruction (cf. Ernest 1999; Even & Tirosh 1995; Graeber 1999).

To understand students' mathematical thinking and learning difficulties requires a deep insight into both the material to be taught and how it is taught. Thus, mathematical knowledge is an essential component of mathematical teaching. Teachers' knowledge of the subject matter is useful for designing the curricula and making the pedagogical content-specific decisions involved (Even 1993; Even & Tirosh 1995).

Mathematical knowledge refers to knowledge about mathematical topics and structures. Such knowledge also influences teachers' views on their students' learning (cf. Fennema & Franke 1992; Thompson & Thompson 1996). Several studies have examined teachers' knowledge about the mathematical curriculum and instruction in general (cf. Marks 1996; Shulman 1987), as well as analyzed teachers' beliefs of a specific topic (cf. Peterson et al. 1989). However, few researchers have taken into account daily mathematical problems which arose in practical mathematical learning situations when analyzing teachers' mathematical knowledge and their views of learning and modes of instruction.

An understanding of what makes the learning of specific topics easy or difficult, and the ways of representing and instructing the subject that promotes students' construction of meaning (Graeber 1999) are also important pedagogical components related to teachers' knowledge and views of learning and instruction. Children catching up in knowing facts and algorithm as well as doing well in assignments do not mean that they understand them completely (Mason & Spence 1999; Rey, Suydam & Lindquist 1995).

For example, a child may be able to manipulate some tools and complete algorithms

but fails to explain why they work and cannot apply them to solving non-routine problems where context is novel. Such difficulty can be found commonly in mathematical learning among children. From a practical standpoint, it is important for teachers to be aware of their students' learning difficulties, when they design their ways of instruction.

It is assumed that a teacher with good grasp of mathematical knowledge could detect and diagnose children's learning difficulties in the learning process (cf. Carpenter et al. 1996; Graeber 1999; Fennema et al. 1996; Nathan & Koedinger 2000). For example, teachers whose students are skilled in solving addition and subtraction problems tended to agree that classroom instruction should help students construct mathematical knowledge rather than learn it passively (Peterson et al. 1989). Moreover, teachers with a good construction of proportionality and idea about students' conceptions tended to focus on mathematical ideas and reflective work with students rather than on procedures, numbers and operations (Thompson & Thompson 1996). Therefore, to explore and identify elementary school teachers' mathematical knowledge, their views of children's learning difficulties as well as their instructional views in daily mathematical teaching and learning situations is an issue of much interest to education researchers and practitioners.

How well students learn from their teachers in the first two grades will have long-term implication on their development in cognitive and mathematical abilities. This research explores teachers' knowledge and their views of children's learning difficulty and instructional practice. The questions investigated in this study are as follows.

1. What are the sources of the main components of teachers' views of children's learning difficulties as well as their mathematics instructional views?
2. Is there an interactive relationship between teachers' mathematical knowledge and their views of children's learning difficulties?
3. Is there an interactive relationship between teachers' mathematical knowledge and their views of mathematics instruction?
4. Is there a relationship between teachers' views of children's learning difficulties and their views in mathematics instruction? Results from the current study would provide new insight into teachers' knowledge and mathematical teaching practices as well as reflection for teachers education.

II. METHOD

This exploratory study was conducted in two stages and employed different instruments for data collection. In addition, follow-up non-structured interviews of four

lower grades teachers in public elementary schools in Taipei were made. The major source of information for the present data came from the questionnaires and interviews.

2.1. First Stage

(a) Instrument & subjects in pilot study

Two techniques for assessing teachers' mathematical knowledge, views of students' learning difficulty and mathematics instructional practices were used in the first two stages. First, in a pilot study, non-structured interviews and questionnaires containing both objective and open-ended questions were used. The Teachers' Knowledge of School Mathematics, Views of Mathematical Learning and Instruction Questionnaire consisted of the following eight different mathematical topics: single-digit addition, two-digit addition, subtraction, multiplication, length measurement, money, time, figure and shape problems. These eight topics mentioned above are essential in the early years of school mathematics. Questions were designed to investigate teachers' basic mathematical knowledge of a specific topic, their views of students' learning difficulties and instructional practices of the forementioned topics. An example of the questions in the open-ended questionnaire is as follows:

A second grader (first semester) explains how he worked on the following problem:

$$\begin{array}{r} 29 \\ + 13 \\ \hline \end{array}$$

He said, "20 plus 10 is 30; and 30 plus 9 is 39. Then add 3 on top of that: 40, 41, 42. So, the answer is 42". However, the teacher put it in another way, "Add one ten to two tens is three tens, and add nine to the three tens is 39. Add three to that is 42".

- (1) Was the student's method of solving the problem consistent with what the teacher required? Please explain the reasoning behind your viewpoints.
- (2) Provide suggestions for instructing students to solve such problems, perhaps, to adapt or create an example and teaching activity.

Twenty-five teachers who were attending a month-long in-service administration program held in the Taiwan Provincial Institution for Elementary School Teachers' Inservice Education participated in the first-stage investigation in December 1996.

(b) Results of pilot study

Integrating the results from teachers' interviews and questionnaire responses, we categorized children's learning difficulties as follows.

