Jour. of the KGS, Vol. 18, No. 3. June 2002, pp. 127~138

A Suggestion of an Empirical Equation for Shear Modulus Reduction Curve Estimation of Sandy Soils

사질토 전단탄성계수 감소곡선 산정을 위한 경험식 제안

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요 지

지진에 의한 지반거동 및 지반-구조물 상호작용 등 지반동역학적 문제분석을 위해서는 정확한 동적 토질전단탄성계수의 획득이 필수적이다. 본 연구에서는 기존의 자료를 조사분석하여 어떤 변형율에서도 활용할 수 있는 사질토 전단탄성계수 감소곡선을 위한 경험식을 제안하였다. 비소성 토질의 전단탄성계수 감소곡선의 위치와 모양은 평균유효구속압에 주로 영향을 받으므로 본 연구에서는 이 영향요소 및 최대전단탄성계수를 이용하여 변형율 증가에 의한 전단탄성계수 감소를 산정할 수 있는 방정식을 형성하였다. 최대전단탄성계수가 측정되면 제안된 식을 이용하여 특정 변형율 및 구속압에서 감소된 전단탄성계수를 산출할 수 있을 것이다.

Abstract

In dynamic analyses such as seismic ground response and soil-structure interaction problems, it is very crucial to obtain accurate dynamic shear modulus of soil deposit. In this study, an extensive data base of available experimental data is compiled and reanalyzed to establish a simple empirical formula for the dynamic shear modulus reduction curve to cover wide range of strain for sandy soils. The proposed empirical equation is to represent the dynamic shear modulus degradation with strain in terms of low-amplitude dynamic shear modulus and effective mean confining pressure, since those factors have the most significant effect on the position and shape of the shear modulus reduction curve for nonplastic soils. If low-amplitude shear modulus is measured, degraded modulus at any shear strain amplitude can be calculated using the proposed equation.

Keywords : Dynamic shear modulus, Effective mean confining pressure, Empirical equation, Modulus reduction curve, Sandy soils

1. Introduction

In analyzing dynamic behavior of soil deposits and soil-structure interaction during strong earthquakes, it is essential to estimate strain-dependent dynamic soil properties. It is known that shear moduli of soils are strongly affected by the shear strain amplitude, especially for strains greater than 10^{-2}%. Consequently, they must be determined as functions of the induced strain in a soil deposit. The maximum shear moduli (G_{max}) can be obtained for shear strain of around 10^{-4}% from in situ test techniques using shear wave velocity measurements. Shear strains induced in surface deposits during strong earthquake motions are estimated to be around 10^{-2}% to 1%

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(Iwasaki and Tatsuoka, 1977), and it is necessary to evaluate strain-dependent dynamic soil properties at those ranges in the laboratory.

Many experimental investigations on dynamic soil properties have been carried out (Kim, et al., 1991), and the results have been formulated for general use (Seed and Idriss, 1970; Hardin and Drnevich, 1972b; Roesler, 1979; Vucetic and Dobry, 1991). Silver and Seed (1971), Khouri (1984), Ishibashi (1992), and Ishibashi and Zhang (1993), whose basic approaches are adopted in this research, proposed unified empirical equations for shear moduli and damping ratios of sand and clay. However, there is still a need for convincing unified relationships between dynamic shear modulus and appropriate state variables and material properties (Rampello, et al., 1994a and b).

In the last two decades, many data have been accumulated and are available and used in general formulations on dynamic soil properties. In this research, an extensive data base of available experimental data is compiled and reanalyzed to establish a simple empirical formula for dynamic shear moduli to cover wide range of strain for sandy soils. The proposed empirical equation will represent the dynamic shear modulus degradation with strain in terms of low-amplitude dynamic shear modulus and effective mean confining pressure.

2. Factors Affecting Shear Modulus Reduction Curves

Even though the general shape of the modulus-strain amplitude curves is generally similar, i.e., decreased shear modulus with increasing strain amplitude, the rate at which the modulus decreases with increasing strain amplitude is not the same for all soils (Hardin and Drnevich, 1972a).

For this reason, it is common to express the cyclic stress-strain behavior by plots of normalized shear modulus (G/G_{max}) as function of cyclic shear strain (γ).

The modulus reduction curve, also called normalized modulus attenuation curves, which describes the variation of the modulus ratio (G/G_{max}) with logarithmic shear strain (γ), has been extensively used to study the strain dependency of shear modulus. The modulus reduction curve and the backbone curve provide the same information. If one of them is defined, the other can be derived. This kind of representation is appropriate since G/G_{max} is dimensionless parameters, which are less affected by various factors such as consolidation stress history (OCR), duration of confinement (time or aging effect), frequency of loading (for earthquake frequencies), and sample disturbance up to moderate strain levels. In addition, the in situ value of strain dependent shear modulus may be estimated from the laboratory G/G_{max} versus curve by determining the in situ low-amplitude

Table 1. Effect of increase of various factors on G/G_{max} of normally and moderately overconsolidated soils (modified from Dobry and Vucetic, 1987; Kramer, 1996)

Increasing Factor	G/Gmax
Confining Pressure, $\bar{\sigma}_0$	stays constant or increases with $\bar{\sigma}_0$, effect decreases with increasing PI
Void Ratio, e	increases with e
Geologic Age, t	may increase with t
Cementation, c	may increase with c
Overconsolidation Ratio, OCR	not Affected
Plasticity Index, PI	increases with PI
Cyclic Strain Amplitude, γ	decreases with γ
Strain Rate($\partial\gamma/\partial t$), or Frequency of cyclic loading	G increases with ($\partial\gamma/\partial t$), but G/G_{max} probably not affected if G and G_{max} are measured at same ($\partial\gamma/\partial t$)
Number of Loading Cycles, N	decreases after N cycles of large γ, (G_{max} measured before N cycles) for clays; for sands, can increase under drained conditions or decreases under undrained conditions

shear modulus from in situ seismic measurements of shear wave velocity (V_s).

