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헬리콥터 시스템의 퍼지 분산 제어기 설계

A Decentralized Control Technique for Experimental Nonlinear Helicopter Systems

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요 약

본 논문은 2 자유도 실험용 헬리콥터 시스템의 제어를 위한 분산 제어기 설계 기법을 제안한다. 분산제어기법은 특히 대규모 제어 시스템에 적합하다고 알려져 있다. 본 논문에서는 Lyapunov 안정도 설계 방법을 이용하여 상호 연결된 TS 퍼지 시스템의 안정도 조건을 유도하고 선형 행렬 부등식을 이용하여 제어기 설계 조건을 공식화한다. 제안된 방법의 유용성을 검증하기 위해, 컴퓨터 시뮬레이션뿐 만 아니라 실험을 통해 그 결과를 도출한다.

Abstract

This paper proposes a decentralized control technique for 2-dimensional experimental helicopter systems. The decentralized control technique is especially suitable in large-scale control systems. We derive the stabilization condition for the interconnected Takagi-Sugeno (TS) fuzzy system using the rigorous tool-Lyapunov stability criterion and formulate the controller design condition in terms of linear matrix inequality (LMI). To demonstrate the feasibility of the proposed method, we include the experiment result as well as a computer simulation one, which strongly convinces us the applicability to the industry.

Key words : fuzzy control, decentralized control, linear matrix inequality, experimental helicopter system

1. Introduction

Most plants in the industry have severe nonlinearity and uncertainties. They thus post additional difficulties to the control theory of general nonlinear systems and the design of their controllers. In order to overcome this kind of difficulties in the design of a controller for an uncertain nonlinear system, various schemes have been developed in the last two decades, among which a successful approach is fuzzy control. Recently, fuzzy control has attracted increasing attention, essentially because it can provide an effective solution to the control of plants that are complex, uncertain, ill-defined, and have available qualitative knowledge from domain experts for their controllers design. There have been many successful applications in the industry to date. In spite of the usefulness of fuzzy control, its main drawback comes from the lack of a systematic control design methodology. To resolve these problems, the idea that a linear system is adopted as the consequent part of a fuzzy rule has evolved into the innovative Takagi-Sugeno (TS) fuzzy systems, which becomes quite popular today[2-8]

On the other hand, although there have been successful applications to real industry, we have wittiness that the conventional control technique is hard to apply to large-scale nonlinear systems, such as power systems spread over distant geographic areas [1]. When a conventional centralized technique is applied for control, the complexity of the analysis grows rapidly as the order of the system increases. This situation motivates us to look for ways to simplify the analysis. If the large-scale system can be viewed as an interconnection of subsystems some of which are strongly connected, while others being weakly connected, we can deals with the simple subsystems with perturbations only.

Motivated by the above observations, this paper discusses a decentralized control technique for a large-scale systems representable by several sub-TS fuzzy systems. The stabilizing controller design condition is derived based on the Lyapunov stability criterion and formulated in terms of linear matrix inequalities (LMIs).

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An application to the experimental helicopter system is also included to show the effectiveness of the proposed method.

2. Interconnected TS Fuzzy Systems

Consider a class of interconnected nonlinear dynamical system of the following form:

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)) + \sum_{j=1, j\neq i}^{r} h_i(u_j(t)) \tag{1}$$

where $i=1,2,\cdots,r$, $x_i(t)\in R^n$ is the state vector, $u_i(t)\in R^m$ is the control input vector in the ith subsystem. The term, $h_i(u_j(t))$ represents interactions among the nonlinear subsystems. The central spirit of the TS fuzzy inference system is quantified by 'IF-THEN' rule base by virtue of the available qualitative knowledge from domain experts. More precisely, the k th rule of the ith sub-TS fuzzy system is formulated in the following form:

$$\begin{aligned} &\cdot\ IF\ z_{i1}(t)\ is\ about\ \Gamma_1^{ik} \text{and} \cdots \text{and} \\ &\quad z_{in}(t)\ is\ about\ \Gamma_n^{ik} \\ &THEN\ \dot{x}_i = A_{ik}x_i(t)B_{ik}u_i(t) \end{aligned} \tag{2}$$

where R_i^k denotes the kth fuzzy inference rule, $z_k(t)$ is the premise variable, Γ_l^{ik}, $k=1,\cdots,q$, $l=1,\cdots,n$, is the fuzzy set of the lth premise variable in the kth fuzzy inference rule.

Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics of this TS fuzzy system(2) is described by

$$\begin{aligned} \dot{x}_i(t) = & \sum_{j=1, j\neq i}^{r}\sum_{k=1}^{q}\alpha_k(z_i(t))(A_{ik}x_i(t) \\ & + B_{ik}u_k(t)) + h_{ij}(u_j(t)) \end{aligned} \tag{3}$$

Assumption 1 The term $h_{ij}(u_j(t))$, which represents the interconnections from ith TS fuzzy system to j th one, is assumed to satisfy the inequality

$$|h_{ij}(u_j(t))| \le \gamma_{ij} \tag{4}$$

3. Stabilizing Controller Design

A decentralized fuzzy-model-based control law is adopted for the stabilization of the interconnected TS fuzzy system of the form:

$$u_i(t) = \sum_{k=1}^{q}\alpha(z_i(t))K_k x_i(t) \tag{4}$$

The closed-loop system of (3) with (4) is describe by

$$\begin{aligned} &\dot{x}_i(t) \\ &= \sum_{j=1, j\neq i}^{r}\sum_{k=1}^{q}\sum_{l=1}^{q}\alpha_k(z_i(t))\alpha_l(z_i(t))(A_{ik}+B_{ik}K_l)x_i(t) \\ &+ h_{ij}(u_j(t)) \\ &= \sum_{j=1, j\neq i}^{r}\sum_{k=1}^{q}\alpha_k(z_i(t))(A_{ik}+B_{ik}K_k)x_i(t) \\ &+2\sum_{j=1, j\neq i}^{r}\sum_{k<l}^{q}\alpha_k(z_i(t))\alpha_l(z_i(t)) \\ &\times\left(\frac{A_{ik}+B_{ik}K_l+A_{il}+B_{il}K_k}{2}\right)x_i(t) \\ &+ h_{ij}(u_j(t)) \end{aligned} \tag{5}$$

Theorem 1 If there exist symmetric and positive definite matrices P_i, some matrices K_{ik}, $i,j=1,\cdots,r.,k$, $l=1,\cdots,q.$, such that the following LMIs are satisfied, then the interconnected TS fuzzy system (3) is globally stabilizable in the sense of Lyapunov, by employing the decentralized TS fuzzy-model-based state-feedback controller(4)

$$\text{a)}\ \begin{bmatrix} \begin{pmatrix} W_iA_{ik}^T + A_{ik}W_i \\ +M^{T_{ij}}B^{T_{ik}} + B_{ij}M_{ik} + Q_i \end{pmatrix} & * \\ W_i & -I \end{bmatrix} < 0$$

$k=1,\cdots,q-1.,$

$$\text{b)}\ \begin{bmatrix} \begin{pmatrix} W_iA_{ik}^T + A_{ik}W_i \\ +M^{T_{ij}}B^{T_{ik}} + B_{ik}M_{il} \\ +W_iA_{il}^T + A_{il}W_i \\ +M^{T_{il}}B^{T_{ik}} + B_{il}M_{ik} + Q_i \end{pmatrix} & * \\ W_i & -I \end{bmatrix} < 0$$

$k=1,\cdots,q-1.,\quad l=k+1,\cdots,q.$

where $M_{ij}=K_{ij}P_i^{-1}$, *and* $*$ *denotes the transposed elements in the symmetric positions.*

Proof : Consider the control Lyapunov functional candidate of the following form:

$$\begin{aligned} V(x(t)) &= \sum_{i=1}^{r}V(x_i(t)) \\ &= \sum_{i=1}^{r}x_i^T(t)P_ix_i(t) \end{aligned}$$

where P_i *is symmetric and positive definite. Clearly* $V(x(t))$ *is positive definite expect for* $x(t)=0$, *and radially unbounded in any neighborhood of zero equilibrium. The derivative of the given control Lyapunov functional candidate with respect to time along the trajectory of the closed-loop system (5) is given by*