1. Difficulty in Understanding mathematical concepts (DU): The mathematical concepts were too abstract for children to understand especially when they involved too many abstract ideas or operational procedures such as subtracting mixed numbers. These difficulties included specific objects, ideas, procedures and algorithms.
2. Difficulty in Operating objects (DO): It is difficult for children to operate concrete objects or materials and figure out the solutions especially when those ideas are complicated or new to them. Though manipulating concrete objects helps children understand mathematical ideas, it is still too difficult to make connections and construct their understanding about events and interactions of some complicated concepts solely through object manipulations. It is hard for children to make inductions of mathematical concepts automatically through manipulating concrete objects.
3. Capable of Imitating Solution but not applying (IS): Children can just imitate teachers' solutions and memorize rules and formulas but fail to comprehend how the rules work. As a result, children can solve problems with similar deep structure by practice-and-drill, but they cannot apply to solve novel problems or extend the reasoning to other situations. In addition, children forget these rules quickly.

On the other hand, the categories of mathematics instructional practices integrated from teachers' views included the following.

1. Teacher Oral Explanation and Demonstration (TOED): Teachers structured mathematical materials, skills or concepts, then transmitted them to students by oral presentation and illustrations as well as demonstrated manipulations in a direct fashion.
2. Constructive and Cooperative Problem Solving (CCPS): Teachers and their students created a constructive atmosphere in which students engaged in cooperative small group discourse and problem-solving before standard mathematical terminology was introduced.
3. Drill-and-Practice (DP): Teachers' instruction focuses on direct instruction and repeated practice which will enable children to master basic skills because they believe that "Practice makes perfect".

2.2. Second Stage

(a) Formal constructed questionnaires and subjects

The pilot study served to build the categories for teachers' views of children's learning

difficulties and for mathematics instructional practices. In the second stage, the formal constructed questionnaires of Teachers' Knowledge of School Mathematics, Views of Mathematical Learning and Instruction were used.

The questionnaire designed in this research consists of the same eight specific mathematical topics as those in the pilot study, and each mathematical question had three subscales, designed to measure interrelated but separate constructs on the knowledge and views of mathematical learning and instruction questionnaire (see Appendix).

1. Subscale of school mathematics knowledge. The scale measures teachers' basic knowledge of elementary lower grades mathematics. Scale of basic knowledge of specific mathematical topics as those in the first stage of the study consisted of 17 items. Each question contained two to three item statements described the related mathematical knowledge. Most mathematical knowledge was taken from instructional booklets of elementary lower grades mathematics textbooks. Teachers judged whether the statements are correct or incorrect according to their knowledge and then made yes or no responses. A high score in this subscale indicates a high level of fundamental mathematical knowledge, while a low score indicate the opposite.
2. Subscale of children's learning difficulty. There were three items designed to measure teachers' views of children's learning difficulties in the eight specific mathematical topics. Each question consisted of three items indicting three kinds of learning difficulties as mentioned in the results of the pilot study. All items used a 4-point Likert scale by indicating strongly agree, agree, disagree, and strongly disagree. A high score on each item reflects high agreement with that kind of children's difficulty in mathematical learning. Conversely, a low score reflects strong disagreement with that particular learning difficulty.
3. Subscale of instructional practice. There were three items designed to measure teacher's views of mathematical instruction in teaching the forementioned eight specific mathematical topics. The three items indicted three kinds of teachers' mathematics instructional views as found in the results of the pilot study. All items used a 4-point Likert scale by indicating strongly agree, agree, disagree, and strongly disagree. A high score on each item reflects strong agreement with that kind of instruction in mathematical teaching. Conversely, a low score indicates strong disagreement with that particular instruction method.

The formally constructed questionnaires as described above were evaluated by 10 experts in mathematics education. The evaluation of validity analysis of scales of children's learning difficulties and mathematics instructional practice was .78 to .98.

A total of 219 teachers of first and second grades from 39 public elementary schools

selected from northern, central, southern and eastern Taiwan were assessed by the structured questionnaire as described above. The mean number of years of teaching experience in elementary school was 13.94 in the sample, and the mean number of years teaching first and second grades was 8.97.

(b) Teachers' interviews

In order to obtain additional information about teachers' views of mathematical learning and instruction, four elementary school teachers who taught in lower grades were interviewed after completing the structured questionnaire. All these four teachers were female and participated voluntarily. Two teachers taught in the first grade and two in the second grade.

The number of years of elementary teaching experience for these four teachers were 10 years (Teacher 1A), 12 years (Teachers 1B), 24 years (Teacher 2A), and 16 years (Teacher 2B). These four teachers' responses on the previous structured questionnaire showed great agreement with DU in their views of children's learning difficulty. In terms of mathematics instructional views, Teacher 1A and Teacher 2A had high agreement with TOED and a medium level of agreement with CCPS, while Teacher 1B and Teacher 2B agreed more to CCPS rather than TOED. All the four teachers expressed low level of agreement with drill-and-practice mode of instruction.

III. RESULTS AND DISCUSSION

3.1. Teachers' mathematical knowledge, their views of children's learning difficulties, and mathematics instructional practice.

The full score of the teachers' mathematical knowledge subscale was 17. The mean and standard deviation of teachers' scores on the subscale was 12.90 and 1.86, respectively. Five teachers got the maximum score of 17, and one teacher got the lowest score of 5. If a teacher's score was higher than or equal to 13, she/he would be defined as a high mathematical knowledge group. If a teacher's score was lower than 13, she/he would be defined as a low mathematical knowledge group. About 65% ($n = 134$) of the teachers were grouped as the high mathematical knowledge group; and 35% ($n = 85$) of the teachers were grouped as the low mathematical knowledge group.

The results revealed that over half of the first and second grades teachers got upper mean score in the evaluation of fundamental knowledge of school mathematics in spite of the limited number of items that measured teachers' mathematical knowledge in the instrument.

