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Experimental Investigations on Tensile Strength of Sand at Low Moisture Contents

저함수비 모래의 인장강도에 대한 실험적 연구

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요 지

새롭게 개발된 직접인장시험기와 시험방법을 이용하여 습윤모래의 인장강도를 측정하였다. 본 연구를 통하여 습윤모래에 인장강도가 존재함을 분명하게 확인 할 수 있고, 저함수비 모래의 인장강도를 간단하고 정확하게 측정 가능함을 알 수 있다. 일반적으로 알고 있는 바와 달리 습윤모래의 인장강도는 영(zero)이 아님을 알 수 있다. 함수비가 (0.5 < w < 4.0%) 증가함에 따라 인장강도는 증가하는 것으로 나타났으며, 상대밀도가 (30 < D_r < 70%) 증가함에 따라 인장강도가 증가하는 경향은 더욱 뚜렷하게 나타났다. 지반공학적 문제에서 습윤모래의 인장강도에 대한 영향을 조사하기 위하여 사질토지반에 설치된 원형의 강성후팅에 대한 수치해석을 실시하였다. 해석결과 미소한 인장강도를 고려하더라도 지반의 안정성이 크게 증가함을 확인할 수 있다.

Abstract

This study shows that tensile strength in moist sand clearly exists due to moisture and it is possible to simply and accurately measure the tensile strength of sands at low moisture contents. These measurements were made through the use of a newly developed direct tension apparatus and technique which are able to produce highly accurate results. The magnitudes of the tensile strengths of these moist and relatively clean sands are not equal to zero, as is widely assumed. Tensile strength increases with increasing moisture content and this trend is more noticeable at increasing relative densities. The influence of tensile strength in geotechnical problems was also examined by considering a simple rigid circular footing in sandy soil. It clearly shows that a small amount of tensile strength can significantly enhance the stability of a geotechnical system.

Keywords : Direct tension device, Moist sand, Moisture content, Relative density, Tensile strength

1. Introduction

The strength of sands is usually considered to be only frictional. For this reason, the vast majority of analysis methods and numerical models dealing with sands consider only the compressive strength of the soil mass and take the tensile strength to be equal to zero. It is also widely observed that most analysis methods that encompass the behavior of sands lead to conservative answers (they underestimate the capacity of the sand to carry loads). This innate conservatism is by-and-large a good thing, of course, but more precise estimates are desirable so that engineers would actually decide on, or design, the margin between working loads and failure, rather than having the analysis methods hide this margin within their own inaccuracies. Even in the case where

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more complex analysis methods can consider both friction and tension (sometimes as cohesion), most modelers will assign a value of zero to the tensile strength of the sand because of a lack of data or due to the perception that cohesion or tensile strength must be insignificant in granular soils.

Recent research at the University of Colorado has focused on developing a method for the rapid, accurate and cost-effective determination of the tensile strength of sands, especially sands at very low moisture contents. This research has led to two important conclusions:

- It is possible to accurately measure the tensile strength of sands at moisture contents similar to those found throughout the unsaturated zone.
- The measured values for tensile strength in sands at moisture contents as low as 0.5% are large enough to have potentially significant impacts on the behavior of moist sands.

The measurement of tensile strength in sands at very low moisture contents is particularly interesting given the fact that the highest stresses seen beneath most structures are imposed on moist sands rather than on saturated or dry sands, yet most existing data on sand behavior are based on either saturated or dry specimens.

This article seeks to outline the use of a device for the measurement of the tensile strength of sands. Some experimental data are presented for the purpose of verifying the repeatability and magnitude of the measurements. This paper also contains results of parametric studies for a circular footing placed on sandy soils, which demonstrate the influence of small amounts of tensile strength on soil-structure systems.

2. Previous Work

Capillary forces induced by interstitial water can substantially control the properties and behavior of an assembly of solid particles. Even at low moisture contents, small amount of water forms water-bridges at contact points, and as the water content increases these bridges become larger and more developed. This results in capillary bonding between particles, giving rise to both cohesion and tensile strength. Capillary bonding generally leads to two force components at low water content levels: 1) the surface tension force acting along the water-particle contact line, and 2) the force due to the difference in the pressures outside and inside the bridge acting on the cross sectional area. The surface tension tends to force the particles together, whereas the force due to the pressure difference can only contribute to particle adhesion if there is a net pressure deficiency within the bridge. Due to the presence of water-bridges between the particles, these two forces act together as a bonding force (Rumpf, 1961; Schubert, 1984; Pierrat and Caram, 1997).