$$\begin{aligned} &\dot{V}(x(t)) \\ &= \sum_{i=1}^{r}\dot{V}_i(x_i(t)) \\ &= \sum_{i=1}^{r}\sum_{j=1, j\neq i}^{r}\sum_{k=1}^{q}\sum_{l=1}^{q}\alpha_k(z_i(t))\alpha_l(z_i(t)) \\ &\times x(t)^T((A_{ik}+B_{ik}K_l)^TP_i \\ &+P_i(A_{ik}+B_{ik}K_l) \\ &+Q_i-Q_i)x_i(t) \\ &+h_{ij}(u_j(t))P_ix(t)+x(t)P_ih_{ij}(u_j(t)) \\ &+h_{ij}(u_j(t))^TP_ih_{ij}(u_j(t)) \\ &= \sum_{i=1}^{r}\sum_{j=1, j\neq i}^{r}\sum_{k=1}^{q}\alpha_k^2(z_i(t)) \end{aligned}$$

$$
\begin{aligned}
&\times x(t)^T((A_{ik}+B_{ik}K_k)^T P_i \\
&+P_i(A_{ik}+B_{ik}K_k)+Q_i-Q_i)x_i(t) \\
&+h_{ij}(u_j(t))P_i x(t)+x(t)P_i h_{ij}(u_j(t)) \\
&+h_{ij}(u_j(t))^T P_i h_{ij}(u_j(t)) \\
&+2\sum_{i=1}^{r}\sum_{\substack{j=1\\ j\neq i}}^{r}\sum_{k=1}^{a}\sum_{l=1}^{a}\alpha_k(z_i(t))\alpha_l(z_i(t)) \\
&\times x(t)^T\left(\left(\frac{A_{ik}+B_{ik}K_l+A_{il}+B_{il}K_k}{2}\right)^T P_i \right.\\
&+P_i\left(\frac{A_{ik}+B_{ik}K_l+A_{il}+B_{il}K_k}{2}\right) \\
&+Q_i-Q_i)x_i(t) \\
&+h_{ij}(u_j(t))P_i x(t)+x(t)P_i h_{ij}(u_j(t)) \\
&+h_{ij}(u_j(t))^T P_i h_{ij}(u_j(t))
\end{aligned}
$$

It suffice to force two sums in the last equality of the above time derivative of the control Lyapunov functional candidate. From the two LMIs, one can easily see

$$
\begin{aligned}
&V(x(t)) \\
&\le \sum_{i=1}^{r}\sum_{\substack{j=1\\ j\neq i}}^{r}(-x_i(t)^T Q_i x_i(t) \\
&+h_{ij}(t)^T h_{ij}(u_j(t)) \\
&+h_{ij}(u_j(t))P_i h_{ij}(u_j(t)) \\
&\le \sum_{i=1}^{r}\sum_{\substack{j=1\\ j\neq i}}^{r}(-\underline{\lambda}_i(Q_i)\|x_i(t)\| \\
&+\gamma_{ij}^2+\overline{\lambda}_i(P_i)\gamma_{ij}^2)
\end{aligned}
$$

where $\overline{\lambda}_i$ and $\underline{\lambda}_i$ denote the maximum and minimum eigenvalue of the matrix. Define a set

$$\Pi_i=\left\{x_i(t)\in R^n \,|\, \|x_i(t)\|_2 \le \frac{\gamma_{ij}^2+\overline{\lambda}_i(P_i)\gamma_{ij}^2}{\underline{\lambda}_i(Q_i)}\right\}$$

which is obvious compact. One can easily agree that $\dot{V}(x(t))$ is negative outside of $\cap_{i=1}^{r}\Pi_i$ as long as $x_i(t)$ is outside of the compact sets. According to the standard Lyapunov stability criterion, we conclude that the states converge to the compact set. This implies the stability of the controlled system in the sense of Lyapunov.

4. Experimental Helicopter System and Its Fuzzy Modelling

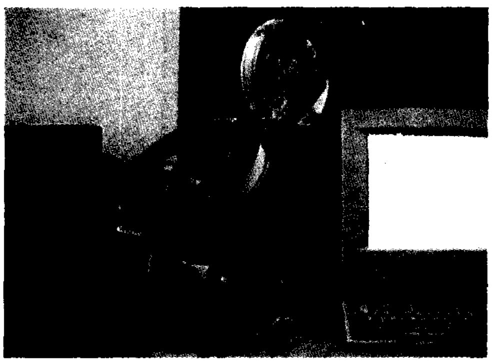

Fig. 1. Experimental 2 dimensional helicopter system.