Tables 1 and 2 present the means and standard deviation of high and low mathematical

knowledge groups with respect to teachers' scores on the three kinds of children's learning difficulties and teachers' modes of instruction, respectively. As seen in Table 1, the most prominent learning difficulty of both high and low mathematical knowledge groups of teachers was DU. In addition, both groups of teachers tended to prefer direct instruction by oral explanation and demonstration as indicated in Table 2.

Table 1. Teachers' views of children's learning difficulties in high and low mathematical knowledge groups.

Views of children's learning difficulties	High group			Low group		
	DU	DO	IS	DU	DO	IS
Mean	28.07	26.90	21.75	28.00	27.31	20.95
SD	3.43	4.71	4.93	2.52	3.33	3.81

Table 2. Teachers' views of mathematics instructional practice in high and low mathematical knowledge groups.

Views of mathematics instructional practice	High group			Low group		
	TOED	CCPS	DP	TOED	CCPS	DP
Mean	23.60	22.03	22.40	23.53	21.47	23.01
SD	4.05	4.59	3.79	3.20	3.44	3.31

3.2. Relationship between teachers' mathematical knowledge and views of children's learning difficulty.

The data was analyzed with two (high versus low mathematical knowledge groups) by three (DU, DO and IS) repeated measures ANOVA. No significant interactive effects were found between teachers' mathematical knowledge and views of children's learning difficulties with $F_{(2,434)} = 1.54$. Teachers' mathematical knowledge did not seem to affect significantly their views of children's learning difficulties. However, the simple effect of teachers' views of children's learning difficulties was significant, $F_{(2,434)} = 222.62$, $p < .05$. Further analysis indicated a much larger difference between DU and IS, as well as between DO and IS. Simple effects of teachers' mathematical knowledge did not reach statistical significance, $F_{(1,217)} = .15$. Teachers in both high and low mathematical knowledge groups agreed that understanding abstract mathematical concepts was the primary difficulty in mathematics learning. The second difficulty for mathematics

learning was comprehending mathematical concepts and making inductions of mathematical concepts through manipulating concrete objects, and the third one is students' imitating teachers' solutions but unable to apply and solve novel problems.

3.3. Relationship between teachers' mathematical knowledge and views of mathematics instructional practice.

The data was analyzed with two (high versus low mathematical knowledge groups) by three (TOED, CCPS and DP) repeated measures ANOVA. As shown in Table 2 and Figure 1, a significant interaction effect was found with $F_{(2,434)} = 5.19, p < .05$. A simple main effect analysis for the three kinds of instructional practice suggested a much larger difference between TOED and CCPS in the high mathematical knowledge group with $F_{(2,434)} = 26.28, p < .05$, whereas there was a much larger difference between TOED and CCPS, as well as between DP and CCPS in the low mathematical knowledge group with $F_{(2,434)} = 28.24, p < .05$. There were no significant differences between high and low mathematical knowledge groups in the three kinds of mathematics instructional practice with $F_{(1,217)} = .00006$. The results illustrated that teachers' mathematical knowledge did not affect significantly their views of instructional practice. Teachers in both high and low mathematical knowledge groups had high agreement with the view of direct instruction. Furthermore, teachers of the low mathematical knowledge group tended to agree more with the view of direct instruction by oral explanation and demonstration as well as drill-and-practice than constructive and cooperative problem solving.

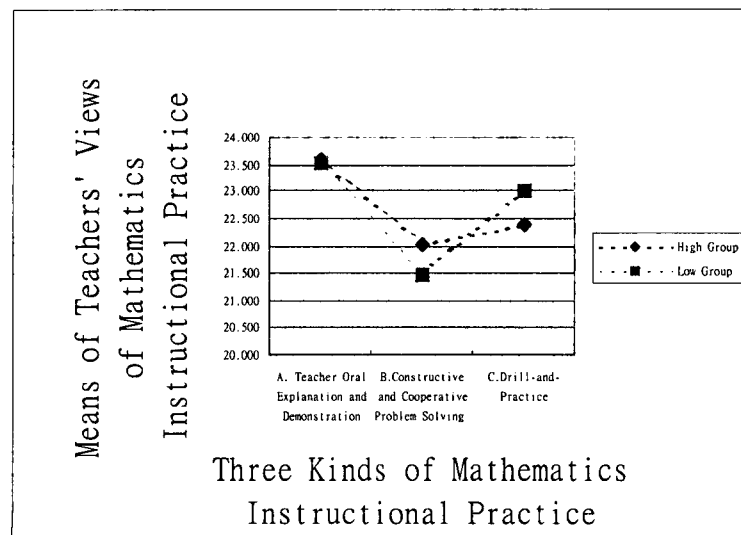


Figure 1. Teachers' views of mathematic instructional practice in high and low mathematical knowledge groups.

3.4. Relationship between teachers' views of children's learning difficulty and mathematics instructional practice.

To examine the relationship between teachers' views of children's learning difficulties and their instructional views, the procedure of multivariate canonical correlation analysis was used. As shown in Table 3, two substantial canonical correlations ($\rho = .473, p < .05$ and $\rho = .293, p < .05$) emerged from this analysis. Two canonical factors were derived from teachers' views of children's learning difficulties and mathematics instructional practice. The canonical variate χ_1 which accounted for 51.81% of the explained variance of teachers' views of children's learning difficulties was responsible for the canonical correlation, $\rho = .473$. Canonical variate χ_2 accounted for 14.95% of the explained variance and the canonical correlation was $\rho = .293$. Teachers' views of children's learning difficulties (X variable) accounted for 16.60% and 0.77% of the explained variance of mathematics instructional views through χ_1 and η_1, χ_2 and η_2 , respectively.

