FRONT PROPAGATION RATE OF DENSITY CURRENTS: DENSIMETRIC FROUDE NUMBER VERSUS DIMENSIONLESS FRONT VELOCITY

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Abstract: In general, two dimensionless numbers are used in predicting the front propagation rate of density currents: the densimetric Froude number and the dimensionless front velocity. The former expresses the front speed in terms of the characteristic length and reduced gravitational acceleration. Previous papers report that the range of this dimensionless number is wide. The other is the dimensionless front velocity, which is a function of the buoyancy flux per unit width. This paper presents the state of the art review of the dimensionless numbers for the front propagation rate of density currents. Values of the densimetric Froude number are found to be consistent when the proper characteristic length is used for normalization. Then, the densimetric Froude number and the dimensionless front velocity are compared by using the experimental data of density currents over a horizontal surface.

Key Words: density current, densimetric Froude number, dimensionless front velocity, front propagation rate

1. INTRODUCTION

A density current is a stratified flow caused by density differences between fluids. The motion of a density current arises from the reduced gravitational acceleration associated with positive or negative buoyancy, which comes from density differences due to dissolved chemicals, suspended materials, or temperature gradients. For this reason, the density current is also known as the gravity current (Simpson, 1982).

Density currents frequently occur in both natural and man-made environments. The book by Simpson (1987) provides an excellent collection of examples. In natural situations, stream intrusions into reservoirs or lakes are important examples of such flows. Turbidity currents, sediment-laden underflows, are known to be the architects of many submarine canyons in the oceans. Other examples include thunderstorm outflows in the atmosphere and snow avalanches Density currents in man-made situations are the ones by water jet injection and hydrostatic jet, which are generated on purpose to dredge the bed and to prevent sediment deposition, respectively. Density currents can also be used in the disposal of mine tailings (Normark and Dickson, 1976) and CO_2 gas (Drange *et al.*, 1993). Fluid flows of a different density become important as concerns on the environment increase.

Density currents can be divided into two types depending on the inflow condition, i.e., the continuous current and the discontinuous current. The former can be seen as a plume followed by a continuous flow, while the latter is a cloud generated by a finite volume from a source. The continuous density current takes place when a certain rate of denser fluid is discharged into a still ambient water, where it propagates downstream. An example is a plume produced by a sediment- laden river entering a lake. The buoyancy flux of the plume may be constant (density plume) or vary with the distance from the source of the flow (suspension plume). On the other hand, the discontinuous turbidity current is generated when a certain amount of denser fluid is discharged into water. The current is fed by a finite amount of dense fluid as an instantaneous source. An example of the discontinuous current is a cloud arising from a single slump such as powder snow avalanches. The buoyancy flux of density clouds is constant, while that of suspension cloud is varying with the distance from the source of the flow.

The density current shows two distinct flow regimes: one is a jet behavior close to the inlet where the inertia force is stronger than the buoyancy force. The other is a plume behavior far from the inlet. Therein the buoyancy is the only force which makes the current flow in the downstream direction. This is reflected in the travel distance versus time curve. That is, for the density current propagating on a slope, the curve consists of two parts generally: an initial non-linear part followed by a linear part. That is, the density current propagates in the longitudinal direction quite rapidly at the initial stage when the momentum dominates, and then it propagates at an almost constant rate after the balance between the buoyancy and the resistance force is attained.

In the practical point of view, one of important aspects in density current studies is the determination of the front propagation rate under different inlet conditions. This is why many previous efforts were devoted to obtaining the front propagation rate through analytical or experimental approaches. The front propagation rates are generally expressed in terms of flow variables at the inlet such as height, flow rate, and buoyancy flux, which constitute dimensionless parameters like the densimetric Froude number and the dimensionless front velocity. If we assume that the densimetric Froude number (Fr_d) is constant, then the front velocity (u_f) is given in terms of the current thickness (h) and the reduced gravitational acceleration (g'), i. e., $u_f = Fr_d (g'h)^{1/2}$. Alternatively, the dimensionless front velocity ($\overline{u_f}$), defined by the front velocity divided by the buoyancy flux per unit width (B_f), can be used. Britter and Linden (1980) showed experimentally and theoretically that the dimensionless front velocity is nearly constant. By using this, the front velocity is ex pressed as $u_f = \overline{u_f}(B_f)^{1/3}$. These relationships can be directly applied to density current routing such as in Alavian and Ostrowski (1992), or they can be used as the front boundary conditions in the numerical computation (Bonnecaze et al., 1993). However, one may be confused to find that these dimensionless numbers vary in a very wide range. For example, Droegmeier and Wilhelmson (1987) listed a table of the densimetric Froude numbers ranging between 0.7 and 1.4. This is thought to come from inconsistent use of characteristic scales, which motivates the current study.