Consider the following nonlinear dynamic equations

$$
\begin{aligned}
J_p\ddot{p}(t)+B_p\dot{p}(t)&=R_pF_p(V_{p(t)})-M_e g(h\sin(p(t)) \\
&\quad +R_c\cos(p(t)))+G_p(\tau_y(V_{y(t)}),p(t)) \\
J_y\ddot{y}(t)+B_y\dot{y}(t)&=R_yF_y(V_{y(t)})+G_y(\tau_p(V_{p(t)}))
\end{aligned}
\tag{6}
$$

which describe the dynamic behavior for a 2- dimensional helicopter system. The experimental system is shown in Fig. 1. This nonlinear system can be viewed as smaller isolated subsystems and their interconnections, thus the decentralized control technique can successfully applied. Let the state vectors and input vectors for subsystems be

$$x_1(t)=\left[p(t)\ \ \dot{p}(t)\ \ 0.2\int_0^t p(\tau)d\tau\right]$$

$$x_2(t)=\left[y(t)\ \ \dot{y}(t)\ \ 0.2\int_0^t y(\tau)d\tau\right]^T$$

and $u_1(t)=[V_{p(t)}]$, $u_2(t)=[V_y(t)]$, then (6) can be represented the interconnected systems

$$\dot{x}_i(t)=f_i(x_i(t))+g_i(x_i(t))u_i(t)+h_i(u_j(t)) \tag{7}$$

where $i,j=1,2.,i\neq j$.

$$f_1(x_1(t))=\begin{bmatrix} x_{12}(t) \\ -\dfrac{B_p}{J_p}x_{12}(t)-\dfrac{M_e g(h\sin(x_{11}(t))+R_c\cos(x_{11}(t)))}{J_p} \\ 0.2x_{11}(t)\end{bmatrix}$$

$$g_1(x_1(t))=\begin{bmatrix}0\\ \dfrac{R_pK_{pp}g}{J_p}\\ 0\end{bmatrix},$$

$$h_1(u_2(t))=\begin{bmatrix}0\\ -\dfrac{K_{py}}{J_p}u_2(t)\\ 0\end{bmatrix},$$

$$f_2(x_2)=\begin{bmatrix}x_{22}(t)\\ -\dfrac{B_y}{J_y}x_{22}(t)\\ 0.2x_{21}(t)\end{bmatrix},$$

$$g_2(x_2(t))=\begin{bmatrix}0\\ \dfrac{R_yK_{yy}g}{J_y}\\ 0\end{bmatrix},$$

$$h_2(u_1(t))=\begin{bmatrix}0\\ -\dfrac{K_{yp}}{J_y}u_1(t)\\ 0\end{bmatrix}.$$

The composite system has one nonlinear term in subsystem

$$1-\frac{M_e g(h\sin(x_{11}(t))-R_c\cos(x_{11}(t)))}{J_p}.$$

Thus Subsystem 1 can be represented as a TS fuzzy system and Subsystem 2 as a linear system.

Subsystem 1

Plant Rules:

R1 : IF $x_{11}(t)$ is about Γ_1,

$$\text{THEN } \dot{x}_1(t)=A_{11}x_1(t)+B_{11}u_1(t)+d_{11}+h_1(u_2(t))$$

R2 : IF $x_{11}(t)$ is about Γ_2,

THEN $\dot{x}_1(t) = A_{12}x_1(t) + B_{12}u_1(t) + d_{12} + h_1(u_2(t))$

$$A_{11} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{Mga}{J_p} & -\frac{B_p}{J_p} & 0 \\ 0.2 & 0 & 0 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{Mgb}{J_p} & -\frac{B_p}{J_p} & 0 \\ 0.2 & 0 & 0 \end{bmatrix},$$

$$B_{11} = B_{12} = \begin{bmatrix} 0 \\ \frac{R_p K_{ppg}}{J_p} \\ 0 \end{bmatrix}, \quad d_{11} = \begin{bmatrix} 0 \\ -\frac{Mga\beta}{J_p} \\ 0 \end{bmatrix}$$