On the other side, the canonical variate η_1 which accounted for 74.28% of the explained variance of mathematics instructional views was responsible for the canonical correlation, $\rho = .473$. Canonical variate η_2 accounted for 8.98% of the explained variance and the canonical correlation was $\rho = .293$. Teachers' views of mathematical instruction (Y variable) accounted for 11.58% and 1.29% of the explained variance of views of children's learning difficulties through η_1 and χ_1, η_2 and χ_2 , respectively. The results suggested that there was a significant relationship between teachers' views of children's learning difficulties and their mathematics instructional practice. Such interrelationship between teachers' views of mathematics instruction and children's learning difficulties was also observed by Petersons et al. (1989) and Fennema et al. (1996).

Table 3. The multivariate canonical correlation analysis of the relationship between teachers' views of children's learning difficulties and their mathematics instructional views.

X variable	canonical variate		Y variable	canonical variate	
	χ_1	χ_2		η_1	η_2
DU	-.824	.566	TOED	-.967	.189
DO	-.754	-.357	CCPS	-.895	-.442
IS	-.553	.011	DP	-.701	-.195
% of explained variance	51.81%	14.95%		74.28%	8.98%
redundancy index	.1158	.0129		.1660	.0077
ρ^2	.223	0.86			
ρ	.473	.293			
p	.05	.05			

Furthermore, an "other comments" part was included in each item of the subscales of children's learning difficulty and mathematics instructional practice. From the collected data and analysis, there was no input written by teachers. This may have been due to two reasons. First, the items described in the questionnaire may have presented sufficient choices of answers and practical situations, so it was not necessary for teachers to write additional comments. Second, most teachers are fully occupied in the classroom setting, so they did not have much time to express any other opinions. However, the follow-up interviews did obtain extra information from the teachers.

IV. RESULTS OF INTERVIEWS OF TEACHERS

It is easy for teachers to express their enthusiasm for constructivism perspectives and students-centered approach. However, there are some different points of views among the four teachers concerning putting theory into teaching practices as observed through in-depth interviews. The main points abstracted from their views about mathematical teaching were as follows.

Teacher 1A expressed: "Mathematical knowledge in curricula of lower grades is not too difficult for children. I found that children with some informal knowledge are able to solve problems. However, to access children's thinking and previous experience is essential in mathematical teaching. I preferred whole-class settings, derived problems from textbooks and posed problems that were related to children's everyday life, thus allowing each child to operate manipulatives and solve problems himself/herself. That way, I could know if every child understands what I mean. I don't think the first graders are mature enough to explain their ideas explicitly. Children sometimes tended to end up chatting with peers when put in small groups instead of having problem-solving and goal-focusing discussions".

Teacher 1B: "It seemed that no matter what kind of classroom setting is adopted, as long as the teachers are focusing on the ideas associated with developmentally appropriate practices which reflect accurately children's constructive capabilities, and children can make sense out of the problem-solving activities and discussions. While having whole-class discussions, where I served as a moderator rather than leader of discussions, I did not use my own mathematical knowledge as a means of guiding the class to decide which solutions should be right. I expected more reasonable solutions figured out from children. I spent more time providing feedback to students' ideas, and students tended to raise more questions for discussion. Sometimes, children expressed their ideas in good mathematical sense and that surprised me".

Teacher 2A: “I agree with using mathematical discourse in constructing knowledge. Divided in small discussion groups, the children could check the solutions with each other in the groups. However, some children are unable to use group interaction to learn to solve the problem, teachers also know less about the experiences of students who have difficulty and whether the kind of responses they received that will help or not help them. In addition, the children spent much time discussing and verbalizing their ideas in the small groups. As a result, teachers did not have enough time to discussing all the problems presented in the textbook and homework. In my case, I hardly have time to discuss the worksheets with children”.

Teacher 2B: “While having the class engaged in cooperative small group problem solving, I moved around the classroom and paid attention to the students’ ideas, and children’s thinking picking up new ideas for problems later. Mathematics of lower grades is all easy and known to teachers. With my many years of teaching experience, I play the role of an intellectual coach and ask specific questions, focusing on some mistakes and misconceptions that commonly confuse students’ understanding. But it did take much more time than direct instruction”. With respect to the reasons accounting for teachers’ less agreement with cooperative small group discussion, Teachers 2B also expressed additionally: “some teachers dislike having small group discussions due to two reasons. First, there is the problem of classroom management. It’s common for a teacher to have more than 30 students in a class and most teachers were not used to students’ high noise level. Second, a teacher would feel embarrassed if he/she cannot answer questions generated from the students’ discussion. It is necessary for a teacher to read more reference books and makes better preparation. Having more knowledge of the subject matter would assure success in leading the children to participate more meaningfully in discussions and problem-solving”.

Integrating the analysis and the results of the interviews, we obtain the following findings.

1. In teaching practice, textbooks also played an important role. Teachers either posed problems that were analogous to textbooks by themselves or picked up students’ ideas as new sources of problems to be posed. When posing problems, teachers with more constructivism perspectives picked up children’s ideas more often than did teachers with direct instruction views.
2. With respect of students’ mathematics learning, four teachers realized that it was even harder for lower grades students to understand mathematical concepts and written symbols, but those with some informal knowledge were able to try various methods and use informal strategies to solve problems. However, young students sometimes cannot verbalize their ideas explicitly. Thus, teachers have to access

students' thinking and their existing knowledge by various activities.

