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Intelligent Fuzzy Controller for Nonlinear Systems

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Abstract

In this paper, we proposed an intelligent digital redesign method for a class of fuzzy-model-based controllers, effective for stabilization of continuous-time nonlinear systems. The TS fuzzy model is used to expend the results of the digital redesign technique to nonlinear systems. The proposed method utilized the recently developed LMI technique to obtain a digitally redesigned fuzzy-model-based controller. The intelligent digital redesign problem is converted to equivalent problem, and the LMI method is used to find the digitally redesigned fuzzy-model-based controller. The stabilization conditions of TS fuzzy model are derived for stabilization in the sense of Laypunov stability. In order to demonstrates the effectiveness and feasibility of the proposed controller design methodology, we applied this method to the single link flexible-joint robot arm.

Key words : Fuzzy control, Linear matrix inequality, Nonlinear system, single link flexible-joint robot arm

I. Introduction

Most of dynamic system in industry framework are formulated by a continuous-time systems. It is natural to design a controller in the continuous-time domain. At the same time, we have been witnessed rapid development of flexible, small size, low-cost microprocessors in the electronics field. Therefore, it is desirable to implement the recent advanced controller in digital[1-3].

There exist three digital design approaches for digital control systems. The first approach, called the direct design approach, is to discretize the analog plant and then determine a digital controller for the discretized plant. The second approach, called the digital redesign approach, is to pre-design an analog controller for the analog plant and then carry out the digital redesign for the pre-designed analog controller. The third approach, called the direct sampled-data approach, is directly design a digital controller for the analog plant, which is still under development. Fuzzy control is feasible for practical system in industry. In this kind of approaches method have the merits and the demerits. Since the first approach has the disadvantage of ignoring inter-sample behavior. The third approach is still under development, stability, reliability and performance are not feasible.

The second approach is the one most often used in industry and is the focus of this paper. Then, in this paper we used second approach, called the digital redesign approach, is to pre-design an analog controller for th analog plant and then carry out the digital redesign for the pre-designed analog controller.

In this paper, we proposed that intelligent digital redesigned fuzzy controller for digital control of continuous-time nonlinear systems. Digital control represent continuous-time models to discrete-time models. Besides, the same control input of continuous- time models as control input of discrete-time models, and the result is same too. We designed the controller of continuous-time model and the controller of discrete-time model using the intelligent digital redesign. The proposed method utilized the recently developed LMI technique to obtain a digitally redesigned fuzzy-model-based controller. The intelligent digital redesign problem is converted to and equivalent problem, and the LMI method is used to find the digitally redesigned fuzzy-model-based controller. The stabilization conditions of TS fuzzy model are derived for stabilization in the sense of Laypunov asymptotic stability.

2. Fuzzy Modeling of Nonlinear Systems

2.1 Takagi-Sugeno fuzzy model

The continuous-time fuzzy model, proposed by Takagi and Sugeno, is described by fuzzy **IF-THEN** rules which locally represent linear input-output relations of nonlinear systems. The i-th rule of T-S fuzzy model is defined by [4]

Plant Rule i :

$$\begin{aligned} &\text{IF } x_1(t) \text{ is } M_1^i \text{ and} \ldots \text{and } x_n(t) \text{ is } M_n^i \\ &\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t) \end{aligned} \tag{1}$$

The final defuzzified output of fuzzy system is inferred by

$$\begin{aligned} \dot{x}(t) &= \frac{\sum_{i=1}^{q} \omega_i(t)(A_i x(t) + B_i u(t))}{\sum_{i=1}^{q} \omega_i(t)} \\ \omega_i(t) &= \prod_{j=1}^{n} M_j^i(x_j(t)) \end{aligned} \tag{2}$$

where, M_j^i $(j=1,2,\ldots,n)$ is i-th fuzzy set, q is the number of rules of this TS fuzzy model, $x(t) \in R^n$ is the

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state vector, $u(t) \in R^m$ is the control input vector, $A_i \in R^{n\times n}$ and $B_i \in R^{n\times m}$ are system matrix and input matrix, $x_1(t), \ldots, x_n(t)$ are premise variables.