The objectives of the present paper are to review various relationships for the front propagation rate of density currents and to establish a consistency in the data in the literature. Then, a better approach between the densimetric Froude number and the dimensionless front velocity will be proposed.

2. DENSIMETRIC FROUDE NUMBER

The relationship describing the front propagation rate of the density current has been sought in terms of its characteristic length and the reduced gravitational acceleration. The length scale can be either head or body height. This relationship, a constant value of the densimetric Froude number, was first referred to as "the specific law of saline fronts" by Keulegan (1957). Herein, two different densimetric Froude numbers are defined, i.e.,

$$Fr_1 = \frac{u_f}{\sqrt{g'h}} \tag{1}$$

$$Fr_2 = \frac{u_f}{\sqrt{g'h_f}} \tag{2}$$

which use the current (or body) height (h) behind the front and the front height (h_f) as their characteristic lengths, respectively.

2.1 Theoretical Studies

Von Karman (1940) applied Bernoulli's theorem to two points along the interface of the two fluids, i.e., one is the stagnation point at the tip of the current and the other is a point at the interface far downstream. It is assumed that the front intersects the bottom boundary at an angle of 60° as shown in Fig 1. Von Karman obtained the front propagation rate (u_f) of a density current as

$$u_f = \sqrt{2g'h} \tag{3}$$

which is identical to $Fr_1 = \sqrt{2}$. However, this relationship comes from an unreasonable analysis because von Karman applied Bernoulli's theorem along the interface, an invalid procedure in a dissipating, rotational flow. In fact, the foremost part of the front consists of a complicated shifting pattern of lobes and clefts, within an intermittent series of billows. Later, however, the above result happened to coincide with the result from a different approach by Benjamin (1968).

Benjamin (1968) realized the mistake in von Karman's (1940) analysis, and applied the

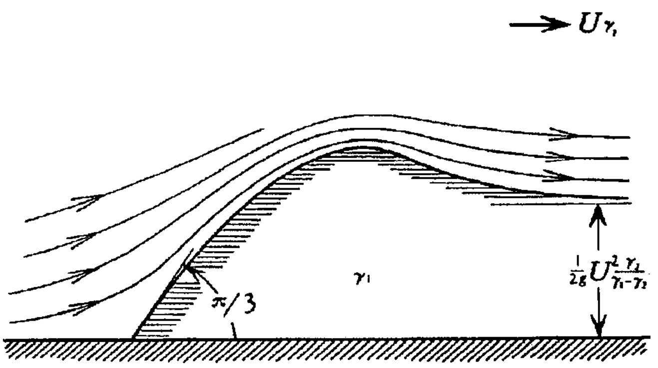

Fig 1. Density Current Creeping on a Horizontal Surface (Von Karman, 1940)

flow-force balance relationship to an energy-conserving cavity flow. He expressed the densimetric Froude number for a propagating gravity current as

$$Fr_1 = \sqrt{\frac{(1-\eta)(2-\eta)}{(1+\eta)}} \quad (4)$$

where η is a fractional depth (=h/H), and H denotes the total depth. Fig. 2 shows the densimetric Froude number Fr_1 by eq. (4) as a function of η for energy-conserving density flows. In the figure, the densimetric Froude number (Fr_1) is $1/\sqrt{2}$ for a density current in a shallow ambient layer of $\eta = 1/2$. In this case, Benjamin (1968) proved analytically that a uniform gravity current progresses steadily without losing energy. For a density current flowing in a deep ambient fluid ($\eta \cong 0$), Benjamin (1968) showed that the densimetric Froude number (Fr_1) becomes $\sqrt{2}$, which is exactly the same as the value obtained by von Karman, eq. (3). Considering a nose lifting of $0.35h_f$ from the bottom due to the friction, which is from the universal shape for the density current head in Middleton (1966), $Fr_1 = 1.14$ is obtained for a deep density current.