3. In terms of instructional setting in the classroom, some teachers did not prefer small groups discussion because students took too much time discussing and verbalizing their ideas, as a result, the schedule was delayed. On the other hand, there were also some teachers who preferred to involve students in mathematical discussion and problem-solving in cooperative small groups rather than whole class discussion.

V. DISCUSSION

This study, on the one hand, contributed to understanding the relationships between teachers' knowledge about school mathematics, their views of children's learning difficulties and mathematics instructional practice. On the other hand, interviews with four teachers about their perceptions of mathematical teaching shed light on teachers' preferred mode of instruction.

These findings in this study illustrated that different levels of mathematical knowledge among teachers did not affect significantly their views of students' learning difficulties and mathematics instructional practice. Furthermore, teachers in both high and low mathematical knowledge groups regarded difficulty in understanding mathematical concepts as the most prominent problem in students' mathematics learning. Teachers also tended to prefer oral explanation and demonstration in mathematical teaching rather than constructive and cooperative discussion and problem solving.

These findings from questionnaire investigation can be supported and explained by the results from interviews. First, there is a general agreement among teachers' views that knowledge for lower grades school mathematics is easy and known to teachers. That is first and second grade teachers may have adequate amount of knowledge about the subject matter, they have basic knowledge about daily mathematical problems that are related to practical learning situations. However, most teachers are more used to the role of an instructor presenting clearly and concisely step-by-step mathematics, while the responsibility of the students is to take notes and do the assigned problems. This seems to support Shulman's notion that mathematical knowledge alone does not translate into teaching practices directly (Shulman 1986). The reasons that influence teachers to have a preference for direct instruction mode than cooperative small group discussions and problem solving as mentioned in the interviews can be summarized as follows.

1. It would take much longer time using small group discussion and problem solving than direct teaching.
2. Lower graders lack the ability to carry on fruitful discussions.

3. Class management became more difficult when students were engaged in small group discussion and problem solving.
4. Fearing that their insufficient mathematical knowledge would make them unable to answer students' questions effectively, teachers are less willing to place emphasis on constructive and cooperative problem solving. Teachers with low mathematical knowledge tended to have a preference not only for direct instruction but also drill-and-practice rather than constructive and cooperative problem solving. These teachers lacked confidence in answering students' questions adequately, and thus adopt either direct instruction or drill-and-practice mode that they can have better control.

Questionnaire investigation revealed that a relationship existed between teachers' views of children learning difficulty and mathematics instructional practice. Results of interviews also showed that different views of instructional practice among teachers led to differences in their modes of posing problems and interacting with students. Moreover, teachers knows better the students' learning difficulty would pay more attention to their ideas and provide more opportunities for each student to try to construct a solution to the problem that was meaningful to him/her during instruction.

Mathematical knowledge not only encompasses a concrete dimension, but also include calculation, situation, application and so on (Ereast 1999). Mathematical concepts are known as a formal abstract system. It is not easy for children to understand and connect mathematical symbols and operations in problem solving. Though such views of children's learning difficulty was also echoed by teachers, most teachers still preferred direct instruction. Even though some teachers hold constructivism perspectives and favor cooperative problem solving, they still use direct instruction in their practical teaching.

On the one hand, teachers think that students must master some computation skills; on the other hand, they fear that many problems are beyond the discussion abilities of their students. Quite many teachers are used to explain and demonstrate in a direct mode, focusing more on content clarity, instructional pace and opportunity for answering questions but without reflective thinking. Teachers can achieve better class management in teacher-centered situations. However, students learning in such situation, have little opportunities to manipulate or express their thinking. Students remember the solutions through repeated practice with different exercises instead of asking questions, summarizing content, clarifying problem or thinking critically. Students reflect only on the relationship between the exercises and their results rather than reflecting on a meaningful conscious level. Such forementioned characteristics in some cases may be associated with effective teaching (Cooney 1999) and some students may benefit from giving high-level explanations in whole-class setting as well as in small groups (Webb

1989). However, teaching that is predominantly teacher telling and demonstrating has been challenged.

The perceived ineffectiveness of meaningful understanding for children's learning has aroused much attention in current research (cf. Peterson et al. 1989; Simon, Tzur, Heinz, Smith & Kinzel 1999). Teachers who adopted the mode of direct instruction and drill-and-practice approaches to mathematical instruction would make it difficult unexpectedly for students to understand mathematical concepts.

Though reform in mathematical instruction have been developed for several years, a key issue for those concepts of mathematical learning do not suffice as a basis for the development of appropriate teaching methods in particular. That is, no epistemology nor learning psychology can serve as the sole basis for the development of a teaching methodology (Simon et al. 1999). Teachers who had limited knowledge of how mathematical concepts are acquired and little practical experience of cooperative construction learning could alienate themselves from the reasonability of the subject matter structure in their teaching (Cooney 1999).

Fundamentally, providing teachers with more research-based knowledge about mathematical learning and problem-solving, as well as supplying a link between the psychology of children's thinking and the mathematical curriculum can help teachers apply what they learned in their classroom teaching. For example, teachers who are aware that children are unable to make induction of mathematical concepts automatically through manipulating objects can try to improve students' learning and reasoning through either teacher-and-student or peers verbal interaction in mathematical discourse (cf. Hick 1996; King 1995).