In order to design a global controller for the TS fuzzy model for original nonlinear system, Tanaka used parallel distributed compensation(PDC) technique[9]. But the PDC technique is not available for the tracking problem. Therefore, we utilized the extended parallel distributed compensation (EPDC) technique for this problem. Using the same premise, the EPDC fuzzy controller in the continuous-time model has the following rules structure[7].

Controller Rule i :

$$\begin{aligned} &\text{IF } x_1(t) \text{ is } M_1^i \text{ and} \ldots \text{and } x_n(t) \text{ is } M_n^i \\ &\text{THEN } u(t) = -K_i x(t) + E_i r(t) \end{aligned} \tag{3}$$

where, $K_i = [k_1^i, \ldots, k_n^i]$ is state feedback gain vector, $E_i = [E_1^i, \ldots, E_n^i]$ is feedforward gain vector in i-th subspace. $r(t)$ is the reference input.

The final deffuzified output of fuzzy controller for Equ. (3) is as follow:

$$u(t) = \frac{\sum_{i=1}^{q} \omega_i(t)(-K_i x(t) + E_i r(t))}{\sum_{i=1}^{q} \omega_i(t)} \tag{4}$$

The EPDC technique are avaliable for the regulation problem and the tracking problem of fuzzy system. Figure 1 shows the concept of EPDC design. Therefore the fuzzy controller shares the same fuzzy sets with the fuzzy system (2). For each rule, we can use linear control design techniques.

The overall closed-loop fuzzy system obtained by combining (2.1) and (2.4) becomes

$$\dot{x}(t) = \frac{\sum_{i=1}^{q}\sum_{j=1}^{q} \omega_i(t)\omega_j(t)((A_i - B_i K_j)x(t) + B_i E_i r(t))}{\sum_{i=1}^{q}\sum_{j=1}^{q} \omega_i(t)\omega_j(t)} \tag{5}$$

2.2 Discretization of the continuous-time TS fuzzy models

We can obtain the discrete-time TS fuzzy model against the continuous time TS fuzzy model, let us described observable continuous-time plant by:

$$\begin{aligned} \dot{x}_c(t) &= A x_c(t) + B u_c(t), \quad x_c = x_0 \\ y_c(t) &= C x_c(t) \end{aligned} \tag{6}$$

where, $x_c(t)$ is the state vector, $u_c(t)$ is the input vector, $y_c(t)$ is the output vector and (A, B, C) are constant matrices of appropriate dimensions. The continuous-time state feedback control law for the system is as follows:

$$u_c(t) = -K_c x_c(t) + E_c r(t) \tag{7}$$

where, the analog feedback gain K_c and the feedforward gain E_c have been given, and $r(t)$ is the reference input, assumed to be a piecewise-constant signal, $r(t) = r(KT)$ for $kT \le t < k(T+1)$ with T as sampling period.

We substitute (6) into (7), obtained by

$$\dot{x}_c(t) = A_c x_c(t) + B E_c r(t), \quad x_c(0) = x_0 \tag{8}$$

where, A_c is $A - BK_c$. The corresponding discrete-time model for $r(t) = r(kT)$ with $kT \le t < k(T+1)$ in (8).

$$x_c(kT+T) = G_c x_c(kT) + H_c u_c(kT) \tag{9}$$

where, $G_c = e^{A_c T}$, $H_c = [G_c - I_n]A_c^{-1}B$.

The fast rate sampled discrete-time model in (9) for $T_N = T/N$, where N is an integer and $r(t) = r(kT)$ for $kT \le t < k(T+1)$ can be written by

$$\begin{aligned} x_c(kT + iT_N) = G_c^{(i)} x_c(kT) + H_c^{(i)} E_c r(kT) \\ \text{for } i = 1, 2, \ldots, N, \end{aligned} \tag{10}$$

where, $G_c^{(i)} = (e^{A_c T_N})^{(i)}$, $H_c^{(i)} = [G_c^{(i)} - I_n]A_c^{-1}B$.

