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Sensitivity analysis of melt spinning process by frequency response

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Abstract

The sensitivity of the final filament to the ongoing sinusoidal disturbances has been investigated in the viscoelastic spinning using frequency response method. Amplification ratios or gains of the spinline cross-sectional area at the take-up to any disturbances show resonant peaks along the frequency regime, where the frequencies at these points directly correspond to the imaginary parts of the successive leading eigenvalues from the linear stability analysis. As shown in Jung *et al.* (1999) and Lee *et al.* (2001), the sensitivity results on the effect of various process conditions such as spinline cooling and fluid viscoelasticity, obtained by dynamic transient simulation have been corroborated in this study. That is, increasing spinline cooling makes the system less sensitive to disturbances, thus stabilizes the spinning. Also, an increasing viscoelasticity for extension-thickening fluids decreases the sensitivity of the spinning, i.e., stabilizing the system, whereas it increases the sensitivity of the spinning of extension-thinning fluids. Furthermore, it has been found in the present study that the inertia force as one of secondary forces causes the system to be more stable or less sensitive to process disturbances.

Keywords : frequency response, sensitivity, spinning, PTT fluids, spinline cooling, fluid viscoelasticity, inertia

1. Introduction

One of the objectives of a polymer processes such as fiber spinning, film casting, and film blowing is to produce a uniform fiber or film with good physical properties at a given take-up speed. Manufacturing thin products, especially at high speeds, is not a trivial task because most processes are subject to many kinds of unexpected disturbances affecting the uniformity of fiber or film. Therefore, the stability and sensitivity analyses of the base flow of the system affected by various disturbances are industrially very important in that they are closely related to productivity, quality control, and profitability of the final products. The issues of stability analysis on polymer processes, focused on the onset of a self-sustained oscillatory instability known as "draw resonance", have been continually explored by many researchers during the last four decades (Pearson and Matovich, 1969; Gelder, 1971; Fisher and Denn, 1976; Hyun, 1978; Petrie, 1988; Larson, 1992; Kim *et al.*, 1996; Hyun, 1999; Jung, 1999; Jung and Hyun, 1999; Jung *et al.*, 2000, Lee *et al.*, 2000, Lee and Hyun, 2001, Chung *et al.*, 2001, etc).

Although the sensitivity problem about the propagation of disturbances is a very important issue in the industry, there have been only a few studies (Kase and Araki, 1982; Devereux and Denn, 1994; Jung *et al.*, 1999; Lee *et al.*, 2001). Traditionally, sensitivity of the system to disturbances is analyzed by frequency response method, measuring the response of the linearized system subjected to small ongoing sinusoidal disturbances (Devereux and Denn, 1994). That is, governing equations are linearized for small sinusoidal disturbances which are resolved into Fourier components about the steady base flow. In this case, the amplification ratio or amplitude of the sinusoidal output variables like fiber cross-section or film thickness to disturbances represents the system response to the persistent sinusoidal variation under the given operating conditions. This response is tested at many frequencies to find the most dangerous one. Then information about the linear dynamical behavior, including the amplitude or gain, and the phase angle in Bode plots, can be extracted from this frequency response analysis.

The alternative endeavors to investigate the sensitivity of the system can be found from other references (Kase and Araki, 1982; Jung *et al.*, 1999; Lee *et al.*, 2001). Kase and Araki (1982) considered the spinning process as a feedback control system, defining transfer functions between disturbances and process variables following the dynamic simulation of a linear transient system. Jung *et al.* (1999) and Lee *et al.* (2001) examined the effects of spinline cooling and viscoelasticity, respectively, on the tension sensi-

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tivity by directly solving nonlinear dynamic governing equations. They found that spinline tension is the key link in relaying disturbances to the process instability.

This study analyzes the frequency response of the spinning process with respect to various disturbances in take-up velocity, extrusion velocity, spinneret cross-sectional area, extrusion temperature, cooling air velocity, cooling air temperature, and so on. The effect of process conditions or material properties on the sensitivity has then been systematically examined, using Phan-Thien and Tanner (PTT) fluids as a reasonable viscoelastic model in portraying extensional flows (Khan and Larson, 1987).

2. Equations for spinning flows

Dimensionless governing equations of the nonisothermal spinning with PTT liquids are as follows.

Continuity equation:

$$\frac{\partial a}{\partial t}+\frac{\partial}{\partial x}(av)=0 \tag{1}$$

where $a\equiv\frac{A}{A_0},\ v\equiv\frac{V}{V_0},\ t\equiv\frac{\tilde{t}V_0}{L},\ x\equiv\frac{\tilde{x}}{L}$

Equation of motion:

$$C_{in}\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)=\frac{1}{a}\frac{\partial(a\tau)}{\partial x} \tag{2}$$

where $\tau\equiv\frac{\sigma L}{2\eta_0 V_0},\ C_{in}\equiv\frac{\rho V_0 L}{2\eta_0}$

Constitutive equation: PTT fluids

$$K\tau+De\left(\frac{\partial\tau}{\partial t}+v\frac{\partial\tau}{\partial x}-2(1-\xi)\tau\frac{\partial v}{\partial x}\right)=\frac{De}{De_0}\frac{\partial v}{\partial x} \tag{3}$$

where $K\equiv\exp(2\varepsilon De_0\tau),\ De_0\equiv\frac{\lambda_0 V_0}{L}$,

$De\equiv De_0\exp\left(k\left(\frac{1}{\theta}-1\right)\right),\ k\equiv\frac{E}{RT_0}$

Energy equation:

$$\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial x}=-St\,v^{1/3}a^{-5/6}(\theta-\theta_a)\left[1+\left(8\frac{v_y}{v}\right)^2\right]^{1/6} \tag{4}$$

where $\theta\equiv\frac{T}{T_0},\theta_a\equiv\frac{T_a}{T_0},v_y\equiv\frac{V_y}{V_0},St\equiv1.67\times10^{-4}\frac{L}{\rho C_p A_0^{5/6}V_0^{2/3}}$

These dimensionless equations are subject to the following boundary conditions.

$a=a_0=1,\ v=v_0=1,\ \tau=\tau_0,\ \theta=\theta_0=1$ at $x=0$ for all t (5)

$V=V_L=r$ at x=1 for all t (6)

where $\tilde{t}$ is time, t dimensionless $\tilde{t}$, $\tilde{x}$ spinline distance, x dimensionless $\tilde{x}$, A the spinline area, a dimensionless A, V the spinline velocity, v dimensionless V, v_y cooling air velocity, T spinline temperature, θ dimensionless T, θ_a dimensionless ambient temperature, σ axial stress tensor, τ dimensionless σ, C_{in} coefficient representing the inertia force term, ρ liquid density, η_0 liquid viscosity at zero strain-rate, r drawdown ratio. Subscripts 0 and L denote spinneret and take-up conditions, respectively.

In the above equations, the several assumptions have been adopted. First, thin filament approximation simplifies this system to one-dimensional model. Second, the origin of the spinning distance coordinate is chosen at the die (extrudate) swell position, meaning all the pre-spinneret deformation history of the liquid is not included in the model. Third, inertia and rheological forces are more dominant than other secondary forces.

3. Frequency response

Frequency response shows the sensitivity of the linearized system to ongoing disturbances, as mentioned above, governing the amplitude response of tiny perturbations around the base flow. For convenience, above governing equations are reduced to the following simple vector form.

$$\underline{R}(\underline{y},\dot{\underline{y}},p)=\underline{0} \tag{7}$$

where $\underline{y}$ is the state vector such as spinline cross-section, spinline velocity, axial stress, and temperature, $\dot{y}$ time derivative of $\underline{y}$, and p any parameter set to be perturbed. The linearization of Eq. (7) around the steady state leads to the following transient linearized equation set.

$$\underline{\underline{M}}(\underline{y}_s,p)\Delta\dot{\underline{y}}+\underline{\underline{J}}(\underline{y}_s,p)\Delta\underline{y}+\underline{F}(\underline{y}_s,p)\Delta p=\underline{0} \tag{8}$$

where $\underline{\underline{J}}\equiv\left(\frac{\partial\underline{R}}{\partial\underline{y}}\right)_{\underline{y}_0,p}$ is Jacobian matrix at the steady state, $\underline{\underline{M}}\equiv\left(\frac{\partial\underline{R}}{\partial\dot{\underline{y}}}\right)_{\underline{y}_0,p}$ mass matrix, and $\underline{F}\equiv\left(\frac{\partial\underline{R}}{\partial p}\right)_{\underline{y}_0,p}$ forcing vector of the residuals to the parameter p evaluated at steady state.

In the frequency response analysis, an ongoing disturbance of the parameter is sinusoidal in time, that is,

$$\Delta p=\zeta\exp(i\omega t) \tag{9}$$

where ω is the frequency of the ongoing disturbance, ζ the complex value of the amplitude of the imposed disturbance, and $i=\sqrt{-1}$. (In this case, ζ was set to unity.) The response of transient solutions approaches a steady oscillation with the same frequency as that of the ongoing disturbance, and a particular solution of Eq. (8) has the following form

$$\Delta\underline{y}=\zeta\underline{k}\exp(i\omega t) \tag{10}$$

where $\underline{k}$ is the complex value representing the amplitude and phase lag of solutions relative to the imposed disturbance. Substituting Eq. (9)~(10) into Eq. (8) leads to the linear complex equation.

$$(i\omega \underline{\underline{M}} + \underline{\underline{J}})\underline{k} = -\underline{F} \tag{11}$$

Amplification ratio or gain, G_i, and phase angle, ϕ_i, can be formulated from the complex response of Eq. (11). (But, the phase lag is not considered in this study.)

$$G_i \equiv \{(Re(k_i))^2 + (Im(k_i))^2\}^{1/2}, \quad \phi_i \equiv \tan^{-1}\frac{Im(k_i)}{Re(k_i)} \tag{12}$$

There are many alternative ways to solve above complex linear system, as presented in Jung and Scriven (2002). It already has been revealed that the most effective algorithm to solve frequency response of one-dimensional system was the "two-shot" method, i.e., integrating the linearized equations over the spinline length to shoot at the take-up boundary condition using two initial guesses. This method has been mainly used throughout this study.

4. Results and discussion

Figure 1 shows the predicted amplitudes of the spinline area of Newtonian fluids at the take-up along the frequency regime to many kinds of sinusoidal perturbations in take-up velocity, extrusion velocity, spinneret area, extrusion temperature, cooling air velocity, and cooling air temperature. All amplitudes show the resonant peaks along with the frequency, frequently encountered with hyperbolic systems (Friedly, 1972). As pointed out in Jung and Scriven (2002), frequencies corresponding to resonant peaks are the same as the imaginary parts of successive leading eigenmodes from linear stability analysis, which are related

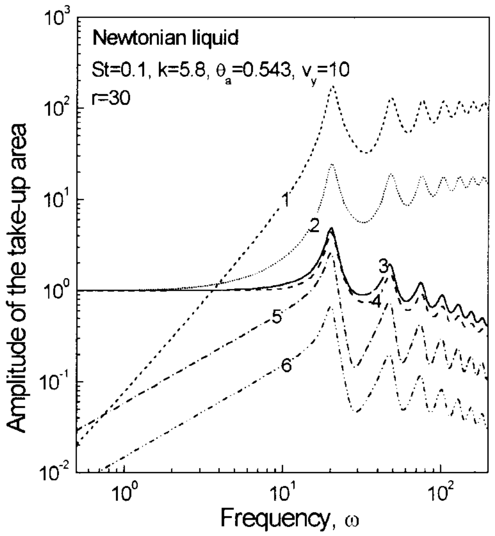

Fig. 1. Amplitudes of spinline cross-sectional area at the take-up for nonisothermal Newtonian spinning where various disturbances are introduced in 1: extrusion temperature, 2: spinneret area, 3: take-up velocity, 4: extrusion velocity, 5: cooling air temperature, and 6: cooling air velocity.

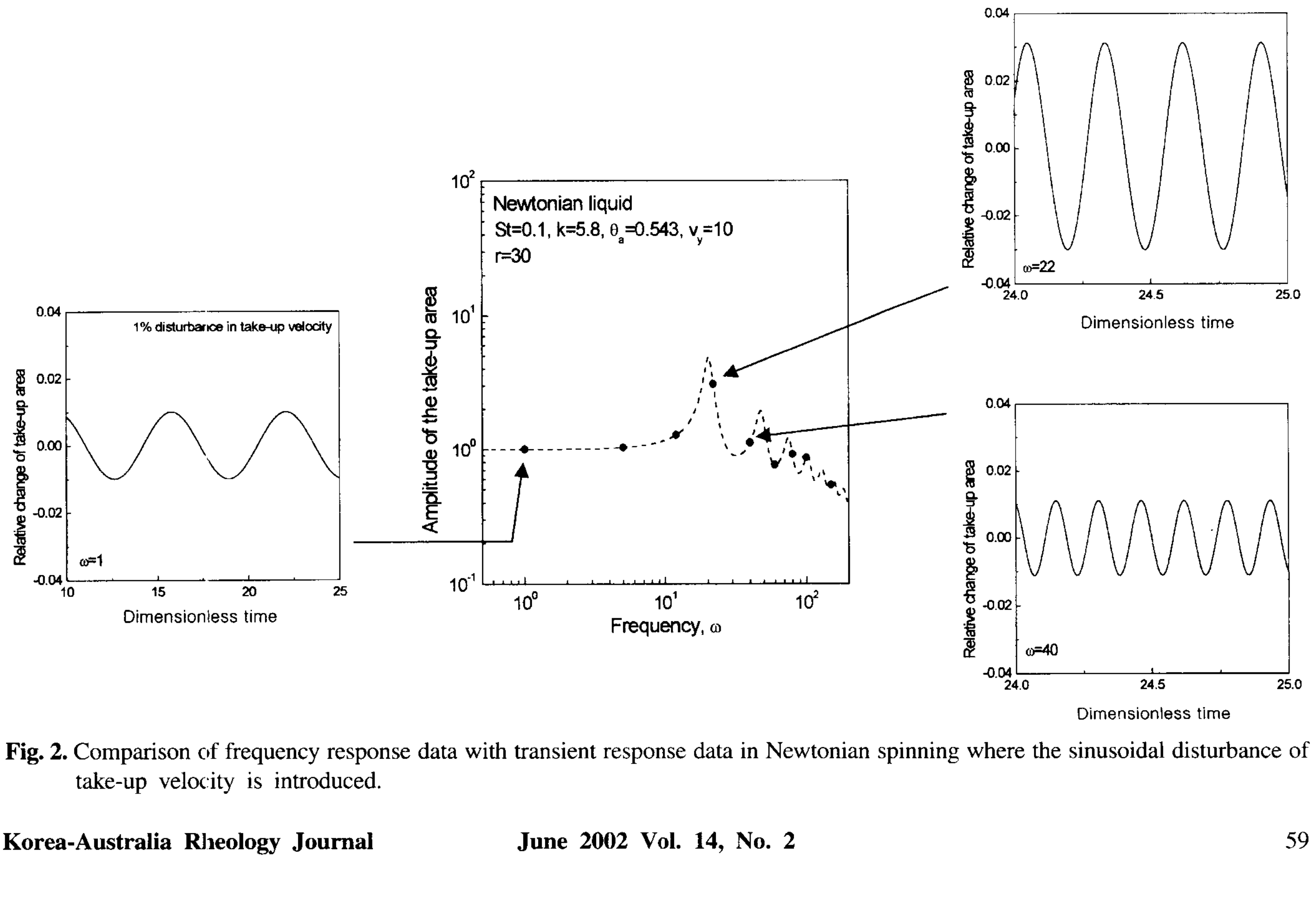

Fig. 2. Comparison of frequency response data with transient response data in Newtonian spinning where the sinusoidal disturbance of take-up velocity is introduced.

to the waves propagating the spinline.

Disturbances changing the mass flow rate – spinneret area (curve 2), take-up velocity (curve 3), extrusion velocity (curve 4) – give unity amplitude in low-frequency region, because the change of the mass flow rate directly alters the final spinline area. This means that the 1% change of perturbation produces the 1% change of output variable. Whereas, disturbances not influencing the mass flow rate – extrusion temperature (curve 1), cooling air temperature (curve 5), cooling air velocity (curve 6) – start from almost zero amplitude in low-frequency region.

The further interpretation of sensitivity data drawn in Figure 1 is as follows. For example, the amplitude of the take-up area to the ongoing oscillatory disturbances in extrusion velocity (curve 4) is gradually decreasing as frequency rises after the most dangerous response at near $\omega = 20.5$, whereas the result for spinneret area disturbance (curve 2) is increasing with frequency and finally attains to almost 15 gain value. This indicates that the tiny change in spinneret hole brings about the considerable filament unevenness, although the system is operated under the stable condition. The actual sinusoidal variation of output frequency response with time is presented in Figure 2, by dynamic transient simulation of above equations Eq. (1) ~ Eq. (6). The transient results of the spinline area at the take-up position with respect to 1% sinusoidal change of take-up velocity is exactly equal to those by frequency response.

The similar behavior of a PTT fluid under the same operating conditions as Newtonian case is observed in Figure 3. But, the amplitudes for the PTT fluid are lower than those of Newtonian fluid throughout the whole frequency due to the fluid viscoelasticity. More descriptive explanation about the effect of viscoelasticity on the sensitivity is presented below.

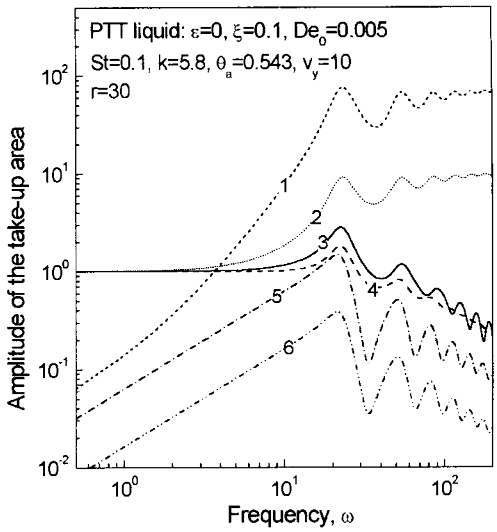

Fig. 3. Amplitudes of spinline cross-sectional area at the take-up for nonisothermal PTT spinning where various disturbances are introduced in 1: extrusion temperature, 2: spinneret area, 3: take-up velocity, 4: extrusion velocity, 5: cooling air temperature, and 6: cooling air velocity.

Also, it has been substantiated that the amplitude variations from sensitivity analysis, as operating conditions or material properties are varied, show the similar analogy with eigenvalue changes from linear stability analysis. For instance, raising inertia ($= C_{in}$) (by increasing the mass flow rate) makes the system more stable and less sensitive (Figure 4).

Jung *et al.* (1999) and Lee *et al.* (2001) established that from the nonlinear dynamic transient simulation, the tension sensitivity was the key link in relaying disturbances

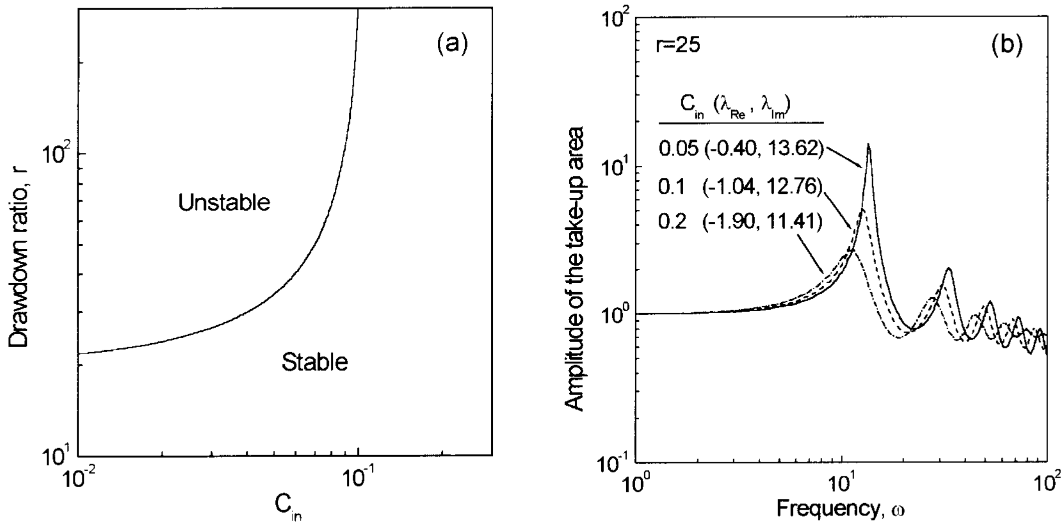

Fig. 4. (a) Critical drawdown ratio plotted against inertia coefficient. (b) The effect of inertia on the sensitivity in Newtonian spinning where take-up velocity disturbance is introduced.

from take-up to the spinneret, examining the effect of various process conditions such as spinline cooling and fluid viscoelasticity on the sensitivity. The same effects can be obtained from the frequency response analysis. Figure 5 showed the effect of spinline cooling on the sensitivity for a PTT fluid, when a disturbance is introduced in take-up velocity. It has been observed that the sensitivity of the spinline area at take-up indeed decreases, as the spinline cooling increases by raising the cooling air velocity (Figure 5(a)) or decreasing the cooling air temperature (Figure 5(b)). So, this result indicates that the spinline cooling makes the spinning less sensitive to any disturbances, thus stabilizes the system.

We also analyze the effect of fluid viscoelasticity on the sensitivity for the spinning of PTT fluids. Grasping the dichotomous behavior of viscoelastic fluids, depending on material parameters (in this case, PTT parameter ξ) – that is, extension-thickening behavior for $\xi < 0.5$, extension-thinning behavior for $\xi > 0.5$ – is very valuable in polymer extensional deformation processes. In contrast with above case dealing with the effect of spinline cooling, the effect of fluid viscoelasticity on the sensitivity has been classified into two different kinds (Figure 6). For extension-thickening fluids, increasing viscoelasticity (De) causes the system to be less sensitive to disturbances, whereas for extension-thinning fluids increasing viscoelasticity brings

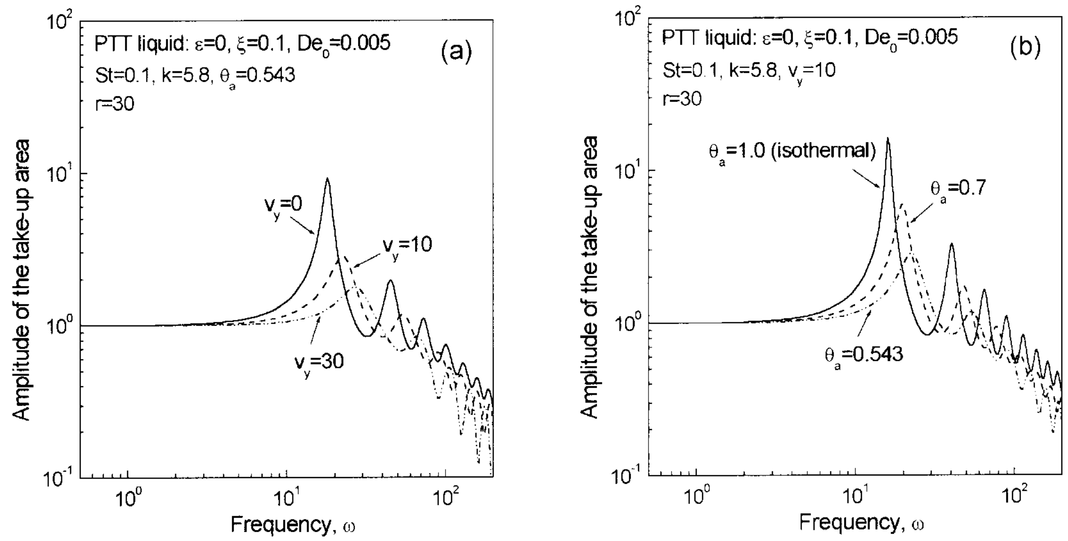

Fig. 5. Effect of spinline cooling (a) cooling air velocity and (b) cooling air temperature on the sensitivity where take-up velocity disturbance is introduced.

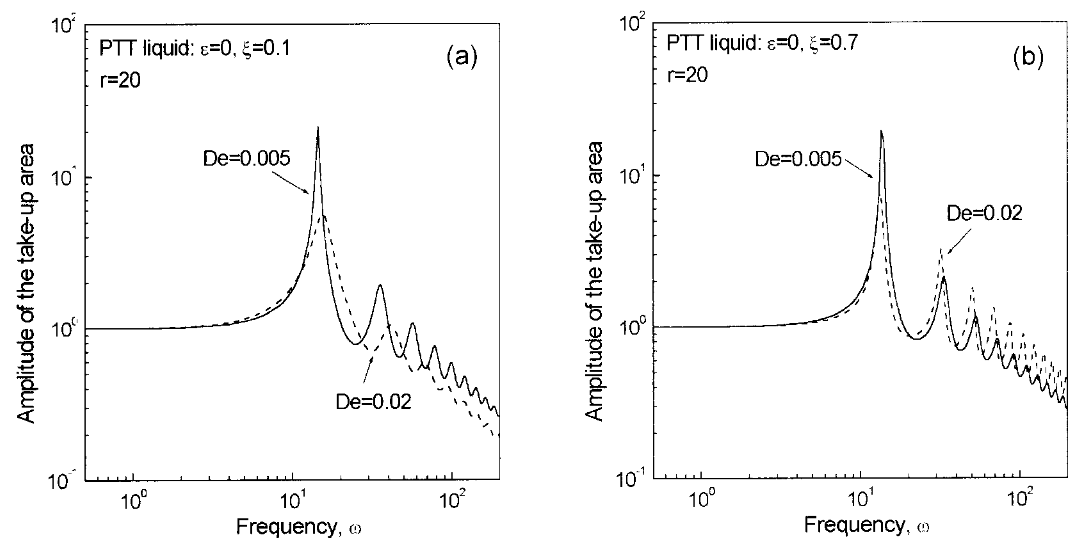

Fig. 6. Effect of fluid viscoelasticity on the sensitivity (a) for an extension-thickening fluid and (b) for an extension-thinning fluid where take-up velocity disturbance is introduced.

about increased sensitivity. This method is a useful tool to analyze the sensitivity of not only the spinning process but also other extensional deformation processes like film casting and film blowing.

5. Conclusions

The sensitivity of the spinline area at the take-up to the various ongoing sinusoidal disturbances has been obtained in the Newtonian and PTT fluids spinning, using frequency response method. Due to the hyperbolic characteristics of the system, amplitudes of the spinline area at the take-up position show resonant peaks along with frequency. Frequencies corresponding to resonant peaks are exactly equal to the imaginary parts of successive leading eigenmodes from linear stability analysis, which are related to the wave properties traveling the spinline. The effect of important process conditions and materials conditions on the sensitivity has been analyzed. Among those considered here, inertia, spinline cooling, and fluid viscoelasticity for extension-thickening fluids make the system less sensitive to any disturbances, whereas fluid viscoelasticity for extension-thinning fluids increases the sensitivity of the system.

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