Subsequently, such efforts can affect teachers' views about mathematical learning and instruction, as well as their classroom practices. Teachers can also compare students' progress under different teaching methods to judge which instructional method may be most effective for a given student. Moreover, through listening to students explaining their problem-solving process provided opportunities for teachers to access students' knowledge (Leung & Wu 2000).

In this way, teachers can pose new questions that are related to students' thinking and allow students to discuss and clarify their thinking. Such questions would provoke students to make sense out of their problem-solving approach. Therefore, more inservice education programs for teachers are needed to challenge seriously the traditional views of direct instruction and drill-and-practice, and to encourage teachers to adopt activities more conducive to flexible student interaction and make mathematics learning more meaningful.

This work represent a general understanding about lower grades elementary school teachers' school mathematical knowledge and their views of mathematical learning and

instruction. There are some suggestions for further investigation. First, one of the limitations of this study was that only eight topics related to mathematical knowledge were used in the questionnaire to assess teachers' basic knowledge of school mathematics. Therefore, the results presented only a partial picture. Similar studies with different teachers and topic areas are needed to assess teachers' knowledge and generate as a complete view of mathematical learning and instruction as possible. Second, the descriptions of children's learning difficulty and mathematical instruction were built on empirical data gathered from 25 teachers through open-ended questionnaires, and followed by in-depth interviews of four teachers. On the other hand, none of the teachers provide voluntarily any ideas or comments in "Other comments" items of the structured questionnaire. More extensive collection of data might reveal additional categories and provide better examples for future studies on understanding more about teachers' conceptual framework for mathematical teaching and the role of teachers in promoting learning. For the purpose of extracting more information from teachers, observations and interviews of more teachers may be more effective than the use of questionnaires. Third, with the increased emphasis on mathematical processes and of the inspirations from social constructivism perspectives of learning, the current mathematical reform expect teachers to play an autonomous role in curriculum enactment (Skott 1999). However, there are some obstacles as indicated above preventing teachers from putting them into practice. It is worthy for researchers to develop a better more understanding of epistemology and learning psychology as well as help teachers create constructive learning situations.

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Appendix: Teachers' Knowledge of School Mathematics and Views of Mathematical Learning and Instruction Questionnaire

(Translated from Chinese)

Question I

First grade students were asked the following word problem in the second semester. "Johnny had 8 marbles. After his dad gave him some more marbles, Johnny ended up with 12 altogether. How many marbles did his dad give him?"

Many students gave the answer as: $8 + 4 = 12$, A: 4 marbles which was different from the expected answer of: $12 - 8 = 4$, A: 4 marbles.

- A. The following are teachers' responses to the above two solution methods. Please indicate whether you agree or disagree to the following.
1. The teacher solves the problem by subtraction. — YES / NO
 2. Subtraction is the opposite operation of addition, therefore, the students' solution method is mathematically logical. — YES / NO
- B. The following are teachers' opinions toward the difficulties faced by lower elementary students when solving the above problem. Please rate your level of agreement according to your teaching experience.
1. Children have difficulties understanding the concepts and operations behind addition and subtraction.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
 2. The above problem is too abstract and therefore children cannot solve it.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
 3. Children can imitate the teacher's way of solving, but cannot understand the concepts behind addition and subtraction.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
 4. Other comments:
- C. The following are teachers' suggestions to parents for helping their children at home. Please rate your level of agreement.
1. Parents should use props or manipulatives found in everyday life to reinforce understanding of the concepts by observations, experimentation and practical experience.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)

2. Parents should use props or opportunities found in everyday life to allow children to have the practical experience and discuss with them alternative ways to solve problems without adhering to methods in textbooks only.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
3. Suggest that parents tell children that both sides of the equal sign should be equal. If one takes 8 from the left, then 8 should also be taken from the right side.
 $8 + () - 8 = 12 - 8$, so $() = 4$. Also, parents should allow more practice opportunities to enhance familiarity.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
4. Other comments:

Question II

A second grader (first semester) explained how he worked on the following problem:

$$\begin{array}{r} 29 \\ +13 \\ \hline \end{array}$$

He said, “20 plus 10 is 30; and 30 plus 9 is 39. Then add 3; 40,41,42. So, the answer is 42”.

However, the teacher reiterated, “Add one ten to two tens is three tens and add nine to the three tens is 39. Add three to that is 42”.

- A. Was the student’s solution method consistent with what the teacher required? The following are teachers’ opinions, do you agree?
 1. The thinking process is the same because the answer comes out the same.
— YES / NO
 2. The thinking process is different because the teacher regards the 10 as a higher digit whereas the child worked on the tens then added the three ones. — YES / NO
 3. The thinking process is different because the teacher used mental calculation and the child added cumulatively. — YES / NO
- B. The following are teachers’ responses to the difficulties involved in solving the above problem. Please rate your level of agreement according to your teaching experience.
 1. Children at this age still do not have the concept of two digit computation and carrying, so they have difficulty with this problem.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
 2. The number of addends are too big for the child to compute and thus it is too difficult to grasp the concept of carrying.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)

3. After drill-and-practice, children should be able to compute accurately, but still cannot understand the concepts of digits and carrying.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
 4. Other comments:
- C. The following are teachers' suggestions for instructing students to solve such problems. Please rate your level of agreement according to your teaching experience.
1. Use props or manipulatives found in everyday life to reinforce understanding of digits and carrying. Teach students to first add the ones, then add the carried one to the two and one in the tens column.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
 2. Divide the students into small groups and use props or opportunities found in everyday life for illustration and practical experience and discuss alternative ways to solve the problem without adhering to methods in textbooks only.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
 3. Teach children to add the ones first, then add the carried "1" to the 2 and the 1 in the tens column recording the sum in the appropriate columns. Provide drill-and-practice opportunities.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree)
 4. Other comments:

Question III

A first grader (second semester) answered the following problem:

There were 9 birds on a tree. There flew over 3 more birds, but 5 flew away. How many are there left on the tree? $9 + 3 = 12$; $12 - 5 = 7$; A = 7 left.