Let the state equation of the continuous-time system in (6) appling to the piecewise-constant input function $u_d(t)$, the results are as follows:

$$\begin{aligned} \dot{x}_d(t) &= A x_d(t) + B u_d(t), \quad x_d(0) = x_0 \\ y_d(t) &= C x_d(t) \end{aligned} \tag{11}$$

where, $u_d(t) = u_d(kT)$ for $kT \le t < kT+T$. The digital control law for the system in (11) with $r(t) = r(kT)$ for $kT \le t < k(T+1)$ is

$$u_d(kT) = -K_d x_d(kT) + E_d r(kT) \tag{12}$$

where, K_d is the feedback digital gain and the E_d is the feedforward digital gain. We substitute (11) into (12) to obtain as follows:

$$\dot{x}_d(t) = (A - BK_d) x_d(kT) + BE_d r(kT) \tag{13}$$

The corresponding discrete-time model of the sampled-data system in (13) is

$$\dot{x}_d(kT+T) = (G - HK_d) x_d(kT) + HE_d r(kT) \tag{14}$$

where, $G = e^{AT}$, $H = [G - I_n]A^{-1}B$.

The process of finding digital gains (K_d, E_d) in (12) from the analog gains (K_c, E_c) in (7) so that the closed-loop state $x_d(t)$ in (13) closely matches the closed-loop state $x_c(t)$ in (8) is called the state-matching digital redesign.

3. Intelligent Controller Design and Stability Analysis

In this chapter, we will consider stability and design issue in the proposed fuzzy model based intelligent controller approach of nonlinear systems. Stability conditions of both fuzzy models and fuzzy control systems can be proposed. The purpose are designing stable fuzzy controllers. More significantly, the stability analysis and controller design problems are reduced to LMI problem.

3.1 Fuzzy Controller Design Using Digital Redesign

In this chapter, we argue that design local controller about i-th subspace fuzzy controller's design method using intelligent digital redesign method.

Consider a controllable and observable analog nonlinear system represented by:

$$\begin{aligned} \dot{x}_c(t) &= A_i x_c(t) + B_i u_c(t) \\ y_c(t) &= C_i x_c(t) \end{aligned} \tag{15}$$

where, $x_c(t) \in R^{n\times 1}$ is the state vector, $u_c(t) \in R^{m\times 1}$ is the input vector.

$$u_c(t) = -K_c^i x_c(t) + E_c^i r_c(t) \tag{16}$$

Through $K_c \in R^{m\times n}, E_c \in R^{m\times m}$ are the feedback gain and the feedforward gain, we obtain the gains using LMI. $r(t)$ is the reference input.

In digital control of continuous-time systems, the continuous-time state-space equations need to be converted into discrete-time state-space equations. In TS fuzzy model-based controller, sampling period for fuzzy modeling and controller design are often assumed to be same.

The state $x_d(t)$ in (16) equals to the state $x_c(t)$ in (8) at each sampling instant, $t = KT_f$.

$$\begin{aligned} \dot{x}_d(t) &= A_i x_d(t) + B_i u_d(t) \\ y_d(t) &= C_i x_d(t) \end{aligned} \tag{17}$$

where, $u_d(t) = u_d(kT)$ for $kT \le t < (k+1)T$.

Also let the digital controls law for the system in (17) with $r(t) = r(kT)$ for $kT \le t < (k+1)T$ be

$$u_d(t) = -K_d^i x_d(t) + E_d^i r_d(t) \quad (kT_f \le t < KT_f + T_f) \tag{18}$$

The analog control input $u_c(t)$ and the digital control input $u_d(t)$ are the same response feature of closed-loop system about dynamic system (15).

The designed closed-loop sampled-data system in (17) and (18) becomes

$$\dot{x}_d(t) = A_i x_d(t) - BK_d x_d(kT) + B_i E_d r(kT) \tag{19}$$

A zero-order hold is utilized in (19). The corresponding discrete-time model of continuous-time system in (17) is

$$\begin{aligned} x_d(kT+T) &= G_i x_d(kT) + H_i u_d(kT) \\ y_d(kT) &= C_i x_d(kT) \end{aligned}$$

where,

$$G_i = e^{A_i T_f}, \quad H_i = \int_0^{T_f} e^{A_i T_f} B_c dt = (G_i - I_n) A_i^{-1} B_i.$$

Let, $u_c(t) = u_d(t)$. Then, the analog control input $u_c(t)$ and the digital control input $u_d(t)$ are the same amount, we obtain as follows:

$$u_c(t) = \sum_{T_f=1}^{N} W_{kf}\, \Phi_{kf} = \sum_{T_f=1}^{N} u_d(KT_f)$$

where, Φ_{kf} : orthnomal series.

