DEVELOPMENT OF 2D DAM BREAK FLOW ANALYSIS MODEL USING FRACTIONAL STEP METHOD

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Abstract: A numerical model for the solution of two-dimensional dam break problems using fractional step method is developed on unstructured grid. The model is based on second-order Weighted Averaged Flux(WAF) scheme with HLLC approximate Riemann solver. To control the nonphysical oscillations associated with second-order accuracy, TVD scheme with SUPERBEE limiter is used. The developed model is verified by comparing the computational solutions with analytic solutions in idealized test cases. Very good agreements have been achieved in the verifications.

Key Words: fractional step method, dam break flow, TVD scheme, Unstructured grid

1. INTRODUCTION

In dam break problems flow would change abruptly in space and time. The conservative form of the governing equations are more appropriate for the problems containing discontinuities in the solution. The problems which involve discontinuities in the solution are known as Riemann problems and numerical schemes essentially solve linearized Riemann problems. To achieve higher order accuracy and higher shock resolution, sophisticated techniques are required to obtain physically reasonable solutions from the Riemann solver. This is achieved by limiting the flux or the dependent variables so that oscillations that are associated with higher order schemes are eliminated.

In this study, the fractional step method is used to solve the shallow water equations on unstructured grid. This method reduces the two-dimensional problem into a sequence of two one-dimensional problems by decomposing the system of governing equation into a product of several simpler operators. Thus, in each fractional step a simple one-dimensional system of equation can be solved, so the computational effort is decreased. Second advantage of this method is that the limitation on time step size is less severe than that for a common two- dimensional explicit scheme. Additionally, there is close relationship between fractional step method and Finite Volume Method(FVM).

2. GOVERNING EQUATIONS

2.1 Shallow Water Equations

The two dimensional shallow water equations in conservative form are given by

$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+S=0 \qquad (1)$$

In equation (1), the vector of conserved variables **U**, the flux vector **E** and **F** in the x- and y-direction and source term **S** can be written as, respectively,

$$U=\begin{bmatrix} h \\ hu \\ hv \end{bmatrix}, \qquad S=\begin{bmatrix} 0 \\ gh(S_{fx}-S_{ox}) \\ gh(S_{fy}-S_{oy}) \end{bmatrix}$$

$$E=\begin{bmatrix} hu \\ hu^2+\frac{1}{2}gh^2 \\ huv \end{bmatrix}, \quad F=\begin{bmatrix} hv \\ huv \\ hv^2+\frac{1}{2}gh^2 \end{bmatrix}$$

where g is the acceleration due to gravity, $S_{fx,y}$ is friction slope, $S_{ox,y}$ is bottom slope, h is water depth, u and v are the velocity in the $x-$ and $y-$direction, respectively.

2.2 Fractional Step Method

It is very difficult to use two-dimensional Riemann solvers in the two-dimensional problem. Even if such solvers were available, the resulting schemes are likely to be too complicated for common use. On the other hand, a number of efficient and robust one-dimensional approximate Riemann solvers have been proposed. By using fractional step method, these can be used to solve the two- dimensional shallow water equations easily.

For homogeneous problems, the fractional step method involves replacing equation (1) by the sequence of two one-dimensional problems. That is,

$$\left.\begin{array}{ll} PDE & : \frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}=0 \\ IC & : U^{n} \end{array}\right\} \overset{\Delta t}{\Leftrightarrow} U_{*}^{n+1/2} \qquad (2)$$

and

$$\left.\begin{array}{ll} PDE & : \frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}=0 \\ IC & : U_{*}^{n+1/2} \end{array}\right\} \overset{\Delta t}{\Leftrightarrow} U_{*}^{n+1} \qquad (3)$$

If we denote $x^{\Delta t}$ and $y^{\Delta t}$, the solution operators for initial value problems of equation (2) and (3) then the approximate solution can be written as

$$U^{n+1}=y^{\Delta t}x^{\Delta t}(U^{n}) \qquad (4)$$

or

$$U^{n+1}=x^{\Delta t}y^{\Delta t}(U^{n}) \qquad (5)$$

The solution obtained by equation (4) and (5) has only first order accuracy in time. The solution that has second-order accuracy can be obtained by using

$$U^{n+1}=x^{\Delta t/2}y^{\Delta t}x^{\Delta t/2}(U^{n}) \qquad (6)$$

or

$$U^{n+1}=y^{\Delta t/2}x^{\Delta t}y^{\Delta t/2}(U^{n}) \qquad (7)$$

For inhomogeneous problems, the source term can be solved with fractional step method using ordinary differential solver

$$\left.\begin{array}{ll} ODE & : \frac{dU}{dt}=-S \\ IC & : U_{*}^{n+1} \end{array}\right\} \overset{\Delta t}{\Leftrightarrow} U^{n+1} \qquad (8)$$

2.3 Finite Volume Method

The FVM is an approach for dealing with general unstructured grid. The major advantages of FVM; ① the ability to use flexible grid, such as triangles or quadrilaterals which suit problems with complex geometries, ② use of an integral conservation law such that the solution may be smooth or discontinuous, and ③ close relationship between fractional step method and FVM.