The rate of modulus reduction depends primarily on the values of G_{max} and on the shear strength of the soil (Hardin and Drnevich, 1972a), and it is somewhat different for cohesive and cohesionless soils. In general, cohesive soils have a wider range of strain dependent modulus reduction curves than sandy soils. Among some factors affecting position and/or shape of modulus reduction curves, it is known that the effective mean principal stress ($\overline{\sigma_0}$) has major effect (Iwasaki, et al., 1978; Kokusho, 1980; Ishibashi, 1992). The influence of various conditions on the modulus ratio of normally consolidated and moderately overconsolidated soils is described in Table 1.

3. Basic Concept for the Analysis

All reported investigations revealed that shear modulus decreases with increase in shear strain amplitude. To find adequate expressions for modulus degradation with shear strain, several empirical equations have been proposed. For instance, Hardin and Drnevich (1972b) proposed an approximate method of computing shear modulus (G) at any strain level (γ) assuming hyperbolic stress-strain relationships. The soil models such as equivalent linear model, bilinear model (Thiers and Seed, 1968), linear viscoelastic (Kelvin) model (Hardin and Black, 1968), hyperbolic model (Hardin and Drnevich, 1972b), and Ramberg-Osgood model (Ramberg and Osgood, 1943) do not include the effects other than shear strain amplitude. Several investigators, however, proposed empirical equations which do include some other factors. Edil and Luh (1978) also developed equations to find reduced shear modulus at a given raw strain level.

In 1971, Silver and Seed performed NGI simple shear tests on air-dried uniform angular quartz sand. Based on the test results, they presented the relationship of shear modulus, shear strain amplitude, confining pressure, and relative density for the tenth loading cycle as shown in Fig. 1. For further analyses, Silver and Seed read off values of the shear modulus at different constant values of shear strain amplitude from Fig. 1, and plotted shear modulus values as a function of the vertical stress as shown in Fig. 2. It is clear that the relationship between shear modulus and the vertical stress can be represented by a straight line on a log-log plot. The effect of shear strain amplitude in reducing shear modulus values is more pronounced at lower values of applied vertical stress than at higher values. This implies that lines of constant shear strain have positive slopes. Moreover, the slope increases as shear strain amplitude increases. This effect can be represented using the equation as:

$$G = K_n(\overline{\sigma}_v)^n \tag{1}$$

where n is the slope of the line on the log-log plot of vertical stress versus shear modulus, and K_n is a constant which gives the value of the shear modulus when the vertical stress is unity or, intercept of the line

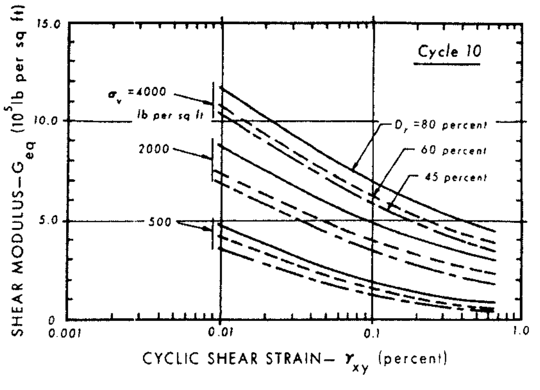

Fig. 1. Effect of vertical stress and relative density on shear modulus of dry sand (Silver and Seed, 1971)

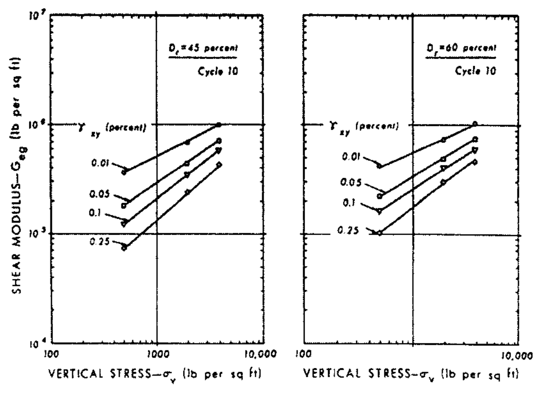

Fig. 2. Relationship between shear modulus and vertical stress at different strain levels (Silver and Seed, 1971)

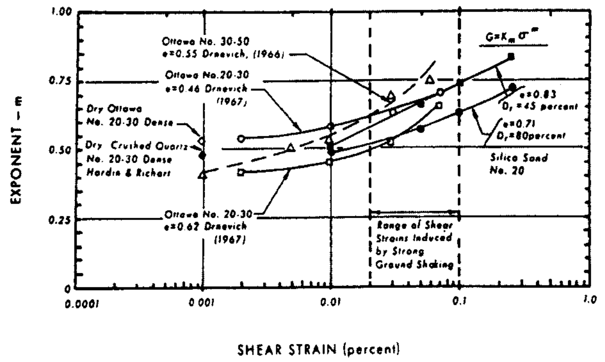

Fig. 3. Effect of shear strain amplitude on measured exponent, *m* (Silver and Seed, 1971)

on the shear modulus axis when $\log \overline{\sigma_v} = 0$. Values of the constant K_n and the exponent n can be readily determined from the plots such as Fig. 2. However, it is important to note that the values of K_n and n are strain dependent, since shear modulus values and its exponent vary with the strain level.