Using (19), W_{kf} is

$$W_{kf} = \frac{1}{T_f} \int_{KT_f}^{(K+1)T_f} u_c(t)\, dt \tag{20}$$

Using the Chebyshev quadrature formula[2],

$$W_{kf} = \frac{1}{N+1} \sum_{i=0}^{N} x_c\left(KT_f + i\frac{T_f}{N}\right) + E_c r(KT_f) \tag{21}$$

where, $x_c(KT_f + i\frac{T_f}{N})$ is

$$x_c\left(KT_f + i\frac{T_f}{N}\right) = G_{cr} x_c(KT_f) + H_{cr} E_c r(KT_f) \tag{22}$$

where, $G_{ir} = e^{A_{ir} T_f}, A_{ir} = A_i - B_i K_c^i$

$$H_{ir} = \int_0^{T_f} e^{A_{ir} T_f} B_i dt = (G_{ir} - I_n) A_{ir}^{-1} B_i.$$

We substitute (22) into (19) to obtain as show in (23)

$$W_{kf} = \frac{1}{N+1} \sum_{i=0}^{N} \left(G_{cr} x_c(KT_f) + H_{cr} E_c r(KT_f)\right) + E_c r(KT_f) \tag{23}$$

In (23), W_{kf} is quantity of analog control input $u_c(t)$. The control input W_{kf} stabilizes the system.

$$x_d(kT_f + T_f) = G_i x_d(KT_f) + H_i u(KT_f) \tag{24}$$

The closed-loop system is shown by

$$\begin{aligned} & x_d(KT_f + T_f) = \\ & G_c x_d(KT_f) + H_c\Big(\frac{1}{N+1} \sum_{i=0}^{N} (G_{cr} x_c(KT_f) \\ & + H_{cr} E_c r(KT_f)) + E_c r(KT_f)\Big) + E_c r(KT_f)) \end{aligned} \tag{25}$$

In (25), overall closed-loop system is as follows:

$$x_d(KT_f + T_f) = \hat{G}_{CN} x_d(KT_f) + \hat{H}_{CN}\, r(KT_f) \tag{26}$$

where, $\hat{G}_{CN} = G_i - H_i K_{dr}^i$, $G_i = e^{A_c T_f}$,

$\hat{H}_{CN} = H_c E_{dr}$, $H_i = (G_i - I_n) A_i^{-1} B_i$.

Because (25) and (26) have same result, then we have

$$\begin{aligned} G_c - H_c K_{dr} &= G_c - H_c K_c \frac{1}{N+1} \sum_{i=0}^{N} G_{cr} \\ H_c E_{dr} &= -H_c\Big(K_c \frac{1}{N+1} \sum_{i=0}^{N} H_{cr} - I_n\Big) E_c \end{aligned} \tag{27}$$

In (27), E_{dr} is the feedforward gain, K_{dr} is the feedback gain. Solving (27) yields the desired digital control gains as

$$\begin{aligned} K_{dr} &= K_c \frac{1}{N+1} \sum_{i=0}^{N} G_{cr} \\ E_{dr} &= \Big(I_n - K_c \frac{1}{N+1} \sum_{i=0}^{N} H_{cr}\Big) E_c \end{aligned} \tag{28}$$

Representing (28) yields the digital control gains as

$$\begin{aligned} K_{dr} &= K_c \frac{1}{N+1}(G_{cr}-I_n)^{-1}(G_{cr}-I_n) + G_{cr} \\ E_{dr} &= E_c - K_c \frac{1}{N+1} \sum_{i=0}^{N} G_{cr} - K_c A_{cr}^{-1} B_r E_c \\ &= E_c + (K_c - K_{dr}) A_{cr}^{-1} B_r E_c \end{aligned} \tag{29}$$

Assuming $N \rightarrow \infty$, the digital gains in (29) becomes

$$\begin{aligned} K_{dr} &= \lim_{N\to\infty} K_c[(N+1)(G_{cr}-I_n)^{-1}(G_{cr}-I_n)] \\ &\quad + \lim_{N\to\infty} K_c\left[\frac{1}{N+1} G_{cr}\right] \\ &= \frac{1}{T_f} K_c A_{cr}^{-1}(G_{cr}-I_n) \end{aligned} \tag{30}$$