Integrating the equation (1) over an arbitrary cell, the basic equation of the FVM obtained using the divergence theorem can be written as,

$$\frac{\partial}{\partial t}\int_A U dA + \oint_S G \cdot n dS = 0 \tag{9}$$

where G is the flux tensor, A and S are the area and boundary of control volume L respectively, and n is the unit outward vector normal to the boundary. Assuming the x-direction as the reference direction and using the rotational invariance of the flux becomes

$$G \cdot n = T_S^{-1} G(T_S U) \tag{10}$$

where T_S, the transformation matrix, namely

$$T_S(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \tag{11}$$

Using the equation (10), the equation (11) becomes

$$\frac{dU}{dt} + \frac{1}{|V|}\sum_{s=1}^{N}\int_{A_s}^{A_{s+1}} T_s^{-1} G(T_s U) dA = 0 \tag{12}$$

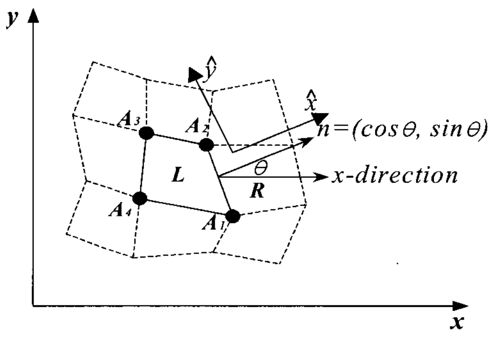


Fig. 1. Quadrilateral finite volume in 2D unstructured domain (Zhao, 1996)

By employing the new conserved values $\hat{U} = T_s U_{\wedge}$ which are aligned with the rotated axises $\hat{x}$ and $\hat{y}$ in Fig. 1, the two-dimensional problems can be dealt with as a sequence of two one-dimensional problems, that is, fractional step method, in transformed ($\hat{x}$, $\hat{y}$) domain.

3. TVD VERSION OF WAF METHOD

3.1 HLLC Approximate Riemann Solver

The HLLC approximate Riemann solver is given as follows

$$U(x,t) = \begin{cases} U_L & for \quad 0 \le S_* \\ U_L^* & for \quad S_L \le 0 \le S_* \\ U_R^* & for \quad S_* \le 0 \le S_R \\ U_R & for \quad S_R \le 0 \end{cases} \tag{13}$$

In order to determine the numerical fluxes in HLLC Riemann solvers we need to estimate the wave speed in equation (10). The estimates of the wave speed, in this study, are given as

$$\begin{aligned} S_L &= \min(u_L - \sqrt{gh_L}, u_* - \sqrt{gh_*}) \\ S_* &= u_* = \frac{u_L + u_R}{2} + \sqrt{gh_L} - \sqrt{gh_R} \\ S_R &= \max(u_L + \sqrt{gh_L}, u_* + \sqrt{gh_*}) \end{aligned} \tag{14}$$

In the view (13) the HLLC flux can be written as

$$E_{i+1/2}^{HLLC} = \begin{cases} E_L & for \ 0 \le S_* \\ E_L^* = E_L + S_L(U_L^* - U_L) & for \ S_L \le 0 \le S_* \\ E_R^* = E_R + S_R(U_R - U_R^*) & for \ S_* \le 0 \le S_R \\ E_R & for \ S_R \le 0 \end{cases} \tag{15}$$

where i+1/2 means the intercell boundary between L and R *in* Fig. 1.