Silver and Seed (1971) reported the values of the exponent n from various investigators at different shear strain amplitude as shown in Fig. 3 (originally, they used notation "m" instead of "n"). They noted that at low shear strain amplitudes, values of n typically are about 0.5 but the magnitude increases for increasing shear strain level. The major disadvantage of procedures described by Silver and Seed (1971) is that the slope and intercept in the plot of shear modulus versus vertical stress are not independent of other parameters, such as relative density as shown in Fig. 2. As discussed in the previous section, this dependency can be minimized by using the shear modulus ratio (G/G_{max}) instead of using shear modulus for the vertical axis in Fig. 2. Therefore, the shear modulus ratio will be used extensively for the ordinate in this paper. Of course, when the ordinate changes, the slope and intercept represent different information.

Iwasaki and Tatsuoka (1977) also described the shear moduli (G) for specimens of clean sands reconstituted in the laboratory by an empirical formula as:

$$G = A(\gamma) \cdot B \cdot \frac{(2.17 - e)^2}{1 + e} \overline{\sigma_0}^{n(\gamma)} \qquad (2)$$

where $A(\gamma)$ and $n(\gamma)$ are soil constants which are a function of shear strain (γ), and B is a parameter showing the influence of grain characteristics and void ratio (e). The value of $A(\gamma)$ was found to decrease with increasing shear strain. On the other hand, the value of exponent, $n(\gamma)$, increased with increasing shear strain. They suggested several values for $A(\gamma)$ and $n(\gamma)$ and several other researchers observed the similar strain dependency of $A(\gamma)$ and $n(\gamma)$ on shear strain (Stokoe, et al., 1978; Porovic and Jardine, 1994).

These findings will be the key argument used in this research to find a new empirical equation for modulus reduction relationship of sandy soils. Many researchers (Hardin and Black, 1969; Hardin and Drnevich, 1972b; Hardin, 1978) found that the low-amplitude shear modulus (G_{max}) of any soil can be expressed using a general form as:

$$G_{\max} = A \cdot (OCR)^K \cdot F(e) \cdot p_a^{1-n} \cdot \overline{\sigma}_o^n \qquad (3)$$

As strain increases, the stiffness of a soil element (G_{max}) starts to decrease. When shear modulus of soil is decreasing, Eq. (3) should be changed for the softening of soil stiffness. As mentioned above, the constant A starts to decrease and the exponent (n) of mean effective principal stress ($\overline{\sigma_0}$) starts to increase as shear strain increases. The equivalent shear modulus (G) may be generally expressed as (Iwasaki, et al., 1978; Tatsuoka, et al., 1978; Kokusho, 1980; Ishibashi, 1981; Khouri, 1984; Ishibashi and Zhang, 1993):

$$G = A(\gamma) \cdot (OCR)^K \cdot F(e) \cdot p_a^{1-n} \cdot \overline{\sigma}_o^{n(\gamma)} \qquad (4)$$

where $A(\gamma)$ is a different value of A from that given in Eq. (3) with shear strain (γ), and $n(\gamma)$ is a different value of n from that given in Eq. (3) with. From Eqs. (3) and (4), shear modulus ratio (G/G_{max}) can be expressed as:

$$\frac{G}{G_{\max}} = \frac{A(\gamma)}{A} \cdot \overline{\sigma_0}^{n(\gamma)-n} \qquad (5)$$

Using Eq. (5) for representing the shear modulus reduction relationship is a reasonable approach, because the position of the shear modulus reduction curves of

Table 2. Summary of references used for sandy soils and index properties

Reference	Test Type*	Name	Strain for G_{max} (%)	Pressure Range (kPa)	Remark
Iwasaki and Tatsuoka (1977)	RC	Toyoura sand	10^{-4}	24~389	subangular, C_u = 1.46, e_{max} = 0.96, e_{min} = 0.64
Iwasaki, et al. (1978)	RC, TS	Toyoura sand	10^{-4}	24.5~196.1	-
Tatsuoka, et al. (1979)	RC, TS	Toyoura sand	10^{-4}	19.6~392.3	G_s = 2.64, e = 0.62~0.70
Kokusho (1980)	CTX	Toyoura sand	10^{-4}	20~300	D_{50} = 0.29mm, C_u = 1.8, e_0 = 0.64
Kokusho and Esashi (1981)	CTX	Toyoura sand	10^{-4}	20~300	e_0 = 0.64
Alarcon-Guzman (1986)	RC/TS	Ottawa sand	10^{-4}	28.6~202.0	e_0 = 0.635, D_r = 40%
Ni (1987)	RC/TS	mortar sand	$<10^{-3}$	82.7~137.9	γ_{dry} = 99.6pcf, e_0 = 0.673
Ladd, et al. (1989)	CTX	Monterey No.0 sand	$<10^{-3}$	25.5~191.5	G_s = 2.65, D_{50} = 0.36mm, C_u = 1.5
Saxena and Reddy (1989)	RC	Monterey No.0 sand	10^{-4}	49~588	G_s = 2.65, C_u = 1.5, D_r = 25%, 60%
Yasuda and Matsumoto (1993)	CTSS	Toyoura sand	$<5\times10^{-4}$	100~400	e_0 = 0.74
Hardin, et al. (1994)	RC	Ottawa sand	10^{-4}	303~3470	e_0 = 0.547
Kanatani, et al. (1994)	CTX	Toyoura sand	$<3\times10^{-4}$	2.9~49.0	G_s = 2.64, C_u = 1.46, e_{max} = 0.975, e_{min} = 0.632
Macari and Ko (1994)	RC	non-plastic marine silt	$<10^{-3}$	13.8~344.5	-
Yamashita and Toki (1994)	CTX, TS	Ishikari sand	10^{-4}	90~270	D_{50} = 0.12mm, C_u = 1.7, e_{max} = 1.416, e_{min} = 0.837
Borden, et al. (1996)	RC/TS	silty sand	10^{-4}	25~100	e_0 = 0.75