Assuming $N \rightarrow 1$, the gains becomes

$$\begin{aligned} K_{dr}^i &= \frac{1}{2} K_c^i (I_n + G_{ir}) \\ E_{dr}^i &= (I_m - \frac{1}{2} K_c^i H_{ir}) E_c^i \end{aligned} \tag{31}$$

We apply the bilinear transform method to (30), then we have

$$\begin{aligned} K_{dr}^i &= \frac{1}{2}(I + \frac{1}{2} K_c^i H_r)^{-1} K_c^i (G_r + I) \\ E_{dr}^i &= (I + \frac{1}{2} K_c H_r)^{-1} E_c \end{aligned} \tag{32}$$

where, K_{dr}^i, E_{dr}^i are the digital feedback gain and the digital feed -forward gain in i-th subspace. Figure 1 is shown intelligent digital redesign. The proposed digital controller can control continuous-time nonlinear dynamic system effectively .

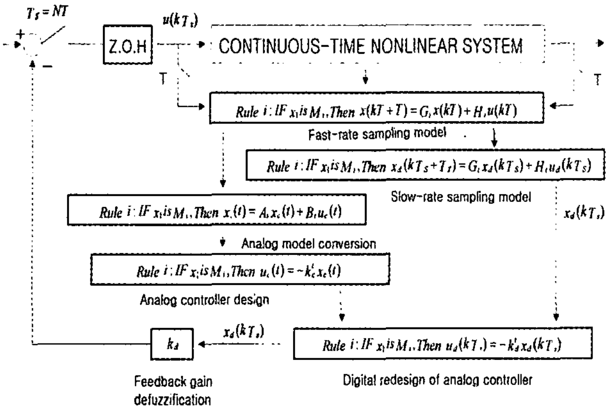

Fig. 1. Concept of intelligent digital redesign

3.2 Stability Analysis and Controller Design Using LMI

Numerically, the LMI problems can be solved very efficiently by means of some of the most powerful tools available to date in the mathematical programming literature. Therefore, recasting the stability analysis and control design problems as LMI problems is equivalent to finding solutions to the original problems. The recasting of stability analysis and design of fuzzy control systems to LMI problems was first considered in [6, 7]. We argue the stability analysis based on LMI. Though controller in (32) is stable in i-th subspace, the overall closed-loop system can be unstable. Therefore, we have to analyze the stability of overall closed-loop system. Stability analysis about overall closed-loop system have to satisfy theorem 1's sufficient condition.

Theorem 1[6]: The equilibrium of a fuzzy system is asymptotically stable in the large if there exists a common positive definite matrix P such that the following two conditions are satisfied.

$$\{A_i - B_i K_i\}^T P + P\{A_i - B_i K_i\} < 0, \quad i = 1, \ldots, q \tag{33}$$

$$G_{ij}^T P + P G_{ij} < 0, \quad i < j \leq q \tag{34}$$

where , $G_{ij} = \dfrac{\{A_i - B_i K_j\} + \{A_j - B_j K_i\}}{2}$

Theorem 1 is not the stability analysis of nonlinear system but the stability analysis of fuzzy system. The design problem for the fuzzy controller have to satisfy conditions of theorem 1. Where, K_i is the feedback gain. If we obtain the common positive definite matrix P, the stability of closed-loop system enable to decision. But the obtaining of common positive matrix P is difficult, therefore the guaranteed stability of fuzzy system is difficult. In other words, the overall closed-loop system is unstable though the local systems are stable. Also, existed PDC method is not the stability analysis of nonlinear system but the stability analysis of TS fuzzy system. Hence, the tracking problem is not referred to this expression. In this paper, to solve this shortcoming, we proposes extension parallel distributed compensation (EPDC). In order to solve these problems, we modify the controller rule of PDC with the same premise in (1) as follows, which is called an EPDC:

Controller Rule i :

$$\begin{aligned} &IF\ x(t) \text{ is } M_1^i \text{ and} \ldots \text{and } x^{(n-1)}(t) \text{ is } M_n^i \\ &THEN\ \ u(t) = -K_i x(t) + E_i r(t) \end{aligned} \tag{35}$$

where, K_i and E_i are feedback gain and feedforward gain in i-th subspace, respectively, and $r(t)$ is the reference input. The local gains are obtained by LMI, In (35), that are stabilized local systems.

$$\begin{aligned} u(t) &= -\sum_{i=1}^{q} \mu_i(x(t)) K_i x(t) + \sum_{i=1}^{q} \mu_i(x(t)) E_i r(t) \\ &= -K(\mu)\, x(t) + E(\mu)\, r(t) \end{aligned} \tag{36}$$

The control input $u(t)$ in (26) stabilize the overall closed-loop system. We obtain the feedback gain and the feedforward gain using LMI. The LMI definition is as follows:

Definition 1 [8]: A linear matrix inequality (LMI) is a matrix inequality of the form.

$$F(x) = F_0 + \sum_{i=0}^{m} x_i F_i > 0 \tag{37}$$

Where, $x^T = (x_1, x_2, \ldots, x_m)$ is variable and symmetric matrix $F_i = F_i^T \in R^{n \times n}$, $i = 0, \ldots, m$ are given. The inequality symbol >0 means that $F(x)$ is positive definite.

The LMI (37) is a convex constraint on x. ie., the set $\{ x \mid F(x) > 0 \}$ is convex. The LMI can represent a wide variety of convex constraint on x such as Lyapunov stability conditions and convex quadratic matrix inequalities. In this thesis, the stability conditions for TS fuzzy model are formulated in LMI framework, since this LMI based stabilizing controller design is very efficient, systematic and powerful technique.

Consider a continuous-time TS fuzzy model, described by the following state space equation.

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(x(t))(A_i x(t) + B_i u(t)) \tag{38}$$

$$\begin{aligned} u(t) &= -\sum_{i=1}^{q} \mu_i(x(t))K_i x(t) + \sum_{i=1}^{q} \mu_i(x(t))E_i r(t) \\ &= -K(\mu)x(t) + E(\mu)r(t) \end{aligned} \tag{39}$$

Substituting (38) into (38) gives

$$\begin{aligned} \dot{x}(t) = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(x(t))\mu_j(x(t))\{(A_i - B_i K_j)x(t) \\ + B_i E_i r(t)\} \end{aligned} \tag{40}$$

The main result on the global asymptotic stability of continuous-time TS fuzzy model is summarized in the following theorem.

Theorem 2. If there exist a symmetric and common positive definite matrix P, some matrices K_i such that the following LMI are satisfied, then the continuous-time TS fuzzy system is asymptotically stabilization via the TS fuzzy model based state feedback controller.

$$\Gamma > 0$$

$$\begin{pmatrix} A\Gamma + \Gamma^T - B\Phi - \Phi^T B^T & BE \\ BE^T & I \end{pmatrix} < 0 \tag{41}$$

If there exist Φ, E and the symmetric positive definite matrix Γ, Conclusionally, the system stabilized asymptotically. Where $P^{-1} = \Gamma, KP^{-1} = \Phi$.

Proof)

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\,u(t) \\ u(t) &= -Kx(t) + Er(t) \end{aligned} \tag{42}$$

Consider the Lyapunov function candidate about the system (42) as follows:

$$\begin{aligned} V &= x^T P x > 0 \\ \dot{V} &= \dot{x}^T P x + x^T P \dot{x} < 0 \end{aligned} \tag{43}$$

If the condition is satisfied, overall closed-loop system can be globally asymptotically stabilized.

$$\dot{x}(t) = (A - BK)x(t) + BEr(t) \tag{44}$$

Consider the Lyapunov function candidate about the system (44) as follows:

$$\begin{aligned} \dot{V}(t) &= ((A - BK)x(t) + BEr(t))^T Px \\ &\quad + x^T P((A - BK)x(t) + BEr(t)) \\ &= (x^T\ r^T)\begin{pmatrix} (A-BK)^T P + P(A-BK) & PBE \\ (PBE)^T & I \end{pmatrix}\begin{pmatrix} x \\ r \end{pmatrix} < 0 \end{aligned} \tag{45}$$

If there exist symmetric positive definite matrices P, K, the closed-loop system is stable in the Lyapunov sense. But the above matrix is QMI, so we need to make the LMI form by changing variables. Pre- and post-multiplying the following matrix both side of the QMI.