Kao (1977) derived the front propagation rate of density currents in generalized situations of a stratified inflow into a stratified ambient fluid. If $\rho(z)$ and $\rho_a(z)$ denote the densities of the dense and ambient fluids, respectively, then the propagation rate of the density current is given by

$$u_f = \sqrt{2\frac{\Delta\rho}{\rho_0}gh + (\beta_a - \beta)gh^2} \quad (5)$$

where $\rho_0 = \rho_a(0)$, $\Delta\rho = \rho(0) - \rho_0$, $\beta_a = -1/\rho_0\,(d\rho_a/dz)$, and $\beta = -1/\rho_0\,(d\rho/dz)$. It is obvious that the densimetric Froude number (Fr_1) from the above formula becomes $\sqrt{2}$ if both of the density gradients, β_a and β, become zero. Various relationships can be generated from eq. (5). For example, the propagation rate of a stratified density current in an unstratified ambient fluid ($\beta_a = 0, \beta \neq 0$) or the propagation rate of an unstratified density current in a stratified ambient fluid ($\beta_a \neq 0, \beta = 0$) can be obtained.

Hay (1983) extended the theoretical analysis by Benjamin (1968) to a density current on an inclined bottom and obtained the following expression for the front velocity:

$$u_f = \sqrt{g'h_h}\,(Fr_2 + \frac{L_x}{h_f}\sin\theta) \quad (6)$$

where L_X is the distance from the nose to the point in the dense layer at which the pressure becomes hydrostatic, and θ is the channel slope angle. Eq. (6) describes the front propagation rate of the density current in terms of the channel slope, and it recovers Benjamin's result if the channel slope becomes zero. However, the use of eq. (6) is limited by the unknown parameter L_X, a flow property which can hardly be determined. The above formula is, in any case, not applicable for steep slopes larger than 5° to 10° because water entrainment is not considered.

Buhler *et al.* (1991) modified Benjamin's (1968) relationship by incorporating non- uniform velocity distribution into the derivation. Buhler et al.'s densimetric Froude number for the density current advancing over a flat surface is given by

$$Fr_1 = \sqrt{\frac{(1-\eta)(2-\eta)}{(2\beta_p - 1 + \eta}} \quad (7)$$

where β_p is a modified momentum correction factor of the intruding layer. Buhler et al. (1991)

obtained good agreement between the above relationship and the experimental results by Britter and Simpson (1978) and Simpson and Britter (1979) with $\beta_p = 1.35$.

It should be emphasized in the analytical analyses given above, severe mixing between the two fluids at the front has been ignored. Also not included is the situation in which the fluid in front of the density current is overrun by the density current and then mixed within it. That is, the theoretical solutions resulted from severe simplifications of the complicated dynamics at the head.

2.2 Experimental Studies

Front propagation rates of density currents were investigated experimentally by Keulegan (1957 and 1958). Lock exchange saline currents (Keulegan, 1957) and saline water intrusions from a tideless sea into a channel (Keulegan, 1958) were generated in a horizontal flume by opening a lock or a barrier containing saline dense fluids. In both cases, Keulegan found that the propagation rate is related with the height of the head and the reduced gravitational acceleration. In the study of the lock exchange currents, he obtained expressions for the densimetric Froude number. This value can be expressed as a function of the front Reynolds number Re_f (= $u_f h_f / \nu$):

$$Fr_2 = \frac{u_f}{\sqrt{g' h_f}} = 0.123 \mathrm{Re}_f^{1/4} \quad \text{for} \quad \mathrm{Re}_f < 860 \qquad (8)$$

$$Fr_2 = \frac{u_f}{\sqrt{g' h_f}} = 0.714 \quad \text{for} \quad \mathrm{Re}_f > 860 \qquad (9)$$

Similar relationships for underflows resulting from the intrusion of sea water into a channel were also found:

$$Fr_2 = \frac{u_f}{\sqrt{g' h_f}} = 0.113 \mathrm{Re}_f^{1/4} \quad \text{for} \quad \mathrm{Re}_f < 1200 \qquad (10)$$

$$Fr_2 = \frac{u_f}{\sqrt{g' h_f}} = 0.705 \quad \text{for} \quad \mathrm{Re}_f > 1200 \qquad (11)$$

Keulegan (1957 and 1958) observed that the initial h_f is approximately equal to half of the total flume depth (H), and the average value of h_f is about $2.16h$.