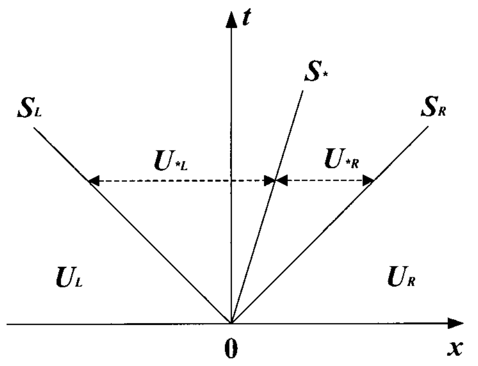


Fig. 2. HLLC approximate Riemann solver structure(Toro, 1999)

3.2 TVD Version of WAF Second Order Scheme

The second-order accurate WAF scheme is given as

$$E_{i+1/2}=\frac{1}{2}(E_i+E_{i+1})-\frac{1}{2}\sum_{k=1}^{N}C_k\Delta E_{i+1/2}^{k} \quad (16)$$

where $E_{i+1/2}^{k}=E(U_{i+1/2}^{k})$ can be replaced by equation (15), C_k is the courant number for wave k of speed S_k and $\Delta E_{i+1/2}^{k}$ is given as

$$\Delta E_{i+1/2}^{k}=E_{i+1/2}^{k+1}-E_{i+1/2}^{k} \quad (17)$$

In order to control the nonphysical oscillation associated with second-order accuracy, TVD scheme is employed. The TVD version of WAF scheme is given as

$$E_{i+1/2}=\frac{1}{2}(E_i+E_{i+1})+\frac{1}{2}\sum_{k=1}^{N}sign(c_k)\psi_{i+1/2}^{k}\Delta E_{1+1/2}^{k} \quad (18)$$

where $\psi_{i+1/2}^{k}$ is a WAF limiter function. In this study the SUPERBEE limiter is used.

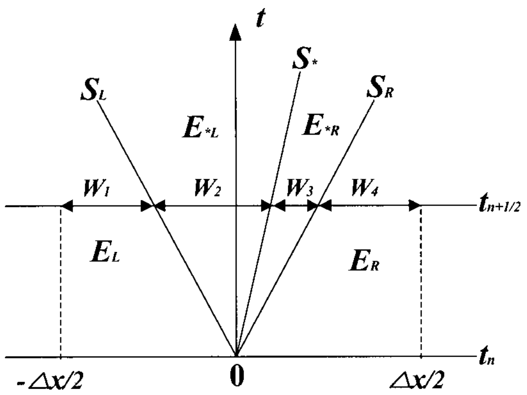


Fig. 3. Numerical flux and weights in WAF method

$$\psi(r,\ c)=\begin{cases} 1 & for \quad 0\le r \\ 1-2(1-|c|)r & for \quad 0\le r\le 1/2 \\ |c| & for \quad 1/2\le 0\le 1 \\ 1-(1-|c|)r & for \quad 1\le r\le 2 \\ 2|c|-1 & for \quad 2\le r \end{cases} \quad (19)$$

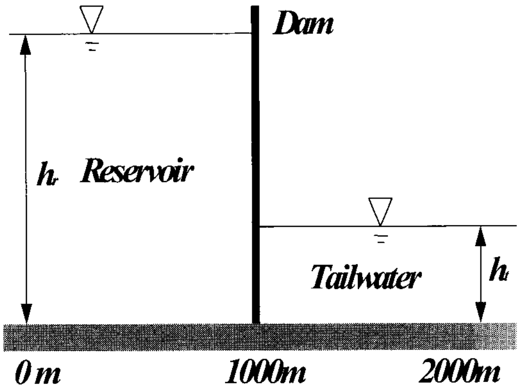
where r is the ratio of the upwind change to the local change. The detail derivation procedures of the equation (16) and (18) are written on Lee and Cho(2001).

4. NUMERICAL APPLICATIONS AND RESULTS

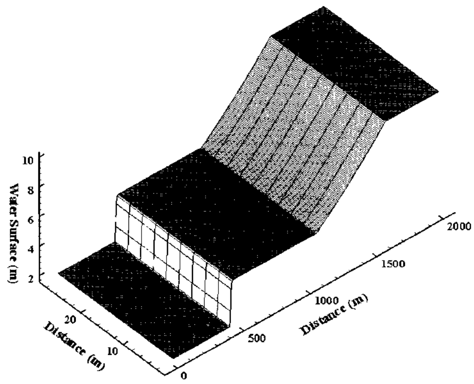
4.1 Idealized Dam Break Problems

To verify the accuracy of developed model, two idealized dam break problems illustrated in Fig. 4(a) are solved on 320×9 cells. The water depth h_r and h_t represent the initial water depth in reservoir and tailwater, respectively.