Let $P^{-1} = \Gamma, KP^{-1} = \Phi$.

$$\begin{aligned} &\begin{pmatrix} P^{-1} & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} (A-BK)^T P + P(A-BK) & PBE \\ (PBE)^T & I \end{pmatrix}\begin{pmatrix} P^{-1} & 0 \\ 0 & I \end{pmatrix} \\ &= \begin{pmatrix} A\Gamma + \Gamma^T - B\Phi - \Phi^T B^T & BE \\ BE^T & I \end{pmatrix} < 0 \end{aligned} \tag{46}$$

If there exist Φ_i, E_i and the symmetric positive definite matrix Γ satisfying the following LMIs.

$$\Gamma > 0$$

$$\begin{pmatrix} A_i\Gamma + A_i\Gamma^T - B_i\Phi_i - \Phi_i^T B_i^T & B_i E_i \\ (B_i E_i)^T & I \end{pmatrix} < 0 \tag{47}$$

Therefore the stability of the TS fuzzy model can be cast as follows:

$$\begin{aligned} u(t) &= -\sum_{i=1}^{q} \mu_i(x(t))K_i x(t) + \sum_{i=1}^{q} \mu_i(x(t))E_i r(t) \\ &= -K(\mu)x(t) + E(\mu)r(t) \end{aligned} \tag{48}$$

4. Illustrative Example

In this chapter, to show the effectiveness of the proposed controller design technique, we simulate the control of the single link flexible joint robot arm. Figure 2 shows mechanism of single link flexible-joint robot arm. The mechanism of this robot is derived by

$$\begin{aligned} I\ddot{q}_1 + MgL\sin(q_1) + k(q_1 - q_2) &= 0 \\ J\ddot{q}_2 - k(q_1 - q_2) &= u \end{aligned} \tag{49}$$

Where, M is the total mass of arm, I is inertia of link, L is length of link, k is spring of inertia coefficient, J is rotor inertia of the actuator, g is gravity constant

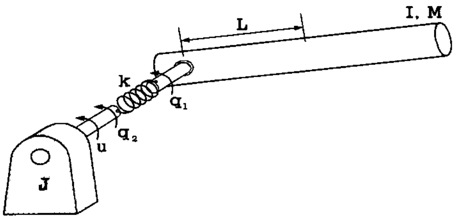

Fig. 2. Single link flexible joint robot arm

TS fuzzy model about system can be obtain by

Plant Rules :

Rule 1: IF x_1 is *about* 0, THEN $\dot{x} = A_1 x + B_1 u$

Rule 2: IF x_1 is *about* π, THEN $\dot{x} = A_2 x + B_2 u$

where,

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{MgL}{I} - \frac{k}{I} & 0 & \frac{k}{I} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J} & 0 & -\frac{k}{J} & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{MgL}{I} - \frac{k}{I} & 0 & \frac{k}{I} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J} & 0 & -\frac{k}{J} & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix}$$

we use the initial parameters as follows:

$M = 1kg$, $I = 1kg - m^2$, $L = 1m$, $k = 1N/m$,

$J = 1kg - m^2$, $g = 9.8m/s^2$

The membership function for Rule 1 and Rule 2 are shown in Fig. 3.

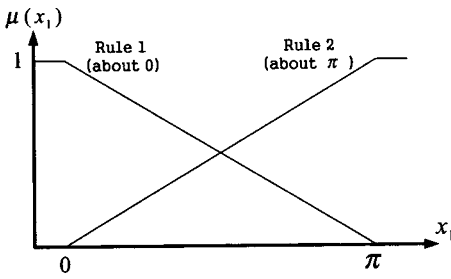

Fig. 3. Membership functions

A common positive definite matrix P that satisfies stability condition defined by Theorem 2 is found to be

$$P = \begin{bmatrix} 812.9896 & 358.0774 & 70.0615 & 6.8172 \\ 358.0774 & 165.2965 & 32.5805 & 3.2415 \\ 70.0615 & 32.5805 & 6.5453 & 0.6460 \\ 6.8172 & 3.2415 & 0.6460 & 0.0700 \end{bmatrix}$$

Then, stability condition of theorem 2 is satisfied.