The experiments by Middleton (1966) were performed by generating density currents in a tilted flume ($S \le 4\%$) and turbidity currents in a horizontal flume by releasing a constant discharge of denser fluid. He verified that the densimetric Froude number is nearly constant, *i.e.*, Fr_2 = 0.75, but may increase slightly with an increase in slope. Similar characteristics between turbidity currents and saline density currents were found. Lower noses, however, were found in the turbidity currents.

Kersey and Hsu (1976) reproduced Keulegan's experiments for conservative currents not only in a horizontal flume but also in flumes with positive and negative slopes. They found that appreciable frictional energy dissipation occurs while the current propagates downstream. They obtained constant values of coefficients in the following relationships :

$$u_f = K_1 \sqrt{g' h_f} \qquad (12)$$

$$u_f = K_3 \sqrt{g' H} \qquad (13)$$

$$u_f = K_4 \sqrt{g'(H - h_f)} \qquad (14)$$

In eqs. (12)-(14), K_1, K_3, and K_4 denote the densimetric Froude numbers associated with different characteristic lengths. Kersey and Hsu estimated constants with the help of the frontal speed by Yih (1965) such as

Table 1. Froude Numbers with Different Length Scales (Kersey & Hsu, 1976)

	K_1	h_f/H	K_3	h_f/H	K_4	h_f/H
horizontal	0.71	0.41	0.45	0.41	0.59	0.41
+ slope	0.64	0.53	0.46	0.53	0.67	0.53
- slope	0.54	0.53	0.39	0.53	0.57	0.53

Table 2. Densimetric Froude Numbers

(a) Cloud Type Density Currents

Author(s)	Dens. Froude Number	Data Source	slope
Keulegan (1958)	$Fr_2 = 0.71$	Lab. Exp.	$\theta \leq 2.29°$
Daly & Pracht (1968)	$Fr_2 = 0.70$	Num. Exp.	horizontal
Kersey & Hsu (1976)	$Fr_2 = 0.71$	Lab. Exp.	horizontal
Huppert & Simpson (1980)	$Fr_1 = 1.19$	Lab. Exp.	horizontal

(b) Plume Type Density or Turbidity Currents

Author(s)	Dens. Froude Number	Data Source	slope
Middleton (1966)	$Fr_2 = 0.75$	Lab. Exp.	horizontal
Buhler et al. (1991)	$Fr_1 = 1.08$	Theoretical	horizontal
Altinakar (1992)	$Fr_2 = 0.63$	Lab. Exp.	$\theta \leq 2.07°$

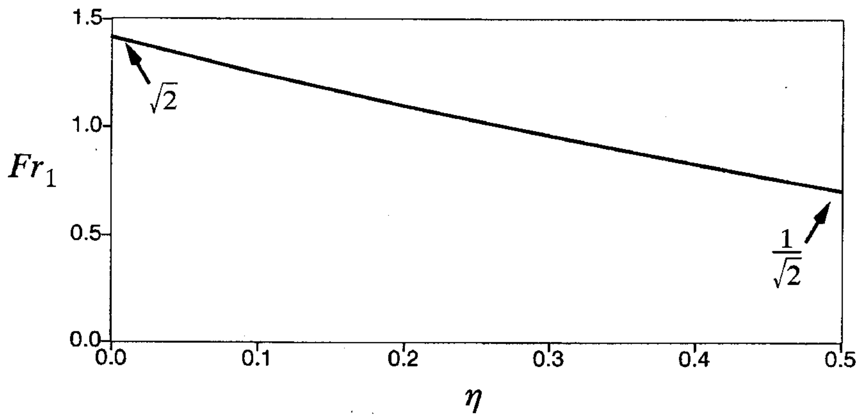

Fig. 2. Densimetric Head Froude Number vs. Fractional Depth (Benjamin, 1968)

$$u_f = \sqrt{\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} g(H - h_f)} \qquad (15)$$

where ρ_1 and ρ_2 denote the density of lighter and heavier fluids, respectively. Kersey and Hsu found that $K_1 = K_4 = 1/\sqrt{2}$ and $K_3 = 0.5$ when $h_f/H = 0.5$. Their experimental results are summarized in Table 1, where good agreement is seen between the theoretical and the measured results. The differences may be explained by the fact that friction and dissipation were ignored during the conversion of potential energy into kinetic energy. Kersey and Hsu (1976) concluded that Keulegan's constants in eqs. (8)-(11) are not truly constant, but varies depending upon the magnitude of the frictional energy loss, the channel slope, and the value of h_h/H.

Huppert and Simpson (1980) proposed a relationship describing the motion of a density cloud as a function of fractional depth, η, based on the experimental observations in Simpson and Britter (1979). They proposed that the densimetric Froude number be given by

$$Fr_1 = u_f / \sqrt{g'h} = 1.19 \qquad \text{for } \eta \leq 0.075 \qquad (16)$$

$$Fr_1 = u_f / \sqrt{g'h} = 0.5\eta^{-1/3} \quad \text{for } 0.075 \leq \eta \leq 1 \qquad (17)$$

A fractional depth, η, larger than 0.075 causes the density current to propagate less rapidly until $\eta = 0.075$, at which point the motion of the density current is no longer controlled by the fractional depth. For the density current in a deep ambient fluid, the turbulent mixing at the head obviously retards the motion as much as the difference between 1.19 from eq. (16) and $\sqrt{2}$ from eq. (4).

In a recent experimental study about weakly depositing turbidity currents, Altinakar *et al.* (1990) obtained a densimetric Froude number $Fr_2 = 0.63$, which is slightly smaller than Froude number for saline currents. This may be due to the loss of buoyancy flux in turbidity currents, i.e., deposition of suspended sediment. They also found that the dimensionless front velocity of a density current is larger than that of a turbidity current with fine sediments, which, in turn, is larger than that of a turbidity current with coarser sediments. The fact that a certain amount of energy has to be spent by the flow to keep the sediment suspended, might explain that a current carrying sediment propagates more slowly.

2.3 Summary

Table 2 summarizes values of densimetric Froude number of density or turbidity currents in the literature. Table 2(a) and (b) list the densimetric Froude numbers of cloud type and plume type density currents, respectively. In Table 2(a), a value from numerical model by Daly and Pracht (1968) is given, and the value by Buhler et al. (1991) in Table 2(b) comes from eq. (5) with $\eta = 0$ and $\beta_p = 1.35$.

Recalling that the front height is about twice the current height, the values of the densimetric Froude number are seen to be fairly consistent. They are even consistent regardless of the type of the currents. That is, the values for density clouds are similar to those for density plumes, and they are close to the mean value from Benjamin's theoretical analysis ($Fr_2 = 0.75$), which is valid for either cloud or plume type density currents. The impacts of slope and ambient water depth do not appear clearly in the values. Note that the considerations of nose lifting (Benjamin, 1968) and non-uniformity of velocity profile (Buhler *et al.*, 1991) result in $Fr_1 = 1.14$ and $Fr_1 = 1.08$, respectively, being similar to the value from experiments. Accounting for the energy loss at the front or water entrainment will lead to the same result. The value by Altinakar *et al.* (1992) is observed to be smaller than others due to the settling effect of suspended particles in turbidity currents.

3. DIMENSIONLESS FRONT VELOCITY

The densimetric Froude number describes the propagation rate of density current in terms of the reduced gravitational acceleration and an appropriate length scale such as head or body height. The use of densimetric Froude number implicitly assumes that the product of the characteristic length and reduced gravity does not change significantly. In other words, ambient water is not entrained into the density current seriously such as the case of cavity flows considered by Benjamin (1968). Whereas the dimensionless front velocity expresses the propagation rate by using the buoyancy flux per unit width. For buoyancy-conserving density currents, the buoyancy flux is conserved regardless of mixing with ambient water.

Dimensional analysis by Britter and Linden (1980) led to the following simple relationship:

$$u_f = (g'q)^{1/3} f(\theta, \mathrm{Re}) \tag{18}$$

for the front velocity after steady motion of a conservative current has been achieved. In eq. (18), q is the discharge per unit width of dense fluid, and $g'q$ (= B_f) is the buoyancy flux per unit width, which is constant for conservative density currents. For turbulent flows with a sufficiently large Reynolds number, the above equation can be written as

$$u_f = B_f^{1/3} f(\theta) \tag{19}$$

Britter and Linden (1980) showed experimentally and analytically that the advancing velocity of density flows is almost independent of the slope θ. This is due to the fact that the gravitational force increases as the slope increases from zero. However, as the slope increases, there is also an increase in water entrainment into the current, which seems to decelerate the body of the current. Based on experimental observations, Britter and Linden (1980) suggested that the dimensionless front velocity $\bar{u}_f$ is a constant for a wide range of slopes such as

$$\bar{u}_f = u_f / B_f^{1/3} = 1.5 \pm 0.2 \qquad 5° \le \theta \le 90° \tag{20}$$

Over this range of slopes, the buoyancy force of the current is large compared with the frictional force, and therefore guarantees the steady motion of the current. Analytically, they expressed the dimensionless front velocity as

$$\overline{u_f} = S_1^{1/2} \left(\frac{\cos\theta}{\alpha} + \frac{\alpha \sin\theta}{2(e_w + c_D)} \right) \left(\frac{\sin\theta}{e_w + c_D} \right)^{-2/3} \tag{21}$$

where S_1 is a profile constant, and α is the ratio of the speed at the level of the nose to the mean speed of the following flow. Experimental data together with eq. (21) are plotted as a function of channel slope in Fig. 3. Therein, values of a = 1.2, S_1 = 0.75, and e_W = θ/1000 are used for the computation of the curve with θ in degrees. It is seen that at high slopes the effect of the bed resistance coefficient, c_D, is negligible because most of the retardation of the current is caused by the water entrainment, whereas the effect of the bed resistance coefficient is noticeable at small slopes.

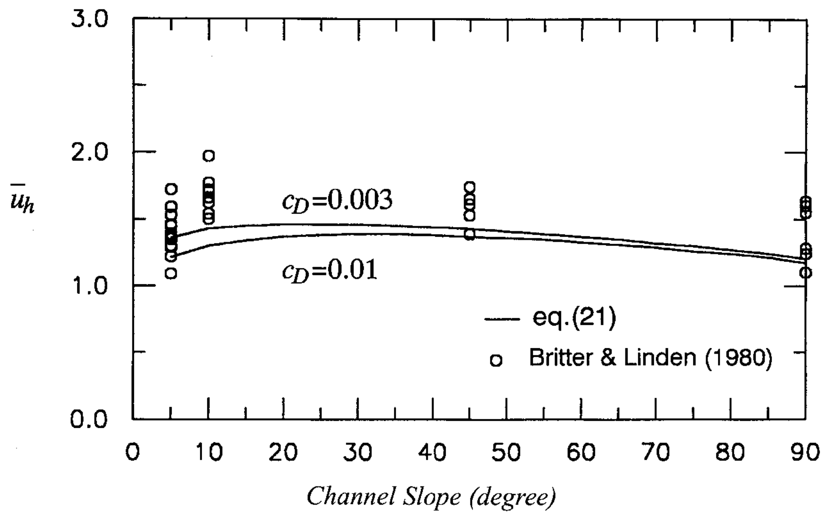

Fig. 3. Dimensionless Head Velocity versus Channel Slope (Britter & Linden, 1980)

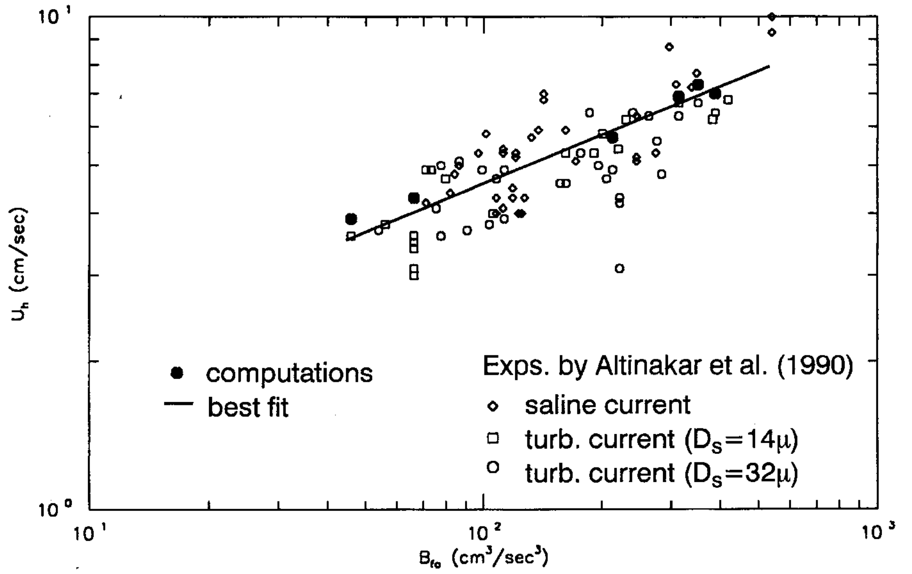

Fig. 4. Head Velocity vs. Initial Buoyancy Flux (Choi & Garcia, 1995)

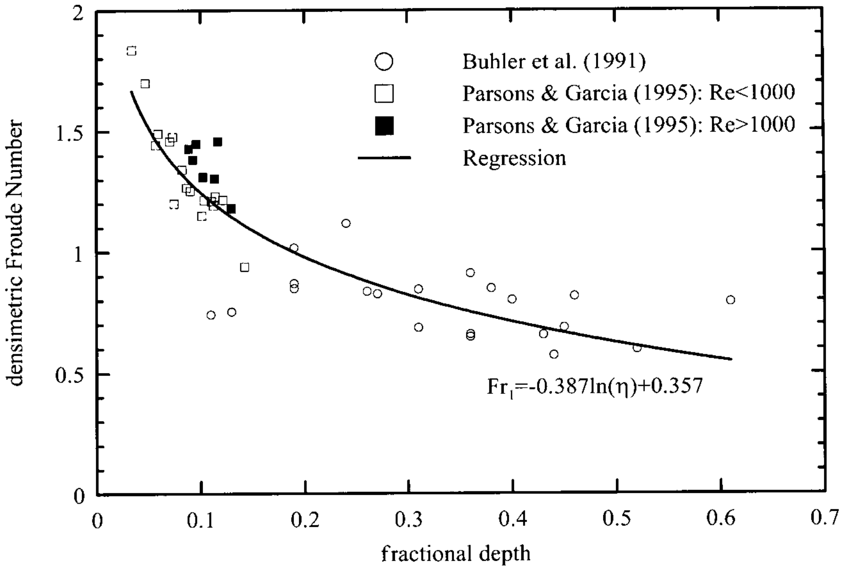

Fig. 5. Densimetric Froude Number versus Fractional Depth

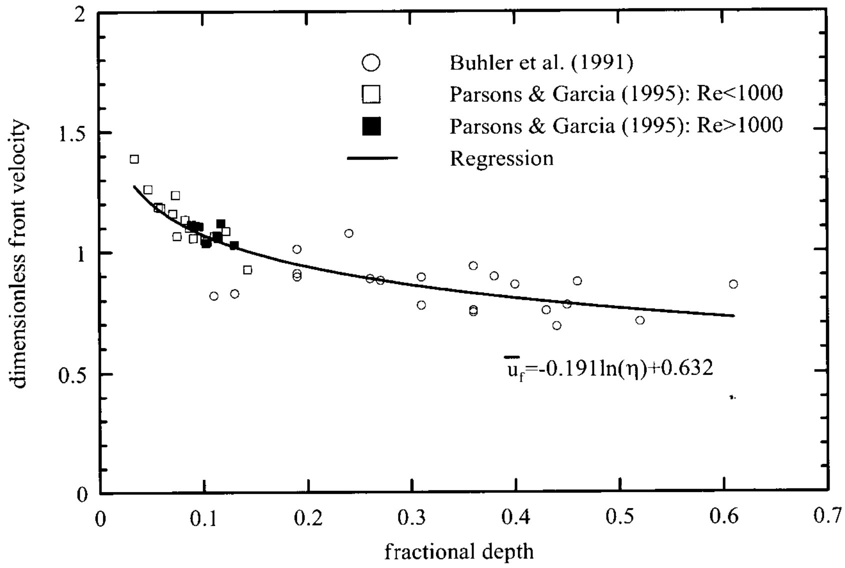

Fig. 6. Dimensionless Front Velocity versus Fractional Depth

Choi and García (1995) plotted buoyancy flux versus front velocity, which are obtained from laboratory observations as well as numerical computations over slope ranges, $0 < S < 0.0362$. The plot is given in Fig. 4. They showed that a constant value of the dimensionless front velocity might still exist for slopes smaller than 5°, however, this constant value is close to unity rather than 1.5.

4. DENSIMETRIC FROUDE NUMBER VS. DIMENSIONLESS FRONT VELOCITY

In this section, two approaches in estimating the front propagation rate, namely the densimetric Froude numbers and dimensionless front velocities, are compared by using the experimental data from Buhler *et al.* (1991) and Parsons and García (1995). Both experiments were carried out on a horizontal surface with conservative density currents. The case of density currents on an inclined surface is not included in this study due to the lack of available data. In Buhler et al.'s (1991) experiments, only the case of static ambient layer is considered. In the experiments by Parsons and García (1995), the front of density currents was freezed with the help of conveyor belt installed at the channel bottom. In this case, the volume flux of the density current from the inlet is the mixing rate, so that the total volume flux is adjusted by adding the mixing rate.

The densimetric Froude number (Fr_1) versus fractional depth is plotted in Fig. 5. The curve from the regression of the data is also shown in the figure. Mean values of the densimetric Froude number from Buhler *et al.*'s (1991), Parsons and García's (1995) with Re < 1000, and Parsons and García's (1995) with Re > 1000 are 0.63, 1.33, and 1.36, respectively. Froude numbers by Buhler *et al.*'s (1991) data are due to larger fractional depth, which clearly tends to retard the front propagation.

Fig. 6 presents the dimensionless front velocity ($\bar{u}_f$) versus fractional depth. Mean values of the dimensionless front velocities from Buhler *et al.*'s (1991), Parsons and García's (1995) with Re < 1000, and Parsons and García's (1995) with Re > 1000 are 0.84, 1.12, and 1.08, respectively. As in Fig. 5, it is seen that the dimensionless front velocity gradually decreases as the fractional density increases. The figure suggests that the dimensionless front velocity for density currents propagating over a flat bottom in deep ambient water is about unity for the fractional depth greater than 0.1. Also, it is seen that the data in Figure 5 are more scattered than those in Figure 6. This indicates that the dimensionless front velocity is more appropriate in predicting the front propagation of the density current over a horizontal surface rather than the densimetric Froude number.

5. CONCLUSIONS

This paper provided a literature survey of the dimensionless numbers for the front propagation rate of density currents. The densimetric Froude number is the front velocity normalized by the reduced gravity and characteristic length. The dimensionless front velocity is the front velocity divided by the cubic root of the buoyancy flux per unit width. A clarification of the characteristic length showed that the values of the densimetric Froude number are consistent. It was also shown that they do not change significantly depending upon the type of density currents. The dimensionless Froude numbers are found to be about $Fr_1 = 1.1$ and $Fr_2 = 0.7$ when the current and body heights are used for characteristic length, respectively. The densimetric Froude number of $Fr_l = 1.1$ is smaller than the theoretical value by Benjamin's (1968) analysis due to severe mixing and energy loss.

Using experimental data available in the literature, the densimetric Froude number and the dimensionless front velocity of density currents are estimated. The data were obtained from the laboratory experiments in Buhler et al. (1991) and Parsons and Garcia (1995), who generated conservative saline currents over a horizontal floor. Both dimensionless numbers are plotted against fractional depth. They show a similar behavior in that they decrease as fractional depth increases due to the dynamic effect of the overlying layer. It was found that the values of the densimetric Froude number deviate more from the mean than the values of dimensionless front velocity. This indicates that the dimensionless front velocity provides a better tool in describing the propagation rate of the density current rather than the densimetric Froude number. Study of the density currents on the inclined surface is remained for further experimental research.

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