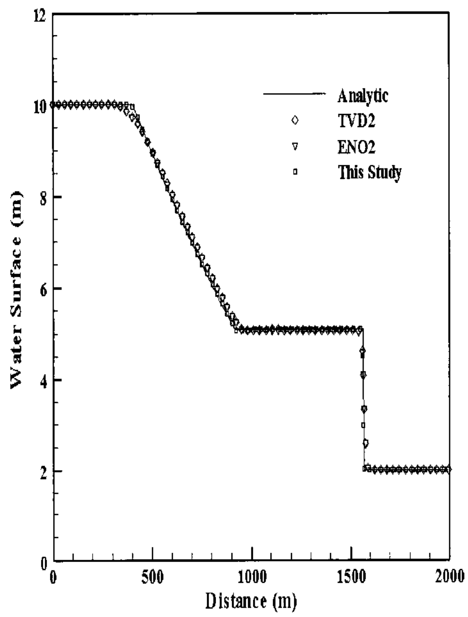
The Fig. 4(c) and (d) describe the solutions obtained for 60 second after dam break. The analytic solutions are given by Tan(1992). The solutions of ENO2 and TVD2 are obtained by using the methods proposed by Kim(2001). As the Fig. 4 (c) and (d), it may be concluded that



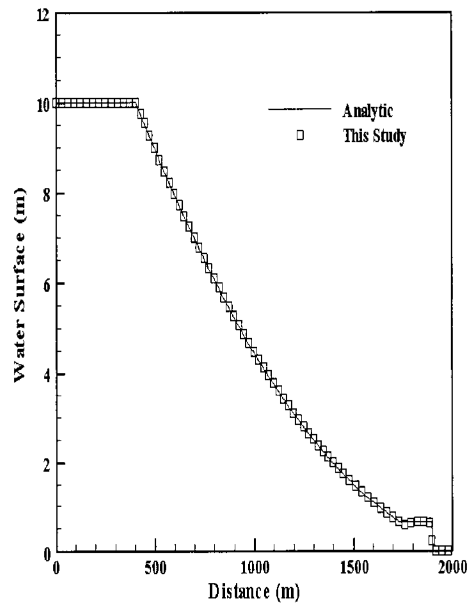
(a) Schematic of problem



(b) 3D view of water surface($h_t / h_r = 0.2$)

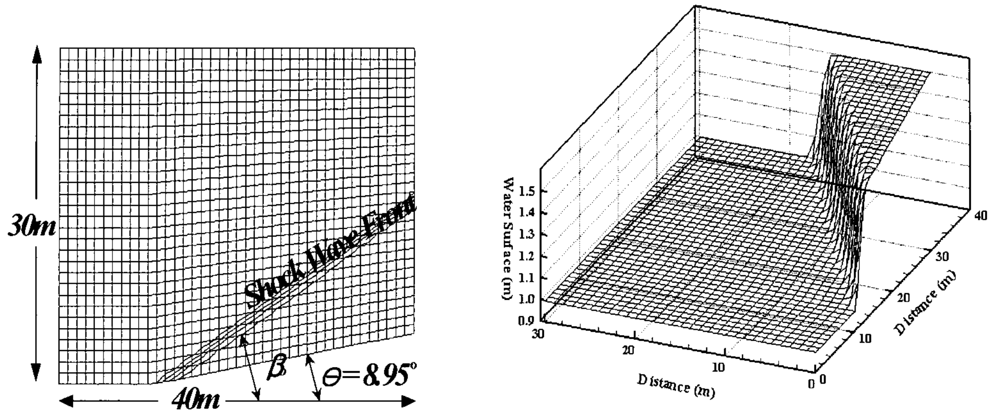


(c) Side view of water surface ($h_t / h_r = 0.2$)



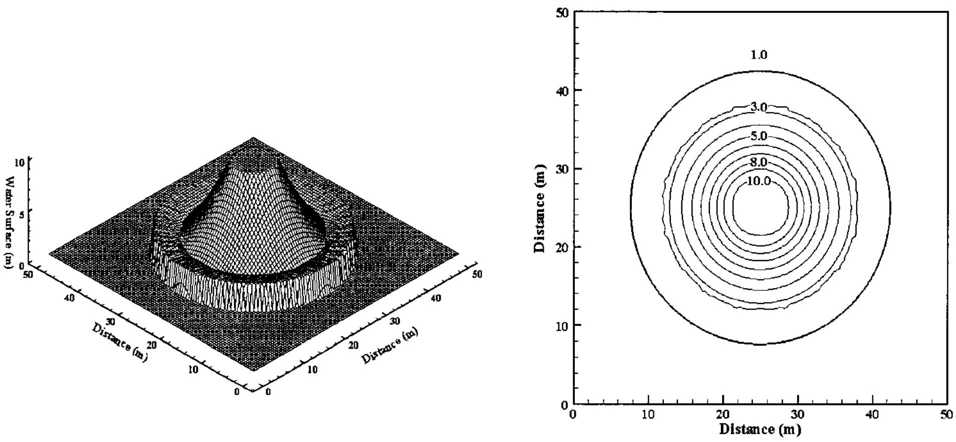
(d) Side view of water surface ($h_t / h_r = 0.001$)