Therefore, overall fuzzy system is stable in Lyapunov sense.

Fig 4 and Fig 5 shows response feature of single link flexible joint robot arm, control input when $r(t) = 0$. The initial conditions is $x_0 = [\pi/6\ 0\ 0\ 0]^T$. Fig 6 and Fig 7 shows response feature of single link flexible joint robot arm, control input when $r(t) = 0.2\sin(t)$.

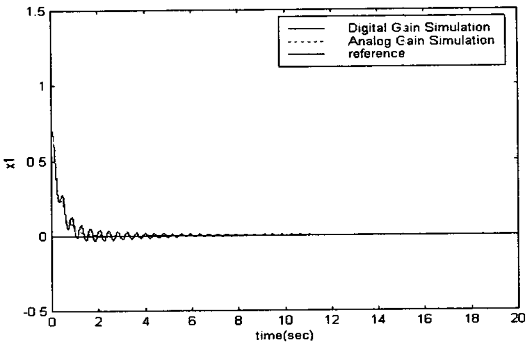

Fig. 4. Response x_1

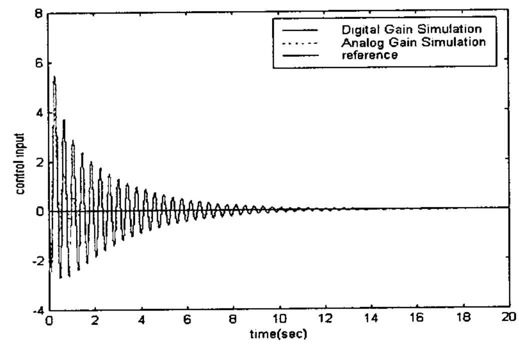

Fig. 5. Control input u(t`)

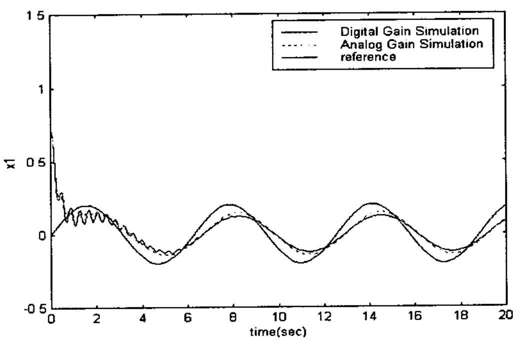

Fig. 6. Response x_1

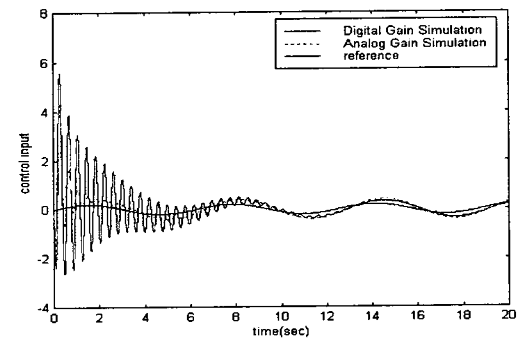

Fig. 7. Control input u(t`)

The proposed controller can know that is possible the tracking problem as well as the regulation problem from figure. The delay phenomenon of Fig 6. and Fig 7 are differences between practical model and computer simulation.

5. Conclusions

In this paper, we propose the design method of intelligent fuzzy controller for nonlinear systems and stability analysis of nonlinear system using LMI. The basic approach are based on intelligent digital redesign using state-mating and Lyapunov stability theory using LMI. Nonlinear system can be represented by TS fuzzy model in each operation point. The continuous-time TS models are represented by the discrete -time TS fuzzy models using state-matching.

The control inputs of the continuous-time TS fuzzy models and the discrete-time TS fuzzy models are equivalent, the results are same. The proposed method are utilized the recently developed LMI technique to obtain a digitally redesigned fuzzy-model-based controller. Simulation result on the single link flexible joint robot arm has convincingly shown the feasibility and effectiveness of the proposed digital redesign method.

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