To measure the tensile strength of soils in the laboratory, several tension apparatuses and experimental techniques have been designed and constructed by Conlon (1966), Bishop and Garga (1969), and Bofinger (1970), respectively. However, these techniques are best suited for measuring the tensile strength of clays, and are not suitable for granular materials because of difficulties in forming and engaging the specimens. In 1991, Perkins developed a direct tension apparatus to measure the tensile strength of a granular material. This device was designed to accommodate a 17.8 cm cubical specimen in a tension box split to form two equal halves, one of which was movable while the other was fixed. Experimental results are expressed as the average stress on the vertical plane of failure versus the displacement of the front box. The critical disadvantage of this apparatus is that the contact between specimen and box, as tension develops across the plane of separation, is not certain because of a flat wall surface; consequently, the uniformity of the stress distribution on the plane of separation is questionable.

3. New Direct Tension Experiments

3.1 Apparatus

The direct tension apparatus used in these experiments is adapted from the device initially described by Perkins

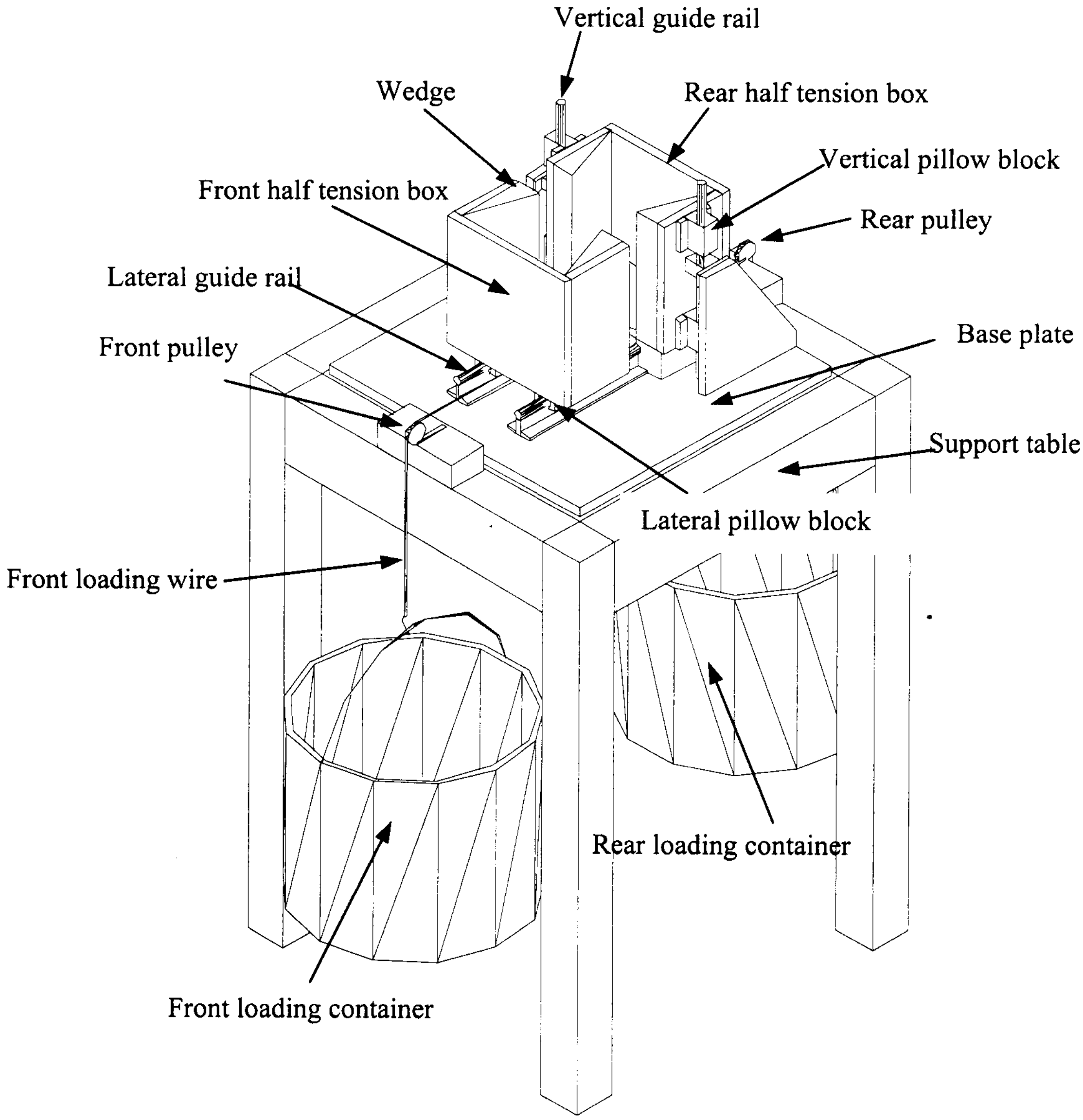


Fig. 1. Schematic diagram of direct tension apparatus