However, the textbook required that the students to work out the problem as follows: " $9 + 3 - 5 = 7$ ".

- A. The following are two ways of approaching mixed-operation problems. Please indicate your agreement or disagreement.
1. First compute multiplication and division components, then work on the addition and subtraction parts. — YES / NO
 2. Operation from the left is always the key. — YES / NO
- B. The following are opinions of some teachers as to why the above problem solve by combined operations is difficult for first graders to understand. Please rate your level of agreement according to your teaching experience.

1. Two-step add/sub problems combined are abstract. Children have difficulties with concept and operation involving two steps ($9 + 3 - 5 = 7$).
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 2. Two-step add/sub problems combined are abstract. Therefore, this problem is difficult for children to grasp the concepts through manipulation.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 3. Students can mimic teachers' operation methods ($9 + 3 - 5 = 7$), but they do not truly understand the concepts.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 4. Other Comments:
- C. The following are teachers' suggestions for instructing students to solve such problems. Please rate your level of agreement according to your teaching experience.
1. Use various teaching tools and manipulatives for observations and actual manipulation. Then, explain the meaning of combined mathematical formulations, followed by teaching children to represent correct mathematical formulations.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 2. Divide students into groups and allow them to use various teaching tools and manipulatives to actually handle the entire solution process, then discuss among themselves the reasonable approach to solving this problem rather than requiring them to imitate the method given in the textbook.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 3. Use methods given in the textbook to teach students directly and let them practice set formulas for solving the problems.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 4. Other Comments:

Question IV

A teacher told the following true story. A second grader brought 2 ten-dollar coins to the store to purchase a book cover. The storekeeper told the child that a cover cost 13 dollars. He quickly made a call at the pay phone to his grandmother saying that he did not have 13 dollars. The grandmother asked, "If you don't have one-dollar coins, how did you make this phone call?" The boy answered, "The phone booth said, 'Deposit 1, 5, or 10 dollar coins', so I used my 10-dollar coin to call".

- A. According to the above problem, some teachers believe that the students must first possess some of the following basic concepts about money in

order to solve the problem. Please indicate agreement or disagreement.

1. Concepts of monetary exchange are needed. That is, 10-dollars (a larger unit) can be exchanged for 10 one-dollars (a smaller unit). — YES / NO
 2. Physically speaking, a five-dollar coin cannot be divided or cut into five one-dollar coins; and stacking five one-dollar coins will not change it into a five-dollar coin. However, five one-dollar coins can be exchanged for one five-dollar coin.
— YES / NO
- B. The following are opinions of some teachers as to why students have difficulty understanding the concept of money. Please rate your level of agreement according to your teaching experience.
1. Children cannot accurately compute problems regarding money because they do not have the clear quantitative concept of money.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 2. Children have difficulties with money exchange problems and thus have difficulties grasping the concepts of money exchange.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 3. Children may compute basic money problems even without clear quantitative concept of money, but they cannot handle practical everyday money problems.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 4. Other comments:
- C. The following are teachers' suggestions for instructing students to solve money exchange problems. Please rate your level of agreement according to your teaching experience.
1. Use diagrams, teaching tools, and real coins to illustrate given problems or mock shopping activities for actual observation and/or manipulation by children along with explanations of money and exchange concepts.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 2. Divide children into groups and allow them to use teaching tools and/or actual coins to discuss and work on problems and play mock money activities, then discuss the solution of monetary exchange problem and concepts.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 3. Provide repetitive instruction on money and exchange concepts and give drill-and-practice problems for children to memorize the different formulas of coin exchange.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).

4. Other comments:

Question V

Many parents of second graders spend the winter vacation teaching their children the multiplication table.

- A. The following are some notions about the multiplication table. Please decide whether you agree or not.
1. The concept of the multiplication table is one of accumulation. — YES / NO
 2. From $(5 \times 2 = 10)$ and $(2 \times 5 = 10)$, we can see that multiplication is transitive.
— YES / NO
- B. There are teachers who believe that memorizing the tables at this stage is too difficult and list the following reasons. Please rate your level of agreement according to your teaching experience.
1. Children have problems with multiplication problems because they have difficulties with the concepts of accumulation and increase in folds (double, triple, ten folds, and the like).
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 2. Children have difficulties understanding multiplication from actual manipulation of objects.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 3. Although children can recite the multiplication table, they do not necessarily understand the concepts behind the table and they may occasionally recall incorrectly.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 4. Other comments:
- C. When faced with parents who are frustrated with children who have difficulties with their multiplication tables, teachers give the following suggestions. Please rate your level of agreement according to your teaching experience.
1. Parents should use teaching tools, diagrams, or actual objects for children to observe or manipulate and at the same time provide explanation of meaning of the multiplication table.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 2. Parents should use teaching tools, diagrams, or actual objects for children to observe or manipulate and at the same time discuss the concept of accumulation

and increase in folds. Also note that the multiplication table works in a certain way and one need not rely on memorization alone.

— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).

3. Parents should teach repetitively the tables to their children and emphasize drill-and-practice.

— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).