$$d_{12} = \begin{bmatrix} 0 \\ -\frac{Mgb\beta}{J_p} \\ 0 \end{bmatrix}, \qquad A_{21} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B_y}{J_y} & 0 \\ 0.2 & 0 & 0 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} 0 \\ \frac{R_y K_{yyg}}{J_y} \\ 0 \end{bmatrix}.$$

and the membership functions for Subsystem 1 are

$$\Gamma_1(x_1(t)) = \frac{a\sin(x_{11}(t)+\beta) - b(x_{11}(t)+\beta)}{(a-b)(x_{11}(t)+\beta)}$$

$$\Gamma_2(x_1(t)) = \frac{a(x_{11}(t)+\beta) - a\sin(x_{11}(t)+\beta)}{(a-b)(x_{11}(t)+\beta)}$$

5. Simulation and Experiment

This section shows a design procedure of the proposed decentralized controller and its experimental validation. In order to avoid the failure of the DC motors, the input voltage limitations is applied, i.e.,

$$|V_p(t)| \le 2.6, \text{ and } |V_y(t)| \le 1.$$

Based on Theorem 1, we obtain the following control gain matrices.

$$K_{11} = [-1.8748 \ \ -2.3129 \ \ -11.0607],$$
$$K_{12} = [-6.2136 \ \ -3.5934 \ \ -16.4346],$$
$$K_{21} = [-5.8513 \ \ -3.3977 \ \ -6.1159].$$

The initial value of the system is set to $x_1(0) = [0\ 0\ 0]^T$, $x_2(0) = [0\ 0]^T$, and the simulation time is 20 sec.. Figure 2 shows the computer simulation of the 2-dimensional helicopter system. The pitch and yaw angles are well guided to zero immediately. The simulation result shows the effectiveness of the proposed control technique. The experiment of the real apparatus is also carried out. The control program is made by using Microsoft Visual Studio. The experiment time is also 20 sec., and the result is shown in Fig. 3. As is expected from the simulation result, the pitch and yaw angles are directly go to origin.

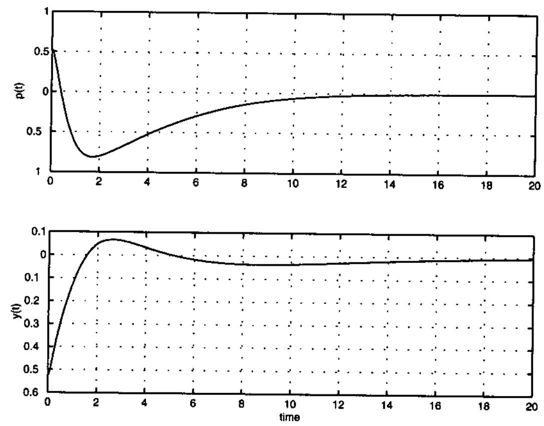

Fig. 2. Simulation result of the control of the 2 dimensional helicopter system

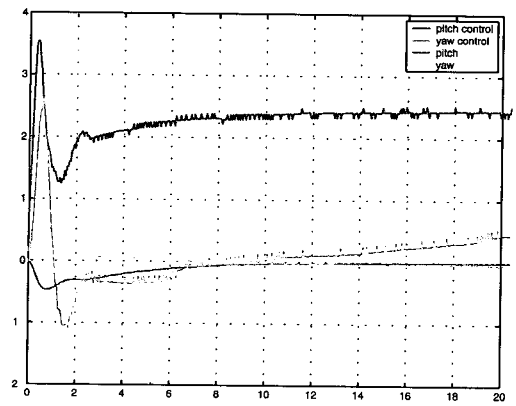

Fig. 3. Experiment result of the control of the 2 dimensional helicopter system.

6. Conclusion

This paper has discussed a decentralized control method for an interconnected TS fuzzy systems. The proposed control strategy is suitable for interconnected large-scale nonlinear systems. The stabilizing controller design condition is derived based on the Lyapunov stability criterion and formulated in terms of linear matrix inequalities (LMIs). The experimental results have successfully validated the theoretical discussion.

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