*RC = resonant column, TS = torsional shear, CTX = cyclic triaxial, CTSS = cyclic torsional simple shear, RC/TS = combined RC and TS

sandy soils is determined predominantly by the effective mean principal stress ($\bar{\sigma}_0$). Eq. (5) is applicable regardless of soil type and it can be seen that at low shear strain, say 10^{-4}%, *A(γ)/A* should be unity and *n(γ)-n* zero. Using these boundary conditions, data analyses were performed to determine the dependency of *A(γ)/A* and *n(γ)-n* on shear strain amplitude. To determine values of *A(γ)/A* and *n(γ)-n* more effectively, logarithmic functions are applied to both sides of Eq. (5) as:

$$\log\left(\frac{G}{G_{\max}}\right) = \log\left(\frac{A(\gamma)}{A}\right) + (n(\gamma) - n)\cdot\log\bar{\sigma}_0 \quad (6)$$

Eq. (6) is a simple equation for a line whose slope is [*n(γ)-n*] with the intercept of log [*A(γ)/A*] in a plot of log($\bar{\sigma}_0$) versus log(G/G_{max}). Using SI units, available data are rearranged to construct plots of log ($\bar{\sigma}_0$)=0 versus log(G/G_{max}), and slopes and the intercept at log ($\bar{\sigma}_0$)=0 or $\bar{\sigma}_0$=1kPa are determined at various levels of shear strain amplitude. After determining *A(γ)/A* and *n(γ)-n* at given strain levels, functions of *A(γ)/A* and *n(γ)-n* with strain amplitude are found by regression analyses.

No attempt was made to formulate the values of *A (γ)/A* and *n(γ)-n* as a function of shear strain amplitude until Khouri (1984) proposed hyperbolic functions to describe the effects of shear strain amplitude. Later, with some modification, similar functions were used by Ishibashi and Zhang (1993) based on the same data points reported by Khouri (1984). Since then, more data have been reported in the literature and a new attempt to describe the shapes of *A(γ)/A* and *n(γ)-n* is possible. Totally different sets of data are used, and the data compiled for this paper and the analyses are presented in the following sections.

4. Analysis and Result

Seventeen sets of shear modulus reduction curves with various effective mean principal stresses ($\bar{\sigma}_0$) were

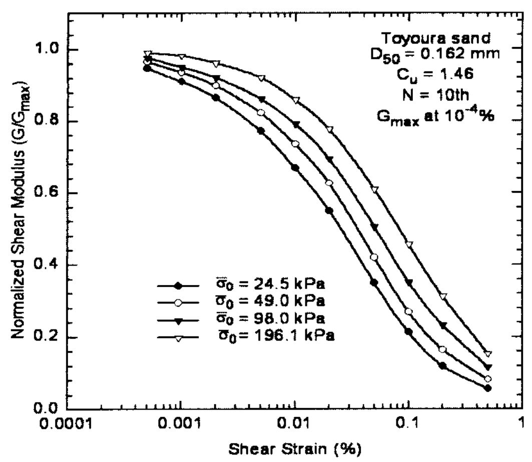

Fig. 4. Normalized shear modulus versus shear strain of toyoura sand (data from Iwasaki, et al., 1978)

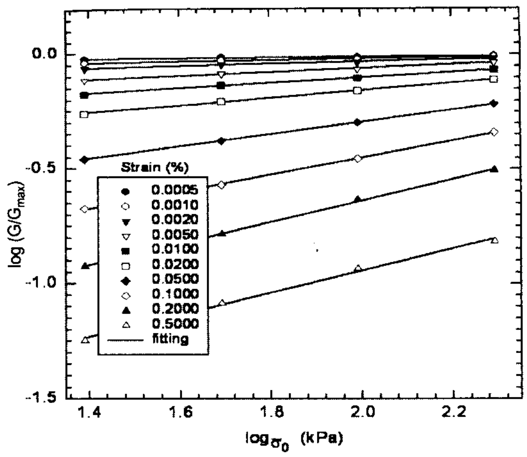

Fig. 5. Log of normalized shear modulus versus log of effective mean principal stress at various shear strains for toyoura sand

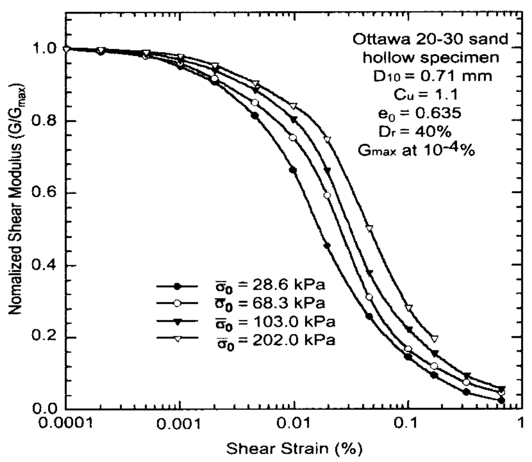

Fig. 6. Normalized shear modulus versus shear strain of ottawa 20-30 sand (data from Alarcon-Guzman, 1986)

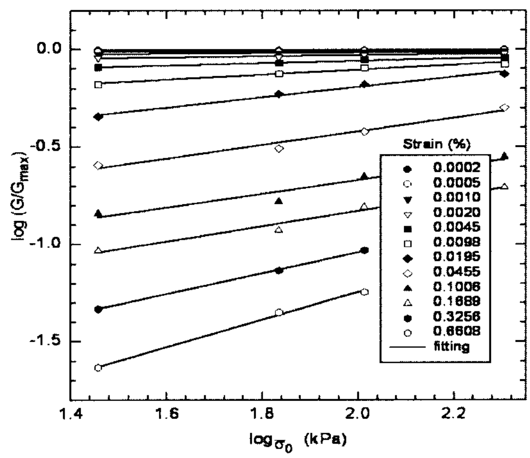

Fig. 7. Log of normalized shear modulus versus log of effective mean principal stress at various shear strains for ottawa sand

collected, manipulated, and analyzed from fifteen references for sandy soils. These references were selected among others because they applied various $\bar{\sigma}_0$ so that the effect of $\bar{\sigma}_0$ on modulus reduction curves could be shown clearly. Procedures explained in the previous section are followed to find the values of $A(\gamma)/A$ and $n(\gamma)-n$, which were defined by Eq. (6), for each data set. Finally, those values are analyzed as a function of shear strain amplitude, so that complete modulus reduction curves for sand can be expressed based on these values.

For this study, shear modulus reduction curves with various confining pressures were sought in the literature. Table 2 shows the references used for sandy soils with information about the soils and the test type. Fig. 4 presents normalized shear modulus (G/G_{max}) versus shear strain (γ) for Toyoura sand reported by Iwasaki, et al. (1978). Based on this figure, shear strain amplitudes are selected, and at a given shear strain, dependency of G/G_{max} on effective mean principal stress is determined. Fig. 5 illustrates the plot of log (G/G_{max}) versus log ($\bar{\sigma}_0$) based on the data by Iwasaki, et al. (1978), where best fitting lines determined using the least-squares method are also shown. The slopes and intercepts of each line are determined by multiple linear regression analyses. Similar procedures are applied to other available data, and regression analyses were performed for each data set. Another example is presented in Figs. 6 and 7 for Ottawa sand reported by Alarcon-Guzman (1986).

All the results from multiple linear regression analyses for each reference are shown in Table 3. The values for $A(\gamma)/A$ are determined by converting the intercepts as $10^{\text{intercept}}$ and the values of $n(\gamma)-n$ are the slopes themselves.

Table 3. Results of multiple linear regression analyses for the determination of A(γ)/A and n(γ)-n of sandy soil

(a) Toyoura sand from Iwasaki and Tatsuoka (1977)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)-n
0.0010	-0.0760	0.0215	0.525	0.8394	0.0215
0.0100	-0.2612	0.0705	0.957	0.5481	0.0705

*average coefficient of determination (r^2) = 0.741

(b) Toyoura sand from Iwasaki, et al. (1978)

Strain(%)	Intercept	Slope	r2*	A(γ)/A	n(γ)-n
0.0005	-0.0516	0.0207	0.993	0.8879	0.0207
0.0010	-0.0886	0.0346	0.983	0.8154	0.0346
0.0020	-0.1311	0.0493	0.991	0.7394	0.0493
0.0050	-0.2286	0.0839	0.996	0.5907	0.0839
0.0100	-0.3377	0.1189	0.998	0.4595	0.1189
0.0200	-0.4849	0.1643	0.998	0.3274	0.1643
0.0500	-0.8304	0.2683	1.000	0.1478	0.2683
0.1000	-1.1883	0.3691	0.999	0.0648	0.3691
0.2000	-1.5708	0.4667	0.999	0.0269	0.4667
0.5000	-1.9034	0.4798	0.996	0.0125	0.4798

*average coefficient of determination (r^2) = 0.995

(c) Dry toyoura sand from Tatsuoka, et al. (1979)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)-n
0.0010	-0.0141	-0.0081	0.506	0.9680	-0.0081
0.0100	-0.0968	0.0150	0.821	0.8002	0.0150

*average coefficient of determination (r^2) = 0.664

(d) Toyoura sand from Kokusho (1980)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)-n
0.0032	-0.0801	0.0198	0.977	0.8317	0.0198
0.0099	-0.1897	0.0551	0.996	0.6461	0.0551
0.0212	-0.3365	0.1053	0.978	0.4608	0.1053
0.0516	-0.6135	0.1939	0.989	0.2435	0.1939
0.0973	-0.8610	0.2612	0.993	0.1377	0.2612
0.1745	-1.1479	0.3237	0.999	0.0711	0.3237
0.2972	-1.4505	0.3896	0.999	0.0354	0.3896

*average coefficient of determination (r^2) = 0.990

(e) Dense toyoura sand from Kokusho and Esashi (1981)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)-n
0.0002	-0.0065	0.0020	0.646	0.9851	0.0020
0.0004	-0.0113	0.0026	0.498	0.9743	0.0026
0.0007	-0.0233	0.0059	0.446	0.9477	0.0059
0.0010	-0.0317	0.0081	0.700	0.9297	0.0081
0.0020	-0.0389	0.0076	0.471	0.9144	0.0076
0.0040	-0.0755	0.0178	0.758	0.8404	0.0178
0.0070	-0.1227	0.0320	0.796	0.7538	0.0320
0.0100	-0.1695	0.0481	0.84	0.6769	0.0481
0.0200	-0.2785	0.083	0.865	0.5266	0.083
0.0400	-0.4724	0.1489	0.887	0.3369	0.1489
0.0700	-0.6953	0.22	0.917	0.2017	0.2200
0.1000	-0.8921	0.2802	0.913	0.1282	0.2802
0.2000	-1.2417	0.3594	0.884	0.0573	0.3594

