Jour. of the KGS, Vol. 18, No. 3. June 2002, pp. 95~104

The Use of Reliability-based Approach to Design Anchored Sheet Pile Walls

신뢰성에 근거한 앵커 널말뚝의 설계방안 연구

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요 지

본 연구에서는 앵커 널말뚝의 설계값 결정과정에 신뢰성이론을 적용한 설계방법을 제안하였다. 본 설계과정에서 먼저 말뚝의 근입깊이는 공학적으로 널리 사용되는 확률론적 수치계산방법에 따라 근입된 지반조건의 불확실성의 정도에 따라 결정되도록 하였다. 여기서 적용되는 확률론적 수치계산방법은 복잡한 계산과정이 필요하다거나 정확한 계산을 위해서는 지반강도 정수들에 대한 광범위한 통계적 분석이 별도로 필요하다든가 하는 번거로움 없이 설계자가 간편하게 사용할 수 있다. 본 연구에서 제안된 설계법의 결과들은 일반적으로 널리 이용되는 다른 결정론적 설계법에 의한 결과들과 호응하는 것으로 나타났다. 본 연구에서는 아울러 널말뚝의 주요 설계입력변수들의 불확실성에 따른 설계결과의 영향을 분석하기 위한 민감도 조사가 실시되었다.

Abstract

In this study, a reliability-based design (RBD) procedure for determining design values for anchored sheet pile wall is proposed considering overturning about the anchor point as the major failure mode. In this design procedure, the depth of embedment of the sheet pile wall is logically chosen in accordance with degrees of uncertainties of design input parameters using approximate probabilistic computation methods. These methods have been successfully used in the geotechnical engineering requiring neither understandings of complex probabilistic theories nor efforts to prepare more data. It was investigated that the design results by the proposed method were compatible with those by commonly used deterministic design methods. Additionally, in an effort to investigate the effects of changes in the degree of uncertainties of major design variables on the design results of the sheet pile wall, a sensitivity analysis was performed.

Keywords : Reliability-based design, Reliability index, Sheet pile wall, Uncertainty analysis

1. Introduction

In designing geotechnical structures, decisions are made under a great deal of uncertainty that may lead finite risks exceeding limit states of the structures. Classically, in order to minimize the risks, conventional factors of safety based on deterministic analyses are commonly used in the design. It is not clear, however, how they relate to the reliabilities for the limit states of geotechnical structures. Uncertainties in the geotechnical design exist in estimating in-situ engineering soil properties and determining subsoil profile and boundary

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conditions (Höeg and Murarka, 1974; Phoon and Kulhawy, 1994). In common deterministic design procedures of anchored sheet piles, the depth of embedment and the force in the anchor rod are determined satisfying equilibrium of moments around the anchor point and horizontal forces regardless of the degrees of uncertainties in design parameters. Therefore, no indication regarding the reliability of the sheet pile wall is given to the design engineer during the design procedure. When design criteria corresponding to a certain level of structural safety of the sheet pile wall are developed, it should be considered how much the variability of the input design parameters is sensitive to the reliability of design outputs. It is the purpose of this study to establish a reliability-based design process for determining the depth of embedment in the anchored sheet pile wall and for assessing the stability of the wall structure in accordance with degrees of uncertainties of design input parameters.

2. Analysis and Design of Anchored Sheet Pile Wall

Figs. 1 and 2 show typical anchored sheet pile walls embedded in sandy and clay soils. The depth of embedment and the force in the anchor rod are obtained by a deterministic design procedure based on free-earth support approach (Das, 1995). In brief, the procedure can be presented as follows;

1. Compute Ka and Kp
2. Compute lateral pressures, p_i
3. Compute lateral force, P_i
4. Determine the theoretical depth of embedment, D_{theory}, so that moment of passive thrust about the anchor point, M_p, is equal to moment of active thrust about the anchor point, M_a
5. Determine the design depth of embedment, D_{design}, multiplying D_{theory} by a certain safety factor
6. Determine the anchor force, F, with which the sum of forces in the horizontal direction is equal to 0.

In this deterministic procedure, the value chosen for the safety factor reflects the engineer's own past experience with similar problems. Improvements in decision-making for selecting appropriate safety factors for sheet pile walls under a variety of soil conditions may be achieved by applying probabilistic principles to the design process. Once the variability of soil strength properties in various ground conditions is characterized with statistical moments and its effect on the stability of the sheet pile wall is quantified, the engineer can assign appropriate safety factors for the wall by well-known probabilistic computation procedures regarding a certain probability of failure that is determined by its experience.

3. Reliability Model Applied to the Design of Anchored Sheet Pile Wall

3.1 Variability of Soil Strength Properties

It is very important to estimate the variability of soil strength parameters involved in producing design outputs.

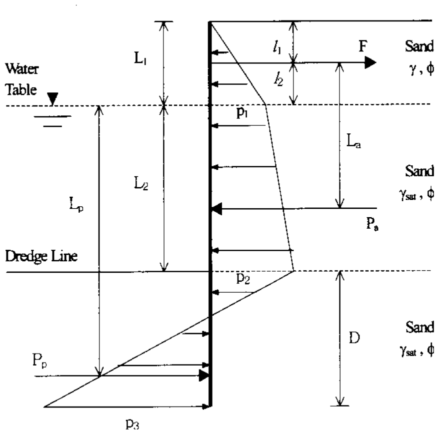

Fig. 1. Embedded in sandy soil

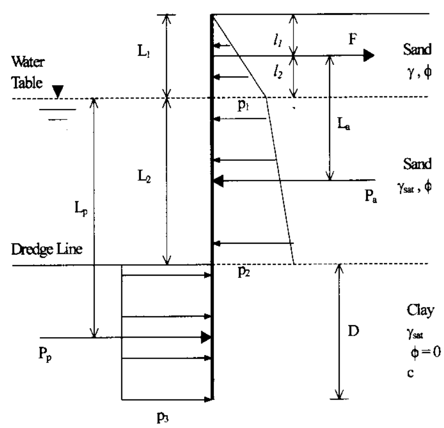

Fig. 2. Embedded in clayey soil

Table 1. Summary of soil variability

Properties	Previous Investigations		This Study
	Range of COV (%)	Reference	COV (%)
Unit Weight	3 - 7	4	5
	1 - 10	7	
Saturated Unit Weight	0 - 10	4	7
Effective Stress Friction Angle	2 - 13	4	15
	3 - 15	7	
Undrained Shear Strength	10 - 35	2	20
	13 - 40	4	
	20 - 65	7	
Vane Shear Test Undrained Shear Strength	15 - 50	2	
	10 - 20	4	

When sufficient data from in-situ and laboratory tests are not available, a possible approach to characterize realistic statistical properties of design soil parameters is to use estimates based on published values, which are most conveniently expressed in terms of the coefficient of variation (COV) (Duncan 2000) :

$$COV = \frac{\text{standard deviation}}{\text{avaerage value}} = \frac{\sigma_x}{m_x} \quad (1)$$

General ranges of COV for common soil strength parameters are summarized in Table 1. It should be noted that the values shown in Table 1 provide only a rough guide for estimating values of COV for any given case and the engineer's judgment must be primarily used in determining appropriate values of COV from published sources. Table 1 also shows the values used in this study.

3.2 Probabilistic Models for the Estimation of Variability of Design Results

In order to estimate the variability of design results in terms of their means and standard deviations, the First Order Second Moment (FOSM) method that involves approximation based on Taylor series expansion (Ang and Tang 1975) and the Point Estimate Method (Rusenblueth 1975) are employed in this study.

3.2.1 First Order Second Moment (FOSM) Model

If the Taylor series expansion for a performance function of several random variables, $g(\underline{x})$, is performed about the mean values of the random variables and only first order terms are retained, approximate mean value and standard deviation of the function can be mathematically expressed as

$$E[g(\underline{x})] \approx g(m_x) \quad (2)$$

$$\sigma^2_{g(x)} = \nabla G^T COV(\underline{x}) \nabla G \quad (3)$$

where,

m_x : vector of the mean values of random variables $\underline{x}$,

$\sigma_{g(\underline{x})}$: standard deviation of $g(\underline{x})$,

∇G : vector of partial derivatives of the function at the mean values of random variables $\underline{x}$, and

$COV(\underline{x})$: covariance matrix of the random variables $\underline{x}$.

When the random variables in the function, $g(\underline{x})$, are assumed uncorrelated, equation (3) can be presented in a simpler form as follows;

$$\sigma^2_{g(x)} = \sum \left(\frac{\partial g}{\partial x_i} \right)^2 \sigma^2_{x_i} \quad (4)$$

It is common in engineering to encounter non-closed forms of the performance functions. When $g(\underline{x})$ is assigned to a non-closed form function, the partial derivatives of $g(\underline{x})$ can be estimated numerically using the finite difference approach as follows:

$$\frac{\partial g(\underline{x})}{\partial x_i} = \frac{\triangle g(\underline{x})}{\triangle x_i} = \frac{g(x_{i+}) - g(x_{i-})}{x_{i+} - x_{i-}} \quad (5)$$

where x_{i+} and x_{i-} represent the random variable x_i taken at some increment above and below its expected values. Theoretically, an extremely small increment gives the most accurate value of the derivative at the expected value, but in practice, one standard deviation increment for each random variable is adequate (Duncan 2000). This FOSM method allows the engineer to see the contribution of each random variable to the total uncertainty in the function, $g(\underline{x})$, by evaluating the partial derivative of the function at its mean value and identify the design parameters that most significantly affect the variability of the function.

3.2.2 Point Estimate Method (PEM)

PEM is the procedure where probability distributions for continuous random variables are considered by discrete equivalent distributions having two or more values (Rusenbluth 1975). In order to obtain the expected value for the performance function, PEM requires all possible combinations of one low and one high value for each random variable for determining various possible g(x)'s. The results are weighted by the product of their associated probability concentrations P_{i+} or P_{i-}, and then summed. The procedure is summarized as:

$$E[g(x)] = \sum (P_{x_1\pm} P_{x_2\pm} \ldots P_{x_n\pm})[Y(x_{1\pm}, x_{2\pm}, \ldots x_{n\pm})] \quad (6)$$

$$E[g(x)^2] = \sum (P_{x_1\pm} P_{x_2\pm} \ldots P_{x_n\pm})[Y^2(x_{1\pm}, x_{2\pm}, \ldots x_{n\pm})] \quad (7)$$

$$\sigma^2_{g(x)} = E[g(x)^2] - (E[g(x)])^2 \quad (8)$$

When performance functions are significantly nonlinear, the PEM may produce better solutions because of its higher order accuracy in the mean value estimate.

4. Reliability Index

When the stability against overturning about the anchor point is assigned as the limit state function of a sheet pile wall, reliability for the wall can be defined as the probability that the passive moment around the anchor point, M_p (=P_p*L_p), is more than the active moment around the anchor point, M_a (=P_a*L_a). This definition can be expressed with the safety margin against overturning, $SM_{O.T.}$, as follows;

$$SM_{O.T} = M_p - M_a \quad (9)$$

As mentioned above, it is reasonable to define major parameters in designing a sheet pile wall as random variables with their means and standard deviations or their complete probability distributions. Once the statistical information for each random variable is obtained, one can calculate means and standard deviations of M_p, M_a, and $SM_{O.T.}$ using the FOSM or PEM. Reliability for $SM_{O.T.}$ can be then characterized by a reliability index β, which is the ratio between the expected safety margin and its standard deviation (Wolff 1995) :

$$\beta = \frac{E[SM_{O.T}]}{\sigma_{O.T}} \quad (10)$$

Fig. 3 graphically illustrates how to obtain β from design parameters that are defined as random variables. The reliability index, β, provides a better indication of how close the stability of a sheet pile wall is related to failure more than is the safety margin alone because it incorporates more information regarding the uncertainty in estimating the safety margin. The sheet pile wall with

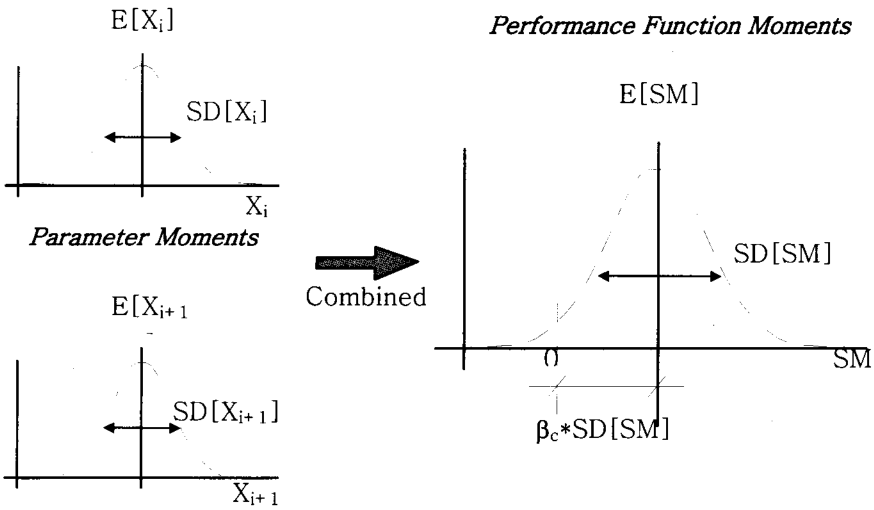

Fig. 3. Graphical illustration of reliability index

larger value of β is farther from failure than that with smaller value of β, regardless of the value of the best estimate of the safety margin.

5. Reliability-based Design of the Depth of Embedment and the Anchor Force

The basic concept in the reliability-based design (RBD) is that the reliability associated with appropriate design values should correspond to a target reliability level representing a certain degree of structural safety. One can estimate the mean values ($m_{D_{theory}}, m_F$) and standard deviations ($\sigma_{D_{theory}}, \sigma_F$) of the theoretical depth of embedment and anchor force incorporating the FOSM or PEM into the deterministic design procedure based on the free-earth support approach. With these mean values and standard deviations, this study proposes a main design equation for determining the design depth of the embedment regarding a desired reliability level:

$$D_{design} = \sigma_{D_{theory}} \cdot \beta_{target} + m_{D_{theory}} \qquad (11)$$

where,

β_{target} : target reliability index,

$m_{D_{theory}}, \sigma_{D_{theory}}$: mean value and standard deviation of the theoretical depth of embedment, and

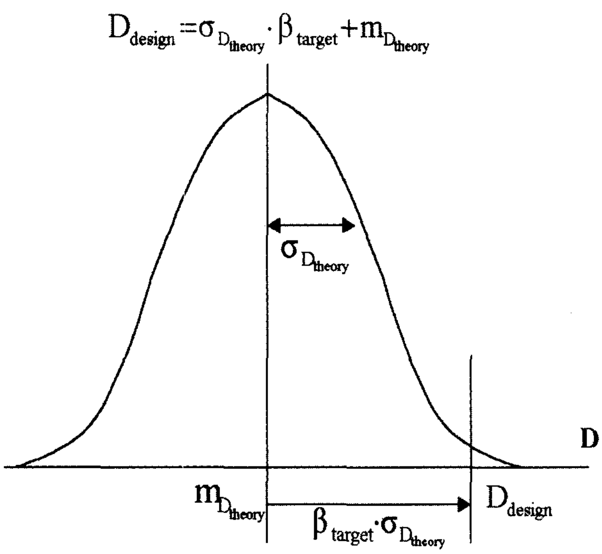

Fig. 4. Conceptual definition of RBD for determining the depth of embedment of the sheet pile wall

D_{design} : Design values of the depth of embedment .

The mean value of the anchor force, m_F, which is computed with $m_{D_{theory}}$, is assigned as the design anchor force, $\mathbf{F_{design}}$, because it is always much more than the value computed with $\mathbf{D_{design}}$. Using equation 11, the engineer can determine the design depth of the embedment with which the reliability for structural safety of a sheet pile wall can satisfy a desired level. Fig. 4 graphically explains this design approach.

At the present time, the best way to reasonably determine the target reliability index is based on the data from case histories (Kulhawy and Phoon 1996). If there are not enough data available to produce a rational target reliability index, the second best way is to depend on reasonable engineering judgments of experienced geotechnical designers. The basic objective of this reliability-based design approach for a sheet pile wall is to economically guarantee that the probability of failure of the wall lies below a target level. If the probability of failure for a sheet pile wall designed using this reliability-based design approach is located far below the target level, the objective is explicitly achieved. However, that design is uneconomical and the reliability concept in the design is misapplied.

Once the design depth of embedment is determined considering a target reliability index, reliability for the stability against overturning about the anchor point should be evaluated employing the equation (9) as a performance function to confirm the structural safety of the wall. If the reliability evaluated indicates that the stability of the wall does not satisfy a desired degree of structural safety, the design depth of embedment should be raised with a higher target reliability index until the stability of the wall reaches the desired degree of structural safety. In the light of the design principles and reliability concepts described above, a RBD procedure for determining the depth of embedment and anchor force of a sheet pile wall is suggested:

Step 1: Identify input data :

- height of the wall
- water table
- strength parameters in backfill and existing

soils

- location of the anchor tie rod
- target reliability index for the stability of the wall against overturning, $\beta_{design\_O.T.}$

Step 2: Calculate the mean values and standard deviations of the depth of embedment and the anchor force.

Step 3: Assign the mean value of the anchor force as the design value.

Step 4: Determine the design values of the depth of embedment based on a target reliability index using equation (11).

Step 5: Check that the reliability for the stability of the wall against overturning is more than $\beta_{design\_O.T.}$.

Step 6: Modify the target reliability index and repeat Steps 3 and 4 until the reliability for the stability of the wall against overturning is equal to $\beta_{design\_O.T.}$

Step 7: Produce final design values of the sheet pile wall.

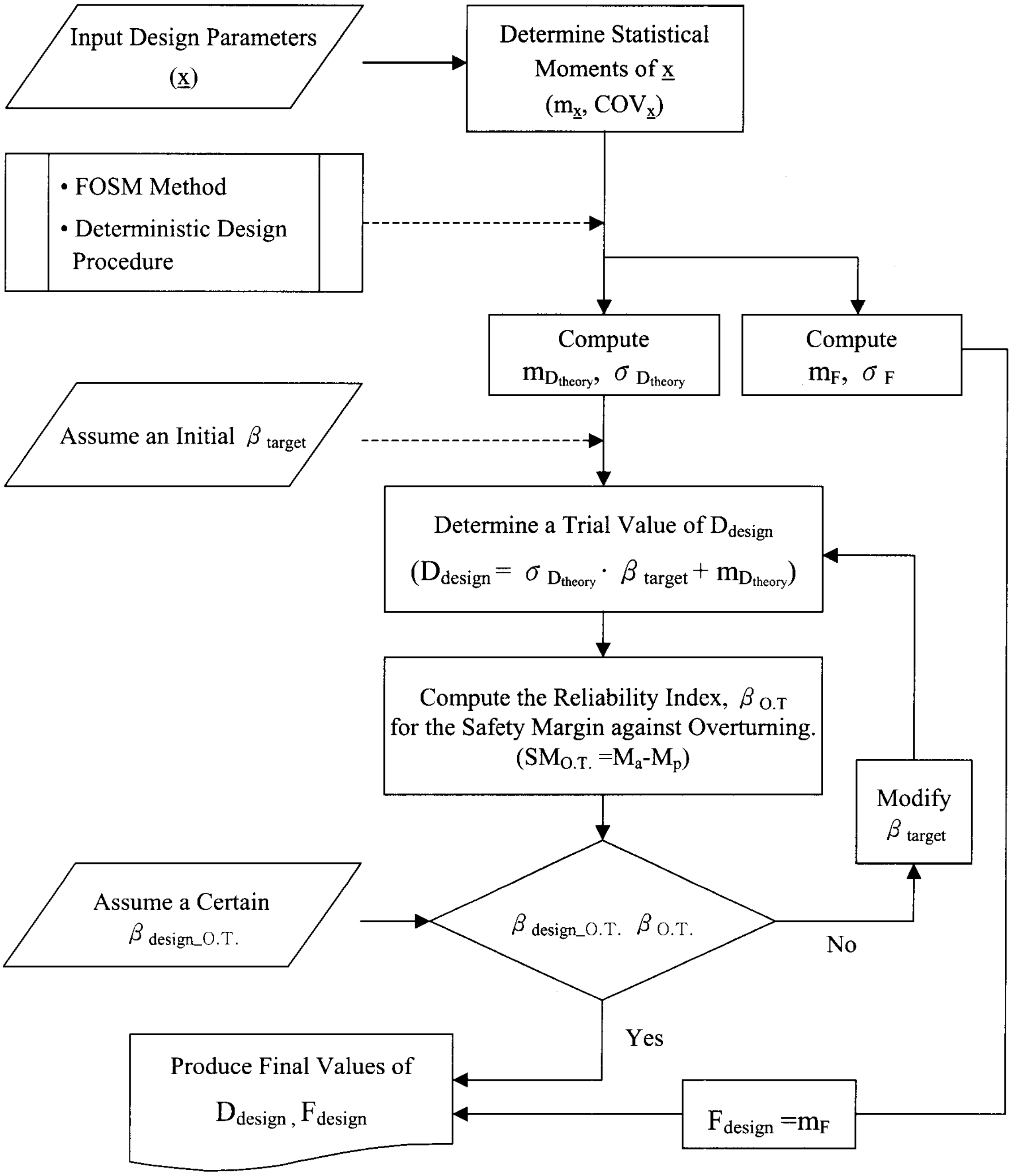

Fig. 5. Flowchart for a RBD procedure for anchored sheet pile wall

The flowchart illustrating this procedure is shown in Fig. 5.

6. Application Example

6.1 Illustration of Proposed RBD Procedure

Applications of the proposed RBD procedure for the sheet pile wall are presented here. Two anchored sheet pile walls embedded in sandy and clay soils as shown in Figs. 1 and 2 were employed to examine the effects of various parameters' uncertainties on the design values and reliabilities of the walls.

Table 2 shows the design input parameters used in these applications. Computations were performed using the FOSM and PEM. In these applications, the desired reliability index for the stability of the wall against overturning ($\beta_{design\_O.T.}$) is assigned as 1.5. The RBD results using FOSM for both anchored sheet piles embedded in sandy and clay soils are presented in Table 3. It seems that the largest contributions to the uncertainties of designing sheet piles embedded in sandy and clay soils are made by soil friction angle and cohesion, respectively. The variations of unit weight and water table do not seem to be of any special relevance with the reliabilities of the walls.

As shown in Table 4, the RBD approaches using the FOSM and PEM produce slightly different design results in both sheet pile walls embedded in sandy and clay soils. This implies that the design procedure does not have high degree of non-linearity. Even though the PEM would be more accurate than the FOSM because of its higher order accuracy in the mean value estimate, the

Table 2. Statistics of major design input parameters for the application example

(a) the wall embedded in sandy soil

Parameters	Type of Distribution	Mean	COV
L_1+L_2	deterministic	8m	–
Depth to the Anchor (l_1)	deterministic	1.5m	–
Level of Water Table (L_2)	Normal	5m	0.1
Angle of Friction	Normal	32°	0.15
Unit Weight	Normal	16kN/m^3	0.05
Saturated Unit Weight	Normal	19.5kN/m^3	0.07

(b) the wall embedded in clayey soil

Parameters	Type of Distribution	Mean	COV
L_1+L_2	deterministic	8m	–
Depth to the Anchor (l_1)	deterministic	1.5m	–
Level of Water Table (L_2)	Normal	5m	0.10
Angle of Friction	Normal	32°	0.15
Unit Weight	Normal	16kN/m^3	0.05
Saturated Unit Weight	Normal	19.5kN/m^3	0.07
Cohesion	Normal	55kN/m^2	0.20

Table 3. RBD results for anchored sheet piles used in the application example

(a) The wall embedded in sandy soil

Design Output	Contribution of the Variances of Design Results (%)				Mean	Stdev	β target	$\beta_{O.T.}$	Dsign Values
	γ	γ_{sat}	ϕ	L_2					
D (m)	0.76	6.00	92.9	0.35	3.19	0.94	2.28		5.32
F kN/m)	2.29	0.39	95.34	1.98	82.6	23.8			82.6
$SM_{O.T.}$ ($\beta_{designO.T.}$=1.50)	0.20	11.43	87.40	0.97	843.5	560.0		1.50	

(b) The wall embedded in clay soil

Design Output	Contribution of the Variances of Design Results (%)					Mean	Stdev	β target	$\beta_{O.T.}$	Dsign Values
	γ	γ_{sat}	ϕ	c	L_2					
D (m)	1.41	6.26	21.18	68.83	2.32	0.58	0.24	2.9		1.27
F kN/m)	4.05	1.87	90.81	0.61	2.67	13.4	13.4			61.5
$SM_{O.T.}$ ($\beta_{designO.T.}$=1.50)	0.86	3.66	5.06	89.20	1.22	645.9	426.8		1.50	

Table 4. Comparison of RBD results using FOSM and PEM

	Embedded in Sandy Soil		Embedded in Clay Soil	
	FOSM	PEM	FOSM	PEM
$m_{D_{theory}}$	3.19	3.36	0.58	0.63
$\sigma_{D_{theory}}$	0.94	0.97	0.24	0.26
β	2.28	2.00	2.90	2.40
D_{design}	5.32	5.30	1.27	1.25
$m_{F_{theory}}$	82.55	86.23	61.51	63.91
$\sigma_{F_{theory}}$	23.81	24.51	13.36	14.57

FOSM can be employed adequate probabilistic tool for the reliability-based sheet pile design application considering its advantages such as simplicity, shorter computation time and capability of identifying contributions of the design parameters to the variability of the design result.

6.2 Comparison with Other Design Methods and Sensitivity Analysis

6.2.1 Overview of Design Methods

Couples of different methods of design that employ the free-earth support approach were used to validate the design results suggested by RBD. Both deterministic and probabilistic methods that are used in common practice are applied to this validation study. The details are presented as follows:

1. Working Stress Design (WSD) Method

In this method, a global factor of safety, F, is applied to the passive pressure (U.S Department of NAVY, 1986). The depth of embedment is obtained by achieving the equilibrium of moments taken about the anchor point :

$$P'_A L_A + P_{WA} L_{WA} = \frac{P'_P L_P}{F} + P_{WP} L_{WP} \quad (12)$$

where,

P'_A and P'_p are the resultant effective active and passive forces; P_{WA} and P_{WP} are the resultant water pressures acting on the active and passive sides of the wall; and L_A, L_P, L_{WA}, and L_{WP} are the moment arms with respect to the anchor point. Normally, F is assumed to be equal to 2 for coarse grained soil (U.S Department of NAVY, 1986).

2. Potts-Burland Methods

Potts and Burland (1983) suggested a sheet pile design method in which a global factor of safety, F_r, is taken as the ratio of the moment of the net available passive resistance to the moment activated by the retained material. The depth of the embedment is obtained from

$$F_r = \frac{P'_{PN} L_P}{P'_{AN} L_A + P_{WN} L_{WN}} \quad (13)$$

where P'_{PN} and P'_{AN} are the net effective passive and active forces and P'_{WN} is the resultant net water pressure. A Value of F_r = 2.0 was recommended for design by Potts and Burland.

6.2.2 Parametric Study

To investigate the effects of changes in the major parametric values and their variations on design results for the sheet pile walls shown in Figs. 1 and 2 suggested by RBD and other design methods, a parametric study was performed. From Table 3 recognizing the parameters that mostly contribute the degree of safety, the variables used in this study are selected as follows;

(i) embedded in sandy soil :

- Soil friction angle (ϕ) = 27, 32, and 37° ; COV_ϕ = 10, 15, and 20%

(ii) embedded in clay soil :

- Soil friction angle (ϕ) = 27, 32, and 37° ; COV_ϕ = 10, 15, and 20%
- Soil cohesion (c) = 45, 55, 65 kN/m^2 ; COV_c = 15, 20, and 25%

6.2.3 Comparison of Design Results

Fig. 6 shows a comparison of the Potts-Burland, WSD, and RBD methods. All methods yield similar depths of embedment penetrating the soils with smaller COV_ϕ's of 10 and 15% except the WDS method for the internal friction angle of 27°. For the sheet pile penetrating the soil with higher COV_ϕ of 20%, however, the RBD method yields the higher depth of embedment than other methods. These results imply that when the sheet pile is embedded on the soil whose design strength parameters

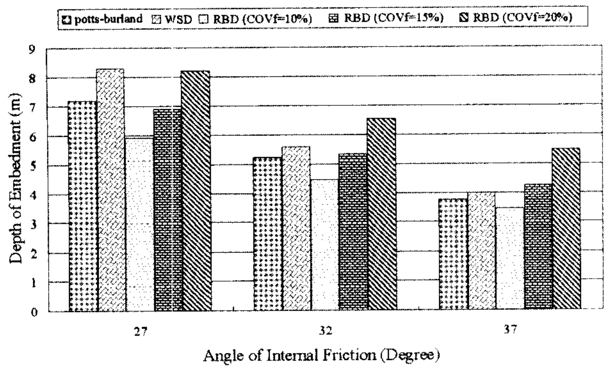

Fig. 6. Comparison of different design methods

are estimated with higher uncertainties, the deeper depth of embedment should be required to satisfy a certain level of reliability for the stability of the wall against overturning than those determined by general deterministic methods.

6.2.4 Sensitivity Analysis

Figs. 7 and 8 illustrate the sensitivity of major soil strength parameters to the depth of embedment analysis in the sheet pile wall embedded in sandy clay soil. In Fig. 7, the variability of internal friction angle of the retained soil significantly affects the design depth of embedment in the case of the sheet wall embedded in sandy soil, while it does not in the case of the wall embedded in clay soil. Figs. 7 and 8 clearly show that for the case of the wall embedded in clay soil, the variability of the cohesion of soil below dredge line contributes more significantly to determining the design values of the depth of embedment than that of internal friction angle of the retained soil. As the shear strength of soil below dredge line decreases and its variability increases, the design values for the anchor force of the sheet pile wall more rapidly increase (Fig. 8). This indicates that the resistance of the wall is more unstable with the high variability of lower shear strength of soil.

7. Concluding Remarks

In this paper, a RBD procedure for determining the design depth of embedment for the anchored sheet pile wall is proposed regarding overturning about the anchor point as the major failure mode. In this design procedure, design values of the depth of embedment are logically determined in accordance with degrees of uncertainties of design input parameters. At the end of the design procedure, it is required that, with designed depth of embedment, the reliability of the wall against overturning about the anchor point should meet a desired level of structural safety. The proposed RBD procedure is founded on incorporating a first order reliability method using Taylor series expansion into a widely recognized deterministic design procedure so that the procedure needs not require more data than is required for conventional deterministic procedures. The FOSM method described in this study can allow the engineer to identify which design parameter's uncertainty most significantly affects the uncertainties of design results by evaluating the partial derivatives of the design model at its most likely values. The results of sensitivity analyses performed in this study indicate that