(1991). The device, shown schematically in Fig. 1, consists of a split acrylic box (17.8 × 17.8 × 17.8 cm) with an open top. One half of the box is secured to the guide rails, thus fixing it in place, while the other half is free to slide on linear bearings that ride on machined guide rails. The linear bearings and guide rails present a very small resistance to movement, requiring a force of only 1.12 N to initiate movement of the free half when empty. This guide system forms the most complex part of what is a fairly simple apparatus.

The system for applying the loads to the specimen is very straightforward. Two containers are attached to the movable half of the container and suspended from a wire and pulley system, one container to the front and the other to the rear. The front container has a known weight when empty, and the rear container serves as a counter-balance to this initial weight so that there is no net load at the beginning of the test. To impose load on the failure surface in this load-controlled apparatus, water is added to the front container very slowly, at a loading rate of about 170 g/min, or 0.03 N/sec.

As can be seen in Fig. 2, the specimens were relatively large, having overall dimensions of 17.8 × 17.8 × 13.9 cm giving the specimen an overall volume of 3,332 cc. Large specimens were chosen for a variety of reasons:

- The larger the specimen, the lower the instrument

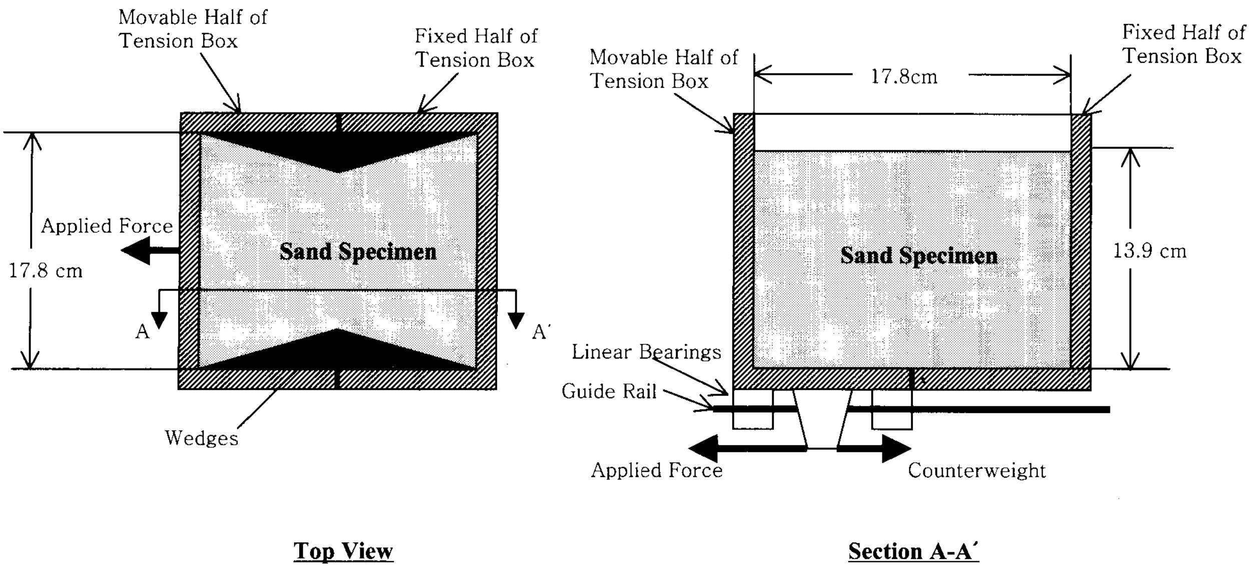


Fig. 2. Schematic diagram of direct tension box

error. For example, the force on the failure surface can be measured to within ± 0.001 N (about 0.1 g of water), making the error in the resulting measured tensile stress ± 0.04 Pa.