*average coefficient of determination (r^2) = 0.740

(f) Ottawa 20-30 sand from Alarcon-Guzman (1986)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)-n
0.0002	-0.0100	0.0041	0.921	0.9772	0.0041
0.0005	-0.0173	0.0054	0.561	0.9609	0.0054
0.0010	-0.0479	0.0168	0.978	0.8956	0.0168
0.0020	-0.0836	0.0272	0.924	0.8248	0.0272
0.0045	-0.1729	0.0572	0.969	0.6716	0.0572
0.0098	-0.3552	0.1250	0.97	0.4413	0.1250
0.0195	-0.7130	0.2598	0.984	0.1936	0.2598
0.0455	-1.1125	0.3461	0.967	0.0772	0.3461
0.1006	-1.3765	0.3534	0.933	0.0420	0.3534
0.1689	-1.6121	0.3917	0.975	0.0244	0.3917
0.3256	-2.1162	0.5386	0.999	0.0077	0.5386
0.6608	-2.6557	0.7060	0.997	0.0022	0.7060

*average coefficient of determination (r^2) = 0.932

(g) Washed mortar sand from Ni (1987)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)-n
0.0003	0.0042	-0.0023	0.611	1.0097	-0.0023
0.0005	0.0072	-0.0042	0.781	1.0166	-0.0042
0.0010	-0.0221	0.0080	0.218	0.9505	0.0080
0.0020	-0.0747	0.0298	0.669	0.8420	0.0298
0.0043	-0.1572	0.0627	0.815	0.6964	0.0627
0.0098	-0.2820	0.1088	0.93	0.5223	0.1088
0.0179	-0.3683	0.1336	0.914	0.4283	0.1336
0.0355	-0.5204	0.1771	0.916	0.3017	0.1771
0.0737	-0.7479	0.2415	0.950	0.1787	0.2415

*average coefficient of determination (r^2) = 0.756

(h) Monterey No.0 sand from Ladd, et al. (1989)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.0021	0.0165	−0.0140	0.897	1.0386	−0.0140
0.0040	−0.0400	0.0017	0.229	0.9121	0.0017
0.0075	−0.1109	0.0209	0.912	0.7746	0.0209
0.0144	−0.2206	0.0551	0.967	0.6017	0.0551
0.0275	−0.3921	0.1149	0.988	0.4054	0.1149

*average coefficient of determination (r^2) = 0.798

(i) Monterey sand at D_r = 25% from Saxena and Reddy (1989)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.0016	−0.0295	0.0002	0.001	0.9344	0.0002
0.0027	−0.0410	0.0014	0.006	0.9099	0.0014
0.0051	−0.0766	0.0144	0.708	0.8383	0.0144
0.0102	−0.1668	0.0468	0.815	0.6811	0.0468
0.0186	−0.2061	0.0552	0.957	0.6222	0.0552
0.0346	−0.2094	0.0425	0.715	0.6174	0.0425

*average coefficient of determination (r^2) = 0.533

(j) Monterey sand at D_r = 60% from Saxena and Reddy (1989)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.0013	−0.0184	0.0052	0.509	0.9586	0.0052
0.0039	−0.1225	0.0405	1.000	0.7543	0.0405
0.0103	−0.1704	0.0504	0.804	0.6754	0.0504
0.0287	−0.1617	0.0356	0.644	0.6891	0.0356

*average coefficient of determination (r^2) = 0.739

(k) Toyoura sand from Yasuda and Matsumoto (1993)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.00100	−0.18832	0.06239	0.779	0.6482	0.0624
0.01000	−0.38015	0.12061	0.956	0.4167	0.1206
0.10000	−1.47444	0.47635	0.995	0.0335	0.4764

*average coefficient of determination (r^2) = 0.910

(l) Ottawa sand from Hardin, et al. (1994)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.0004	−0.0069	0.0014	0.914	0.9842	0.0014
0.0007	−0.0050	0.0006	0.878	0.9886	0.0006
0.0010	−0.0149	0.0034	0.738	0.9663	0.0034
0.0020	−0.0276	0.0058	0.755	0.9384	0.0058
0.0040	−0.048	0.0098	0.842	0.8954	0.0098
0.0060	−0.0649	0.0124	0.78	0.8612	0.0124
0.0090	−0.1163	0.0242	0.863	0.7650	0.0242
0.0145	−0.1888	0.0391	0.897	0.6474	0.0391

*average coefficient of determination (r^2) = 0.834

(m) Toyoura sand from Kanatani, et al. (1994)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.0003	−0.0025	0.0003	0.007	0.9943	0.0003
0.0008	−0.0254	0.0078	0.313	0.9433	0.0078
0.0020	−0.0865	0.0333	0.776	0.8194	0.0333
0.0050	−0.1998	0.0767	0.884	0.6312	0.0767
0.0100	−0.3467	0.129	0.967	0.45	0.1290
0.0300	−0.7487	0.2776	0.973	0.1784	0.2776
0.0600	−1.1853	0.4582	0.996	0.0653	0.4582
0.1000	−1.6834	0.7100	0.909	0.0207	0.7100

*average coefficient of determination (r^2) = 0.728

(n) Alaskan NP silt (OCR = 10−250) from Macari and Ko (1994)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.0007	0.0013	−0.0010	0.787	1.0031	−0.0010
0.0010	−0.0068	0.0011	0.563	0.9844	0.0011
0.0020	−0.0545	0.0183	0.829	0.8822	0.0183
0.0040	−0.1228	0.0393	0.942	0.7537	0.0393

*average coefficient of determination (r^2) = 0.780

(o) Alaskan NP silt (OCR = 6−150) from Macari and Ko (1994)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.0008	−0.0002	−0.0009	0.171	0.9996	−0.0009
0.0010	−0.0057	0.0009	0.137	0.9869	0.0009
0.0020	−0.0388	0.0115	0.871	0.9145	0.0115
0.0040	−0.1056	0.0323	0.975	0.7842	0.0323
0.0060	−0.1659	0.0516	0.960	0.6824	0.0516

*average coefficient of determination (r2) = 0.623

(p) Ishikari sand from Yamashita and Toki (1994)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.0010	−0.0272	0.0034	0.013	0.9394	0.0034
0.0100	−0.2083	0.0511	0.936	0.6191	0.0511
0.1000	−0.6655	0.1544	0.987	0.2160	0.1544
0.2000	−0.9472	0.2287	0.992	0.1129	0.2287
0.5000	−1.4535	0.3633	0.968	0.0352	0.3633

*average coefficient of determination (r^2) = 0.779

(q) Piedmont residual silty sand from Borden, et al. (1996)

Strain(%)	Intercept	Slope	r^{2*}	A(γ)/A	n(γ)−n
0.0010	−0.0511	0.0147	0.998	0.8890	0.0147
0.0034	−0.1797	0.0538	0.954	0.6611	0.0538
0.0101	−0.4160	0.1272	0.994	0.3837	0.1272
0.0366	−0.7346	0.2162	0.999	0.1843	0.2162
0.0996	−0.9967	0.2939	1.000	0.1008	0.2939

*average coefficient of determination (r^2) = 0.989

Coefficients of determination (r^2) are given for each regression line and averaged to illustrate the overall fitting. Using the results of multiple linear regression analyses, it is possible to draw plots for $A(\gamma)/A$ and $n(\gamma)-n$ as functions of shear strain amplitudes. Fig. 8 shows the plot of $A(\gamma)/A$ versus shear strain amplitude, and an equation from nonlinear curve fitting as:

$$\frac{A(\gamma)}{A}=\frac{1.007}{1+\left(\frac{\gamma(\%)}{0.0146}\right)^{0.9908}}\cong\frac{1}{1+68\gamma(\%)} \qquad (7)$$

with r^2 = 0.925, which indicates a reasonable correlation between $A(\gamma)/A$ and shear strain. Here, shear strain must be in percentage. Several other functions were tried to express the best fitting, but the log form produces the best result. The value of $A(\gamma)/A$ is 1.0 at the strain level of 10^{-4}%. By definition it is true because $A(\gamma)/A$ represents the stiffness degradation with shear strain amplitude at $\log(\bar{\sigma}_0)$= 0 or $\bar{\sigma}_0$=1kPa and it was assumed that there is no stiffness reduction in low-amplitude shear strain (10^{-4}%). From the result of curve fitting, the value of $A(\gamma)/A$ is found to be about 0.015 (1.5%) at the shear strain of 1%.

Similarly, nonlinear curve fitting is tried to find the relationship between the values of $n(\gamma)-n$ and shear strain amplitude. Fig. 9 shows the plot of $n(\gamma)-n$ versus shear strain amplitude, and a curve fitting equation as:

$$n(\gamma)-n=0.684-\left[\frac{0.685}{(1+26.2\gamma(\%))^{0.504}}\right] \cong 0.68\left[1-\frac{1}{(1+26\gamma(\%))^{0.5}}\right] \qquad (8)$$

with r^2 = 0.823, which indicates that the correlation between $n(\gamma)-n$ and shear strain is weaker than that between $A(\gamma)/A$ and shear strain, but still tolerable. Here again, shear strain must be in percentage. As opposed to $A(\gamma)/A$, the modified hyperbola produces the best result among several other functions. The value of $n(\gamma)-n$ is zero at the strain level

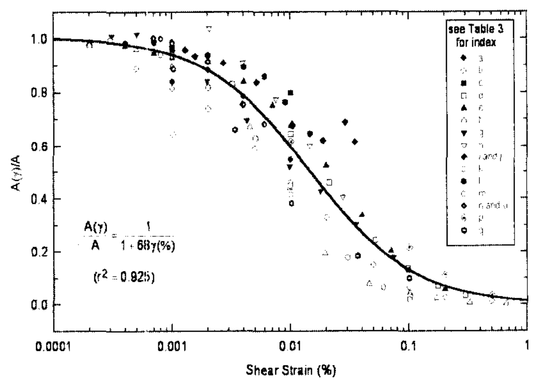

Fig. 8. Relationship between A(γ)/A and shear strain amplitude

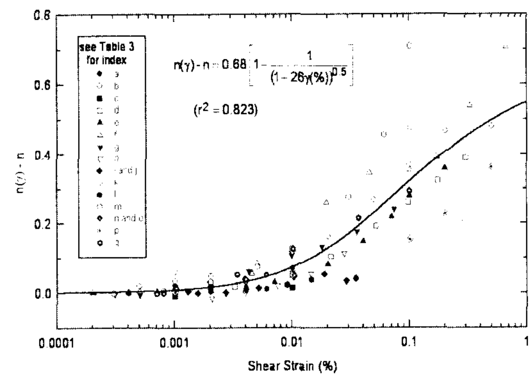

Fig. 9. Relationship between n(γ) − n and shear strain amplitude

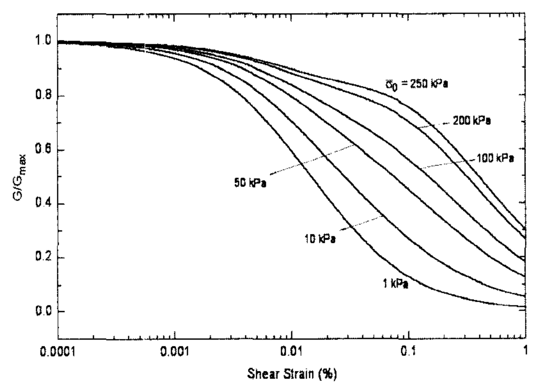

Fig. 10. Modulus reduction curves for sandy soils based on regression analyses

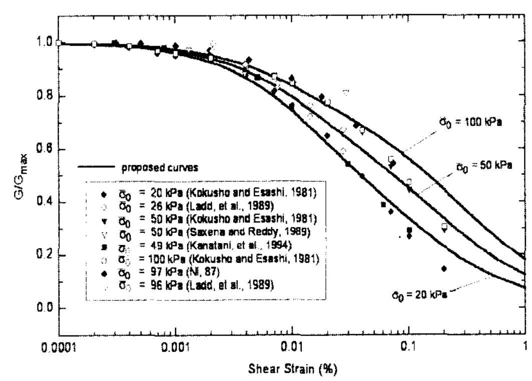

Fig. 11. Comparisons of proposed modulus reduction curves with reported values for sandy soils at confining pressures between 20 and 100kPa

of 10^{-4}% by definition. The value of $n(\gamma)-n$ represents the change in the exponent of effective mean principal stresses with shear strain amplitude, and $n(\gamma)$ equals n at low-amplitude shear strain. From the result of curve fitting, $n(\gamma)-n$ is found to be about 0.549 at the shear strain of 1%. Therefore, the change in the exponent of $\bar{\sigma}_0$ with shear strain amplitude will be easily determined using Eq. (8), if the exponent of $\bar{\sigma}_0$ at low-amplitude is given. If n value for low-amplitude shear modulus is, say, 0.5, Eq. (8) gives n = 0.5 at 10^{-4}% of strain amplitude and n = 1.049 at 1% of strain amplitude. Porovic and Jardine (1994) reported the range of n was 0.5 to 1.0 when shear strain increased from low amplitude to 1%. Yu and Dakoulas (1993) stated that n value might range from 0.35 to 0.90 for different soils. Hryciw and Thomann (1993) presented the n value of cohesionless soils varying a range from 0.39 to 0.72. They also showed that n of unloading and reloading will be different from n on virgin loading.

Substituting Eqs. (7) and (8) into Eq. (5), the final equation for modulus reduction curve for sandy soil is expressed as:

$$\frac{G}{G_{\max}} = \left(\frac{1}{1+68\gamma(\%)}\right) \cdot \bar{\sigma}_0^{\left[0.68\left(1-\frac{1}{(1+26\gamma(\%))^{0.5}}\right)\right]} \quad (9)$$

The shear modulus ratio (G/G_{max}) is now not only the function of shear strain amplitude (γ) but also the function of effective mean principal stress ($\bar{\sigma}_0$). For actual calculation, the unit of kPa should be used for $\bar{\sigma}_0$, and percentage for γ.

Fig. 10 presents shear modulus reduction curves generated using Eq. (9) at various $\bar{\sigma}_0$ for sandy soils. Curves are not perfectly smooth because they are based on the regression analyses. Since data used in the analysis cover mostly about up to 300kPa, Fig. 10 is not for very high confining pressures. Fig. 11 illustrates the comparison of proposed modulus reduction curves with several reported values at confining pressures from 20 kPa to 100 kPa, where a reasonable trend can be observed.

5. Conclusions

It is well known that the adequate estimation of shear modulus and its degradation with strain in soils are very important to the analysis of all soil vibration problems. A single value of modulus is inappropriate because it depends on strain amplitude, state of stress, and other environmental conditions. Shear modulus reduction curves versus shear strain are convenient ways of studying strain dependency. Among several factors, the effective mean principal stress ($\bar{\sigma}_0$) has the most significant effect on the position and shape of the shear modulus reduction curve for nonplastic soils. This factor is used to find a new empirical equations for sandy soils. It is obvious that experimentally determined shear moduli are usually more accurate than empirically determined ones, provided that the experimental methods are appropriate. However, empirical correlation can be useful to confirm the data obtained from laboratory and field tests. Empirical correlations are sometimes better than laboratory and field tests for several reasons. Firstly, empirical equations are easy and fast to use. Secondary, laboratory tests are not always possible to cover vast areas of interest, and retrieving perfect samples can be extremely costly, if not virtually impossible. Thirdly, field tests can not be fully performed in a wide strain range of interest.

In this study available experimental data on dynamic shear moduli of various sandy soils are compiled and reanalyzed. Those data were used to propose a simple empirical equation for modulus reduction curves of sandy soils as indicated in Eq. (9). The proposed closed-form solution presents the dynamic shear moduli in terms of maximum dynamic shear modulus, shear strain amplitude, and mean effective confining pressure.

If low-amplitude shear modulus (G_{max}) is known, degraded modulus at any shear strain amplitude and confining pressure can be calculated using the proposed equation which has a simple expression. Although the availability of experimental data is limited and the result predicted by the empirical equation is based on the reported values, and therefore, cannot be more accurate than the values used for the analyses, the formula fits those data reasonably and can be employed to determine the shear modulus reduction curves for sandy soils in dynamic analyses such as seismic ground response and soil-structure interaction problems.

Acknowledgements

The result of this research is part of the residual strength measurement system development for post-liquefied granular soils project sponsored by the Multi-disciplinary Center for Earthquake Engineering Research from 1994 to 1998. The financial support from the MCEER is gratefully acknowledged.

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(received on May 2, 2002, accepted on Jun. 8, 2002)