4. Other comments:

Question VI

First grade, second semester. After teaching and evaluations of the unit on measurements of lengths, the teacher asked the students the following question:



This is a line formed by placing three nails end to end. James used 12 of such nails to make a line after which he discovered that he made the length of a pencil. Then, James used 12 paper clips to form a line which equaled the length of a plastic straw. The question is, “Which is longer? The pencil or the straw”.

To facilitate understanding, the teacher used actual items as well as illustrations on the blackboard.

- A. Some teachers are of the opinion that in order to solve such a problem, one must apply certain mathematical concepts. Please indicate agreement or disagreement.

1. To solve this problem, children must have the concepts of the length of the unit and develop the idea of iterative nature of measurement. — YES / NO
2. Children should combine repeatedly many of the same length of the unit to specify the quantity of the item being measured. — YES / NO

- B. Some teachers have the following explanations for the children’s difficulties. Please indicate the level of agreement according to your teaching experience.

1. Children have difficulty comparing the length of a specified unit and understanding what is used as the unit of measurement. For instance, children have difficulty comparing and transiting the length of the pencil and the straw (or the lengths of the paper clip and the nail).
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).

2. Children have difficulties making comparison between the length of units of the paper clip and the nail even from actual manipulation of objects.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 3. After thorough and repetitive explanation by the teacher, students should be able to solve such problems, but still cannot understand completely the connection between comparing lengths.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 4. Other comments:
- C. The following are teachers' suggestions for teaching unit comparison in length measurements. Please indicate your level of agreement according to your teaching experience.
1. Use various teaching tools and manipulatives for observations and actual manipulation. And, clarify the significance of comparing lengths by direct comparison of objects and by indirect comparison using smaller units to form new lengths for comparison.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 2. Divide students into groups and allow them to use various teaching tools and manipulatives to actually handle the entire solution process, then discuss among themselves the direct and indirect approaches to comparing lengths.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 3. The teacher provides the explanation of direct and indirect comparison of lengths. Give drill-and-practice problems for better comprehension.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
 4. Other comments:

Question VII

Lower elementary students read a clock at 9:55 as 10:55. Even after remedial instruction, the students continue to read it incorrectly.

- A. Please indicate whether the following descriptions of the running of the clock hands are accurate or not.
1. The turning of the hands signify measurement of angle rotation. — YES / NO
 2. In order to understand the turning of the clock hands, students must first understand the spatial relationship and distance between the marks. — YES / NO
- B. The following are reasons proposed by teachers to explain students' difficulties with the above problem. Please indicate your level of agreement

according to your teaching experience.

1. Students do not understand the concept of angle rotation and relationship between the hour hand and the minute hand as the two turn. It is a common mistake to read the time as "10:55" simply because the hour hand is closer to the "10".
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
2. Students lack practical experience manipulating the hands on the clock. Therefore, they cannot easily tell the correct time.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
3. Children easily make mistakes when telling time because they just memorize the way of reading clock rather than understanding the way to tell time.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
4. Other comments:

C. The following are teachers' suggestions for instructing children how to tell time. Please rate your level of agreement according to your teaching experience.

1. Give each student a clock with adjustable hands to manipulate, observe and read, then explain the way of reading clocks.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
2. Divide students into groups and allow them to use a clock with adjustable hands to work on and discuss the right way to tell time.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
3. Explain to the students that even though the hour hand is near the 10 mark, but the minute hand is at 55, so it is not yet 10. It is only 9:55. Provide plenty of time and opportunity for practice on reading clocks.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
4. Other comments:

Question VIII

In the first-grade curriculum, students are introduced to various geometric shapes not just triangles, squares, rectangles, and circles only. They learn to recognize and name the shapes.

A. The following are descriptions of squares and rectangles. Please indicate their accuracy in description.

1. Rectangles and squares are both four-sided, so they can be called a "quadrilateral".
— YES / NO
2. The attributes of rectangles and squares possess similar qualities that indicate a

relationship of comprising. — YES / NO

B. Some teachers give the following explanations for the students' difficulties in solving problems involving geometric shapes. Please indicate the level of agreement according to your teaching experience.

1. Students have difficulties with understanding the interrelationships among all the different geometric shapes. And, various geometric shapes and their different sizes may also confuse the students in their attempt to categorize the shapes.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
2. There are too many different geometric shapes, not to mention shapes with similar appearance. They visually confuse the children, thus preventing them from categorizing accurately the shapes.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
3. Students should be able to distinguish among the four more common shapes (triangles, squares, rectangles, circles), but have difficulty with the less familiar and less commonly used geometric figures.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
4. Other comments:

C. When faced with students who have difficulties categorizing geometric shapes, teachers give the following suggestions. Please rate your level of agreement according to your teaching experience.

1. Use various teaching tools of different shapes and use everyday objects of different shapes (round clock, rectangular desk, square handkerchief, triangular traffic sign, etc) in categorizing activities. Allow children to manipulate, observe and categorize. Teachers also need to explain how to categorize by the different attributes of the shapes.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
2. Divide the children into groups and allow them to manipulate, observe, categorize and discuss how to categorize according to the different attributes of the shapes. Provide them with various teaching tools of different shapes and use everyday objects of different shapes (round clock, rectangular desk, square handkerchief, triangular traffic sign, etc) in categorizing activities.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
3. Introduce the textbook method of categorizing the various geometric shapes and provide drill-and-practice problems to facilitate familiarity.
— 4 (strongly agree), 3 (agree), 2 (disagree), 1 (strongly disagree).
4. Other comments: