The Effect of Random Point Excitation on the Vibration Level of Plates

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When a mechanical structure is driven by stationary wide band random point forces, the resulting vibration depends upon the number, location, and joint statistical properties of the exciting forces. In this study, under the assumption of light damping, an approximate procedure for analyzing plates is briefly outlined. The effects of number, location and correlation of the force field on the vibration level are then investigated for various cases in which random point forces with band limited white noise are applied, and the optimal spacing between input forces that produces a relative minimum in the vibration response is predicted.

Key Words : Correlation Coefficient, Auto Correlation, Cross Correlation, Band Limited White Noise, Random Vibration, Random Point Excitation, Optimal Spacing

1. Introduction

When a deterministic structure is excited by a randomly fluctuating load, the dynamic response can generally be considered to be a random process in space and time. The analysis of the dynamic response of continuous structures under random excitation was initiated in connection with aerospace problems involving jet noise excitation and buffetting in the middle of 1950's. Today, the advance of dependable measuring and peripheral equipments including transducers, FFT analyzers, data acquisition devices, powerful computers and softwares make it possible to obtain more accurate experimental data and to predict random vibration responses. Thus, vibrational problems in various engineering fields can be analyzed and solved by the application of the theory of random vibration.

The theory of random vibration has been developed by researchers such as Crandall (1963). The vibration phenomena at resonant frequences wese investigated by Lee (1976), Kulvets (1977), Itao (1978), Crandall (1979), and Zha (1983). Peng (1987) worked on the acoustic radiation in plates excited by multiple random point forces. Oh and Keltie (1989) investigated the effect of source correlation of wide band excitation on plate responses.

In 1990's, random vibration problems were continually investigated by researchers. Abdelnaser and Singh (1993) solved the randomly forced vibration problem of simply supported cross ply laminated composite plates. Kim, Kang, and Kim (1993) studied the random vibration of the coupled identical beams subjected to band limited white noise. Chang (1994) developed finite element formulations to analyze random vibration of nonlinear hysteretic plates. Kim and Chung (1995) proposed a method to determine optimal reference spectrum in random vibration control problems. Xu and Miles (1996) estimated the power spectral densities of the random bending strains at all points on the structures by the numerical differentiation of cross spectral densities of the vibration measured by two transducers. Han, Bernhard, and Mongeau (1997)

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developed two methods for calculating the power input to vibrating beams and plates excited by multiple discrete random forces. Bonilha and Fahy(1998) proposed a probabilistic treatment of the vibration field generated by the random vibration. Shankar(1998) obtained the mean square vibrational energy levels of connected structures under certain conditions of coupling and excitation by random uncorrelated force using statistical energy analysis methods. Kang and Kim(1998) investigated the vibration and sound radiation characteristics of a simple and bar stiffened plate using numerical and experimental techniques.

In this study, under the light damping assumption, an approximate procedure for analyzing random vibration of a plate is briefly outlined and the optimal spacing between input forces that produce a relative minimum in the vibration response is predicted. A uniform plate is excited by four or more point forces where time histories are simple functions of stationary wide band random processes with known cross correlations. This study concentrates on reducing the vibration level in structures under the action of random point forces by adjusting the positions of the forces.

2. Random Vibration of a Finite Thin Plate

Consider a simply supported rectangular homogeneous plate, whose sides are of lengths L_x and L_y.. This plate is acted upon by N transverse forces $f_v(t)$ located at positions $x=a_v$ and $y=b_v$.. The equation of motion for the transverse plate displacement $W(x, y. t)$ may be written as

$$D\nabla^4 W + C\frac{\partial W}{\partial t} + \rho h \frac{\partial^2 W}{\partial t^2} = \sum_{v=1}^{N} \delta(x-a_v)\,\delta(y-b_v)\,f_v(t) \qquad (1)$$

where ρ is the mass per unit area and C is the viscous damping per unit area. The operator ∇^4 is $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2$. For a flat rectangular plate of thickness h, the bending modulus is $D=\frac{Eh^3}{12(1-\nu^2)}$ where E is Young's modulus and ν is Poisson's ratio.

The differential equation of motion for the undamped structure within the periphery of the plate is given by

$$D\nabla^4 \Psi_{jp} = \rho h \omega_{jp}{}^2 \Psi_{jp} \qquad (2)$$

where the natural modes Ψ_{jp} (x, y) for a rectangular plate satisfy the eigenvalue problem, and can be determined by the boundary conditions as follows

$$\Psi_{jp} = 2\sin\left(\frac{j\pi x}{L_x}\right)\sin\left(\frac{p\pi y}{L_y}\right) \qquad (3)$$

ω_{jp} is the natural frequency corresponding to $\Psi_{jp}(x, y)$

$$\omega_{jp} = \left(\frac{D}{\rho h}\right)^{\frac{1}{2}}\left[\left(\frac{j\pi}{L_x}\right)^2+\left(\frac{p\pi}{L_y}\right)^2\right] \qquad (4)$$

The modes also satisfy the orthogonality condition

$$\int_0^{L_x}\int_0^{L_y} \rho h \Psi_{jp}\Psi_{kq}\,dx\,dy = M\delta_{jk}\delta_{pq} \qquad (5)$$

where $M=\rho h L_x L_y$ is the total mass of the plate. The asymptotic value of the average frequency spacing between successive natural frequencies is

$$\triangle\omega = \left(\frac{D}{\rho h}\right)^{\frac{1}{2}}\frac{4\pi}{L_x L_y} \qquad (6)$$

When the excitation is the unit impulse at position $x=a_v$, $y=b_v$ on the plate

$$f(x, y, t) = \delta(x-a_v)\,\delta(y-b_v)\,\delta(t) \qquad (7)$$

We can obtain a representation of the unit impulse response $h(x, y, t;a_v, b_v)$ in terms of the natural modes

$$h(x, y, t;a_v, b_v) = \sum_{j,p} g_{jp}(t;a_v, b_v)\,\Psi_{jp}(x, y) \qquad (8)$$

When Eqs. (7) and (8) are inserted in Eq. (1), we obtain one equation for the sequence of unknown time functions $g_{jp}(t;a_v, b_v)$.

$$\sum_{j,p}\left[D\nabla^4\Psi_{jp}g_{jp} + C\Psi_{jp}\frac{\partial h_{jp}}{\partial t} + \pi h\Psi_{jp}\frac{\partial^2 h_{jp}}{\partial t^2}\right] = \delta(x-a_v)\,\delta(y-b_v)\,\delta(t) \qquad (9)$$

By using the orthogonality condition of Eq. (5), both sides of Eq. (9) are multiplied by a particular mode, and integrated over x from 0 to L. Then the sequence of uncoupled equations for time functions $g_{jp}(t;a_v, b_v)$ can be obtained.

$$M\left[\omega_{jp}^2 g_{jp}+C/\rho h\frac{dg_{jp}}{dt}+\frac{d^2 g_{jp}}{dt^2}\right]=\Psi_{jp}\delta(t) \quad (10)$$

$j=1, 2, \cdots$

The solutions to Eq. (10), with $g_{jp}=0$ and $\frac{dg_{jp}}{dt}=0$ for $t<0$, is

$$g_{jp}(t;a_v, b_v)=h_{jp}(t)\,\Psi_{jp}(a_v, b_v) \quad (11)$$

where $h_{jp}(t)$ is the unit impulse response function for j-th and p-th mode given by

$$h_{jp}(t)=\begin{cases}0, \\ \dfrac{\exp\left(-\frac{\beta}{2}\right)}{M\left[\omega_{jp}^2-\left(\frac{-\beta}{2}\right)^2\right]^{\frac{1}{2}}}\sin\left[\omega_{jp}^2-\left(-\frac{\beta}{2}\right)^2\right]^{\frac{1}{2}}t, \end{cases}$$

$t>0 \quad (12)$

where β is the modal bandwidth

$$\beta=\frac{C}{\rho h} \quad (13)$$

The unit impulse response function is now obtained by substituting Eq. (11) into Eq. (8) :

$$h(x, y, t;a_v, b_v)=\sum_{j,p}\Psi_{jp}(x, y)\,\Psi_{jp}(a_v, b_v)\,h_{jp}(t) \quad (14)$$

By using the Fourier transform, we obtain

$$H(x, y, t;a_v, b_v)=\sum_{j,p}\Psi_{jp}(x, y)\,\Psi_{jp}(a_v, b_v)\,H_{jp}(\omega) \quad (15)$$

where $H_{jp}(\omega)$ is the unit complex frequency response function for j-th and p-th mode :

$$H_{jp}(\omega)=\frac{1}{m(\omega_{jp}^2-\omega^2+i\beta\omega)} \quad (16)$$

The response space-time cross-spectral density, $S_w(x, y, \omega)$ can be expressed in terms of the excitation space-time cross-spectral density and the unit complex frequency response functions of the structure

$$S_\omega\omega(x, y, \omega)=\sum_{j,p}\sum_{k,q}\Psi_{jp}(x, y)\,\Psi_{kq}(x, y)\,H_{jp}(\omega)\,H_{kq}^*(\omega)\,\Phi_{jpkq}(\omega) \quad (17)$$

where the superscript "$*$" denotes the complex conjugate. $\Phi_{jpkq}(\omega)$ is the modal excitation cross-spectral density function

$$\Phi_{jpkq}(\omega)=\sum_{\mu=1}^{N}\sum_{v=1}^{N}\Psi_{jp}(x_1, y_1)\,\Psi_{kq}(x_2, y_2)\,S_f(a_\mu, b_\mu, a_v, b_v, \omega) \quad (18)$$

The cross-spectral density function of the applied forces is given by

$$S_f(a_\mu, b_\mu, a_v, b_v, \omega)=\delta(x_1-a_\mu)\,\delta(y_1-b_\mu)\,\delta(x_2-a_v)\,\delta(y_2-b_v)\,S_{\mu v}(\omega) \quad (19)$$

where N is the number of the applied forces and $S_\mu(\omega)$ is the power spectral density between μ-th and v-th forces. The mean square displacement at a location (x, y) may be written as

$$E[W^2(x, y)]=\sum_{j,p=1}^{\infty}\sum_{k,q=1}^{\infty}\Psi_{jp}(x, y)\,\Psi_{jp}(x, y)\int_{-\infty}^{\infty}H_{jp}(\omega)\,H_{kq}^*(\omega)\,\Phi_{jpkq}(\omega)\,d\omega \quad (20)$$

The summation of the mean square displacement may be thought as an indication of the vibration level of the entire plate. It can be obtained by integrating the mean square displacement over the entire surface.

$$W=\int_0^{Ly}\int_0^{LxE}[W^2(x, y)]\,dxdy=\int_0^{Ly}\int_0^{Lx}\sum_{j,p=1}^{\infty}\sum_{k,q=1}^{\infty}\sum_{\mu,v=1}^{N}\Psi_{jp}(x_1, y_1)\,\Psi_{kq}(x_2, y_2)\int_{-\infty}^{\infty}H_{jp(\omega)}H_{kq(\omega)}^*\,\Psi_{jp}(a_\mu, b_v)\,S_{\mu v}(\omega)\,d\omega dxdy \quad (21)$$

3. Approximate Solution Based on Mode Summation

An approximate solution for the vibration level of the plate is proposed to simplify the calculation. Two assumptions are made, which are particularly popular and useful in random vibration analysis. The first assumption is that the damping of the plates is light enough to have quite sharp bandwidth of half power points. Then it can be said that the resonance frequencies are sufficiently separated, so that the off-diagonal coupling terms may be neglected. This implies omitting all terms in the fourfold modal summation except those for which $j=k$ and $p=q$. In the case of a square plate, the mode degeneracy is a symmetrically occurring phenomenon, since every mode Ψ_{jp} $(j \neq p)$ has a mate Ψ_{pj} with the identical natural frequency as given in Eqs. (3) and (4). The important contributions to the dynamic response come not only from the terms representing modal auto-correlations but also from the terms representing modal cross-correlations arising from those modes with $j=k$ and $p=q$.

The second assumption is the white noise approximation, which implies that the auto and cross spectral density functions $S_{\mu v}(\omega)$ are slowly varying with respect to the frequency relative to the rapidly varying mode response functions $H_{jp}(\omega)$, especially near the frequencies $\omega=\omega_{jp}$. The modal sum Eq. (20) for the mean-square displacement then becomes

$$E[W^2(x, y)] = \sum_{j,p=m}^{n} \sum_{\mu,v=1}^{N} \Psi_{jp}^2(x, y)\, \Psi_{jp}(a_\mu, b_\mu)\, \Psi_{jp}(a_v, b_v) \int_{-\infty}^{\infty} |H_{jp}(\omega)|^2 S_{\mu v}(\omega)\, d\omega \tag{22}$$

where the assumption over the mode number j and p is confined within the range where the condition $\omega_1<|\omega|<\omega_2$ is satisfied, in which ω_1 and ω_2 represent lower and upper cut-off frequencies of the excitation band, and m and n represent the lower and upper resonant mode numbers within the given band.

In the case of band-limited white noise spectra, the spectral density for these forces may be written as

$$\begin{aligned} S_{\mu\mu}=S_{vv}&=\begin{cases} S_0, & \omega_1<|\omega|<\omega_2 \\ 0, & otherwise \end{cases} \\ S_{\mu v}=S_{v\mu}&=\begin{cases} rS_0, & \omega_1<|\omega|<\omega_2 \\ 0, & oherwise \end{cases} \end{aligned} \tag{23}$$

Where r is the excitation correlation coefficient $(-1 \leq r \leq =1)$. The integral in Eq. (22) has the value $W_1^2=\frac{\pi S_0}{m^2\beta}$ which is independent of ω_{jp}. The mean square displacement in the rectangular plate is thus approximated by the modal sum

$$E[W^2(x, y)] = W_1^2 \sum_{j,p=1}^{N} \sum_{\mu,v=1}^{N} \frac{16}{\omega_{jp}^2} \sin^2\frac{j\pi x}{L_x} \sin^2\frac{p\pi y}{L_y} \sin\frac{j\pi a_\mu}{L_x} \sin\frac{p\pi b_\mu}{L_y} \sin\frac{j\pi a_v}{L_x} \sin\frac{p\pi b_v}{L_y} \tag{24}$$

In the case of a simply supported square plate $(L_x=L_y=L)$, every mode Ψ_{jp} for $j \neq p$ has a mate Ψ_{pj} with the identical natural frequency. The plate response, including both the auto-correlation terms $(j=k,\ p=q)$ and modal cross-correlation terms $(j=q,\ p=k)$ may be written as

$$\frac{E[W^2(x, y)]}{W_1^2} = A(x, y) + B(x, y) \tag{25}$$

with

$$A(x, y) = \sum_{j,p=1}^{N} \sum_{\mu,v=1}^{N} \frac{16}{\omega_{jp}^2} \sin^2\frac{j\pi x}{L} \sin^2\frac{p\pi y}{L} \sin\frac{j\pi a_\mu}{L} \sin\frac{p\pi b_\mu}{L} \sin\frac{j\pi a_v}{L} \sin\frac{p\pi b_v}{L}$$

$$B(x, y) = \sum_{j,p=1}^{N} \sum_{\mu,v=1}^{N} \frac{16}{\omega_{jp}^2} \sin^2\frac{j\pi x}{L} \sin^2\frac{p\pi y}{L} \sin\frac{p\pi x}{L} \sin\frac{j\pi y}{L} \sin\frac{j\pi a_\mu}{L} \sin\frac{p\pi b_\mu}{L} \sin\frac{p\pi a_v}{L} \sin\frac{j\pi b_v}{L}$$

where $A(x, y)$ and $B(x, y)$ are due to the modal auto and cross correlations, respectively. The integration of $B(x, y)$ over the entire plate vanishes due to the orthogonality of the modes. Therefore the vibration level of the entire plate becomes

$$\begin{aligned} W &= \int_0^L \int_0^L E[W^2(x, y)]\, dxdy \\ &= W_1^2 \int_0^L \int_0^L A(x, y)\, dxdy \\ &= \frac{4\pi L^2 S_0}{m^2\beta} \sum_{j,p=m}^{N} \sum_{\mu,v=l}^{N} \sin\frac{j\pi a_\mu}{L} \sin\frac{p\pi b_\mu}{L} \sin\frac{j\pi a_v}{L} \sin\frac{p\pi b_v}{L} \frac{1}{\omega_{jp}^2} \end{aligned} \tag{26}$$

Eq. (26) may be written as

$$W = Wa + Wc$$

where the term Wa is produced by the source auto correlation,

$$W_a = \frac{4\pi L^2 S_0}{m^2\beta} \sum_{j,p=m}^{N} \sum_{\mu,v=l}^{N} \sin^2\frac{j\pi a_v}{L} \sin^2\frac{p\pi b_v}{L} \frac{1}{\omega_{jp}^2}$$

and the term Wc is produced by the source cross correlation. Thus,

$$W_c = \frac{4\pi L^2 S_0}{m^2\beta} \sum_{j,p=m}^{N} \sum_{\mu,v=l}^{N} \sin\frac{j\pi a_\mu}{L} \sin\frac{p\pi b_\mu}{L} \sin\frac{j\pi a_v}{L} \sin\frac{p\pi b_v}{L} \frac{1}{\omega_{jp}^2}$$

where N is the number of applied forces.

4. Numerical Results and Discussion

In this study, attempts were made to determine an optimal spacing distance between input forces to produce a relative minimum in the vibration level under the excitation of band limited white noise point forces. In the numerical analysis a square aluminum plate with dimensions of 2m × 2m and thickness of 1cm was chosen. Four point, and eight point ring forces were chosen and the excitation was generated in the from of the band limited white noise using octave bands centered at 500 Hz, 1 KHz, and 2 KHz.

The effects of the multiple forces may be introduced most simply by considering the case of four point forces when the excitation is an octave band centered at 500 Hz. Fig. 1 shows that the variation of the correlation coefficients ($-1<r<1$) may change the total mean-square vibration level W of the plate by adding or subtracting the vibration level due to the source cross-correlation, Wc. When the driving forces are uncorrelated ($r=0$), the vibration level of the plate W reduces to a superposition of the distributions which result when each force acts alone. The vibration level of the plate due to the source auto-correlation has a constant value, the difference between W and Wc, and is independent of the correlation coefficient. In the extreme cases ($r=\pm1$) the vibration level of the plate W is uniformly increased or decreased by 19% compared to the case when r is equal to zero.

Vibrational responses of plates simultaneously depend upon the geometry including the boundary conditions, the material, and the loading condition. Thus the change of one or more of them will cause the change of the responses. When the detailed relationship between the input parameters and vibrational responses are obtained, more effective vibration control can be achieved. In this study, as a preliminary step, optimal arrangements of point forces that produce minima in the total plate response is investigated. In Fig. 2, a particular case is shown in which four point forces were located at a radial spacing R on the main diagonals of the plate. The total plate response was calculated for different R values and the results are shown in Fig. 3. It can be said that an optimal force spacing exists that produces a relative minimum in the vibration level. By adjusting the spatial distance R between the applied forces, by alternately adjusting the support positions, the magnitude of the vibration level of

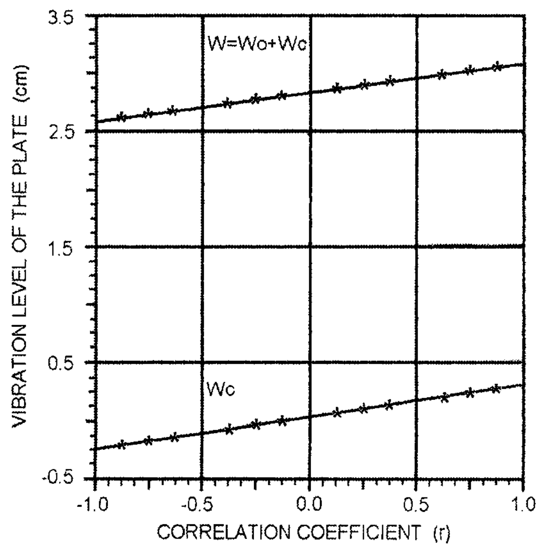

Fig. 1 Vib. level vs. correlation coefficient(r)

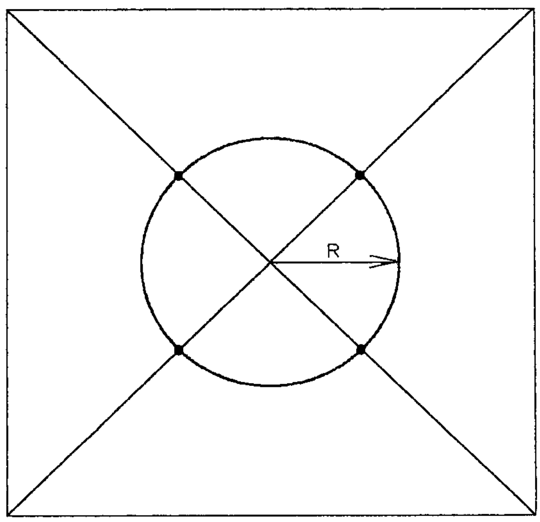

Fig. 2 Four point forces acted on a diagonal of a square plate

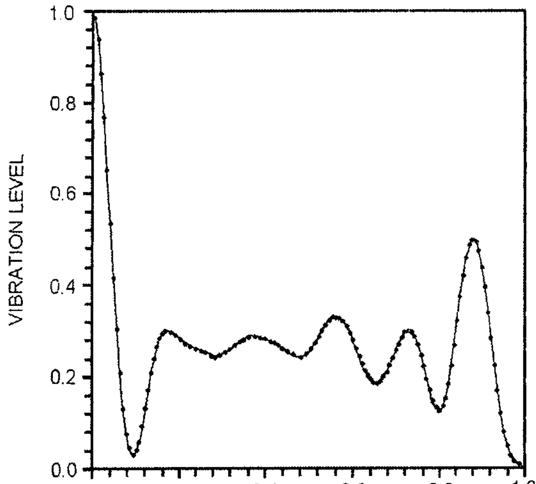

Fig. 3 Vib. level vs. R/Rmax

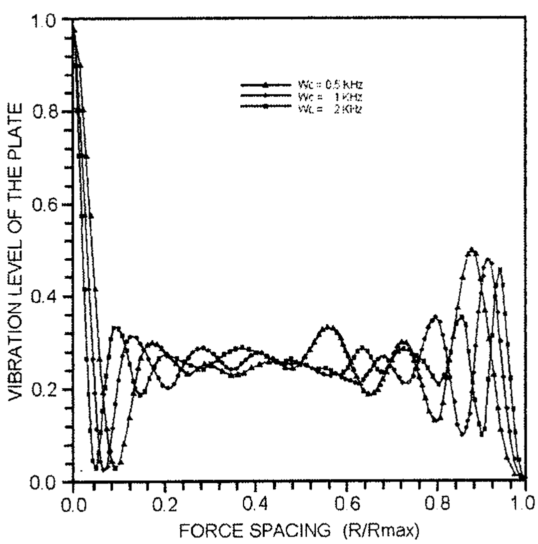

Fig. 4 Vib. level vs. R/Rm

the plate can be controlled to achieve a dramatic reduction, relative to the maximum value corresponding to when the point forces are located at the center of the plate.

Figure 4 shows the total vibration level of the plate for the octave band centered at 500 Hz, 1 KHz, and 2 KHz. The figure shows that the global behaviors of vibration response levels at different frequency bands are similar. Rapid reductions of the vibration level take place at the position of 5 to 15% away from the center

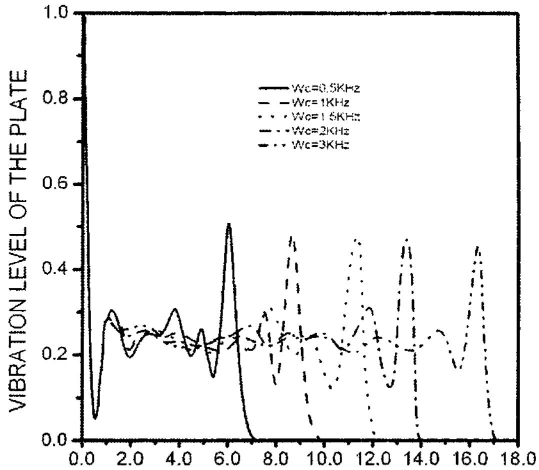

Fig. 5 Vib. level vs. KbR

and at the position of 80 to 90% of the maximum spacing. The spacing between the positions at which the rapid reduction takes place tends to become largeer as the center frequency increases.

In this study dimensionless value $K_b\cdot R=\left(\frac{\rho\omega_c^2}{EI}\right)^{\frac{1}{4}}\cdot R$ is defined and in the limited range of central frequencies from 0.5 kHz to 3 kHz, numerical works ase performed. The vibration level curves show dramatic reductions at the same value of 0.6 for this dimensionless parameter near the center of the plate, for different frequency octave bands as shown in Fig. 5. This shows the possibility that a new dimensionless number could be defined to control plate vibration after further study. Although the optimal spacing distance falls at the same non-dimensional value for certain range of central frequencies, further study on the dimensionless value is required before this dimensionless parameter could be generalized as a design parameter. To generalize the dimensionless parameter, the geometric conditions of the plate including the boundary conditions, as well as its mechanical property and load condition should be simultaneously analyzed and confirmed in the future study.

Figure 6 shows 8 point applied forces to simulate a continuously distributed ring force field. Theoretically the more point forces are used, the

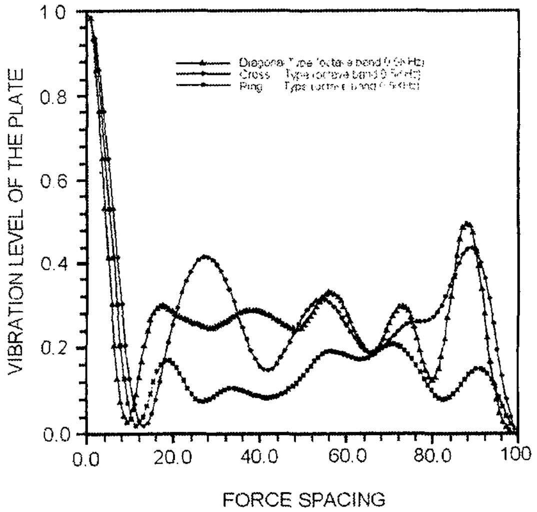

Fig. 6 Ring force acted on a square plate

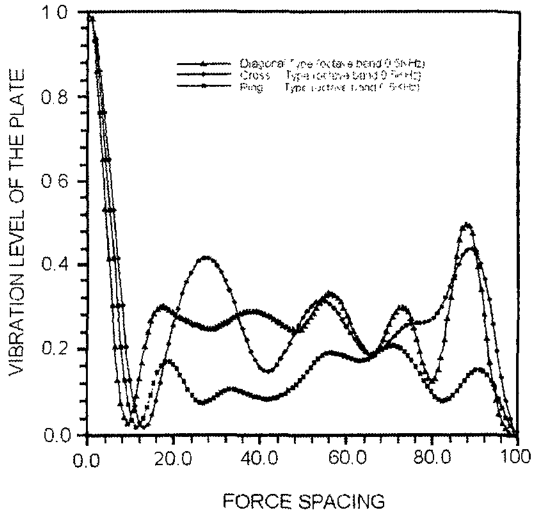

Fig. 8 Vib. level by change of force type

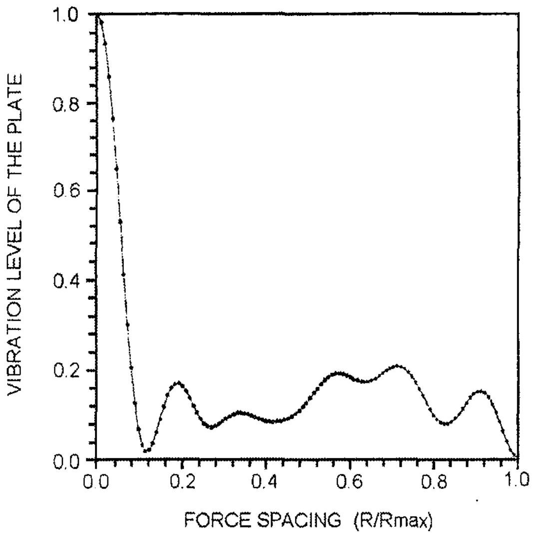

Fig. 7 Vib. level vs. R/Rmas(r=+1)

better the resulting simulation would be, provided the total input force is kept the same. Referring to Peng(1987), eight point forces are enough to produce the same effect as a distributed ring force.

In Fig. 7, vibration level is shown with respect to the force spacing in the case of the ring type octave band excitation, with the center frequency at 500 Hz. The horizontal axis represents the force spacing normalized by the maximum spacing and the vertical axis represents the vibration level normalized by the maximum vibration level when the concentrated force is applied at the center of the plates. Similar to the case of four point forces, great reduction takes place when the plate is excited near the center of the plate.

Vibration levels for two point cross, four point diagonal, and eight point ring excitations are shown in Fig. 8, which shows that the response reduction of the ring force is larger than others. When the number of applied point forces increases, the response reduction capability in the vibration level of the plate is also enhanced.

5. Conclusion

In this study, the specific case of a finite square plate is investigated, for four point diagonal and eight point ring force excitation cases. The spatial distributions of multiple correlated point forces have significant influence on the vibration level of the plate. In the case of a band limited white noise excitation, the spatial distance between the applied forces affects the vibration level of the plate. The vibration level of the plate is influenced by the spacing distance, positions, and types of the applied excitation. For a given excitation band, there was found an optimal spacing distance of the applied forces which leads to a relative minimum in the vibration level of the plate. The scheme could be utilized as a form of structural vibration control.

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