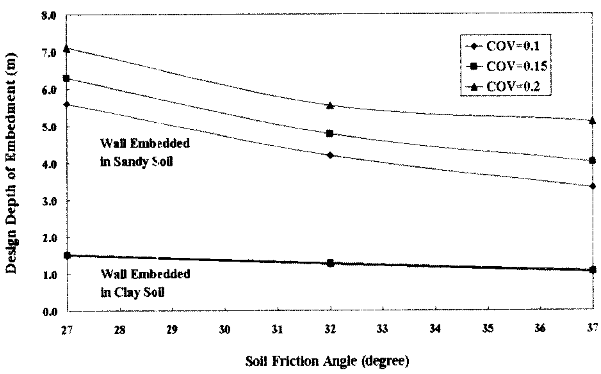

Fig. 7. Sensitivity of soil friction angle to design depth of embedment

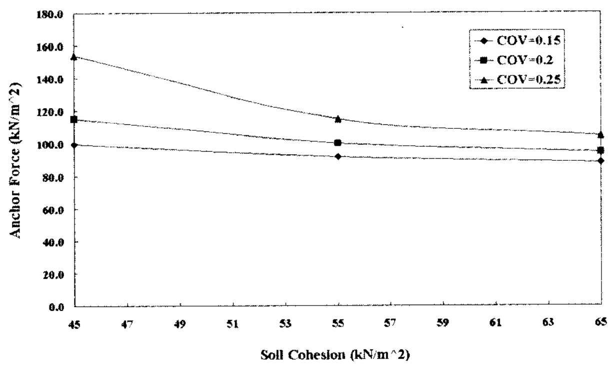

Fig. 8. Sensitivity of soil cohesion to design anchor force in clay soil

the friction angle of backfill soil and the cohesion of clay soil below dredge line are the largest contribution factors to the variability of design values for anchored sheet pile walls embedded in sandy and clay soils, respectively. Relatively, the variations of unit weight and water table little affect the reliability of anchored sheet pile walls.

Acknowledgement

The authors would like to thank the Advanced Highway Research Center in Hanyang University sponsored by Korea Engineering and Science Foundation for its providing financial support for this study.

References

1. Ang, A. and Tang, W. (1975, 1984), *Probability Concept in Engineering Planning and Design Volume-I and II*, John Wiley and Sons, Inc.
2. Cherubini, C., Garrasi, A., and Petrolla, C. (1992), "The Reliability of an Anchored Sheet-pile Wall Embedded in a Cohesionless soil", *Canadian Geotechnical Journal*, Vol.29, pp.426-435.
3. Das, B. M. (1995), *Principles of Foundation Engineering*, 3rd Edition, PWS Publishing Co., Boston, 1995.
4. Duncan, J. M. (2000), "Factors of Safety and Reliability in Geotechnical Engineering", *Journal of the Geotechnical and Geo-environmental Engineering*, ASCE, Vol.126, No.4, pp.307-316.
5. Höeg, K., and Murarka, R. P. (1974), "Probabilistic Analysis and Design of a Retaining Wall", *Journal of the Geotechnical Engineering Division*, ASCE, Vol.100, No.GT3, pp.349-365.
6. Kulhawy, F., and Phoon, K. K. (1996), "Engineering Judgment in the Evolution from Deterministic to Reliability-Based Foundation Design", *Proceedings of Uncertainty in Geologic Environment: From Theory to Practice*, ASCE, pp.29-48.
7. Phoon, K. K. and Kulhawy, F. H. (1994), "Practical Reliability-Based Design Approach for Foundation Engineering", *Transportation Research Record 1546*, Transportation Research Board, National Research Council, Washington, D.C., pp.94-99.
8. Potts, D. M., and Burland, J. B. (1983), "A Parametric Study of the Stability of Embedded Cantilever Retaining Walls", *TRRL Report SR 813*, Transport and Road Research Laboratory, Crowthorne, UK.
9. Rusenblueth, E. (1975), *Point Estimates for Probability Moments, Proceedings*, National Academy of Science, USA, Vol.72, 1975, pp.3812-3814.
10. U.S Department of the NAVY (1986), *Design Manual 7.2. Foundations and Earth Structures*, NAVFAC, Washington, D.C.
11. Wolff, T. F. (1995), *Geotechnical Reliability of Dam and Levees Embankments*, A Technical Report prepared for U.S. Army Engineer Waterways Experiment Station, Geotechnical Laboratory, Vicksburg, MS.

(received on Apr. 27, 2002, accepted on Jun. 10, 2002)