Fig. 4. Idealized Dam Break Problem



(a) Computational grid & contours (b) 3D view of water surface

Fig. 5. Oblique hydraulic jump



(a) 3D view of water surface (b) Contour of water surface(m)

Fig. 6. Circular dam break problem

the computational solutions are very good agreement with the analytic solutions.

4.2 Oblique Hydraulic Jump

Let the alignment of the vertical sidewall of a channel change inward into flow by angle θ, as shown in Fig. 5(a). At the inlet, the water depth is 1.0m and the inlet velocity is 8.75m/s. The vertical sidewall length is 40m, the inlet width is 30m and inward wall angle θ is 8.95°. The flow is supercritical at the inlet and oblique hydraulic jump occurs at the transition from supercritical to subcritical flows. The Fig. 5 show the computed results that $\beta_0 \approx 30^\circ$ and the water depths at the subcritical region are between 1.498m and 1.507m on 40×30 computational domain. The analytical solution predicts water depth at subcritical region is 1.5m and the shock wave front angle $\beta_0 = 30^\circ$ ((Masayuki, 2000). It may also be concluded the accuracy of this developed model is excellent.

4.3 Circular Dam Break Problem

Another dam break problem is tested using 100×100 square cells. In this case, the initial condition of the cylinder radius is 10.0m and its water depth is 10.0m. The outside region water depth of the circular dam is 1.0m and the velocities u and v are 0.0m/s, respectively. The Fig. 6 describes the computational solutions obtained for 0.69 second after dam break.

5. CONCLUSIONS

A numerical model for the solution of two-dimensional dam break problems is developed on unstructured grid. Using fractional step method, two-dimensional shallow water equations are treated as one-dimensional problems. Thus, it is possible to simulate computational hydraulic problems with higher computational efficiency. The one-dimensional problems are solved using a second-order Weighted Averaged Flux scheme with HLLC approximate Riemann solver. The numerical oscillations which are common with second-order numerical scheme are controlled by exploiting SUPERBEE limiter. Some idealized problems are solved using the model and very accurate and stable solutions are obtained. It can be concluded as an efficient implement for the computation of dam break problems, especially those problems concerning discontinuities, subcritical and supercritical flows and complex domain.

REFERENCES

Zoppou, C. and Roberts, S. (2000). "Numerical solution of two-dimensional unsteady dam break." *Applied Mathematical Modelling*, Vol. 24, pp. 457~475.

Zhao, D.H. Shen, H.W. Lai, J.S. and Tabios, G.T. Ⅲ (1996). "Approximate Riemann solvers in FVM for 2d hydraulic shock wave modeling." *Journal of Hydraulic Engineering*, Vol. 122, No. 12, pp. 692~702.

Eleuterio F. Toro (1999). *Riemann solvers and numerical methods for fluid dynamics*. Springer

Kim, W. and Han, K.Y. (2001). "Numerical analysis of transcritical flow in open channels using high-resolution scheme I : model development." *Journal of Korea Water Resources Association*, Vol 34, No. 1, pp. 45 ~55.

Lee, J.W. Cho, Y.S. Yoon, K.S. and Yoon, T.H (2001). "Development of TVD numerical models: I .linear advection equation." *Journal of Korea Water Resources Association*, Vol. 34, No. 2, pp. 177~186.

Lee, J.W. and Cho, Y.S. (2001). "Development of TVD numerical models: II . shallow water equations." *Journal of Korea Water Resources Association*, Vol. 34, No. 2, pp. 187~195.

Fujihara, Masayuki and Borthwick, G.L. (2000). "Godunov-type solution of curvilinear shallow-water equations." *Journal of Hydraulic Engineering*. Vol. 126. No. 11. pp. 827~836.

Weiyan,, Tan (1992). *Shallow water hydrodynamics*. Elsevier.

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