- While every effort was taken to ensure uniform specimens, complete uniformity is not possible. This was of particular concern in this case, since small variations in moisture content led to large variations in measured tensile strength. For large specimens, local variations in specimen consistency, for instance, density and water content, have a smaller effect on the overall behavior of the specimen, assuming that inconsistencies would be of the same size in both large and small specimens.
- The surface area of the boundaries decreases in relation to the specimen volume as the specimen volume increases. This would indicate decreased boundary effects for larger specimen.

Taken in combination, these factors lead to significantly more accurate results for large versus small specimens, given a particular apparatus type and specimen preparation procedure.

Because of concerns about slippage of the specimen along the vertical boundaries where the normal stresses could be quite low, triangular wooden wedges were added to the side walls. Wedges having angles larger than the dilatancy angle of the sand, as determined through triaxial testing, were selected to reduce movement of the soil particles relative to the box and to achieve a uniform stress distribution on the plane of separation. The dilatancy angle of F-75 Ottawa sand is about 17° for confining pressures in the range of 1.3 to 68.9 kPa (Batiste, 1998; Sture et al., 1998). Thus, wedges having angles of 20° were selected to kinematically constrain the sand near the wall during the motion. These wedges were covered with sandpaper to allow them to engage the sand specimens without slippage. These blocks had the added

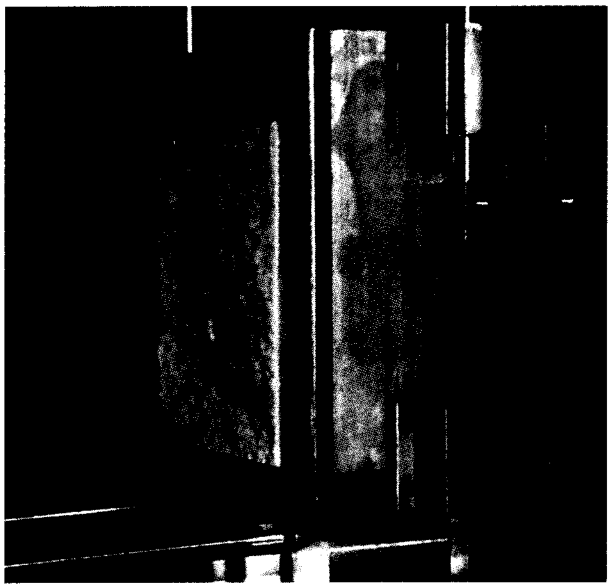


Fig. 3. Photograph of vertical failure surface in F-75 moist sand

advantage of acting to force a highly vertical, flat failure surface along the split between the container halves, as shown in Fig. 3.

3.2 Specimen Preparation

Specimens were prepared using a washed F-75 Ottawa silica sand produced by the Ottawa Silica Company. F-75 is a fine-grained natural quartz sand of uniform gradation with a mean particle size of 0.22 mm (see Fig. 4) and maximum and minimum void ratios of 0.805 and 0.486, respectively. Parameters such as moisture content and density were also varied in the testing program.

Specimen preparation presented two significant challenges; creating specimens with a uniform but low moisture content, and placement of the specimens at a predictable, measurable and repeatable density. Initially, attempts were made to mix the water with the sands using conventional manual methods, but this proved too slow and resulted in non-uniformity. Additionally, since the mixing took too long by hand, the sand tended to dry out during mixing, leading to unpredictable final moisture contents. Mixing the specimens in a 6-litre industrial bread-dough mixer proved much more efficient, saving considerable time and leading to highly homogenous specimens with very predictable moisture contents. Considerable care had to be taken throughout specimen preparation and testing to ensure that the sand did not have an opportunity to dry out, and speed turned out to be the key factor. Basically, letting significant time lapse during any step of the testing process leads to specimen drying and thus was avoided.

Placement of the specimens at a predictable, measurable and repeatable density was the second major problem. Conventional methods call for creating sand specimens by raining the sand through either air or water. However, raining of the specimens through air was not possible due to the moist, clumpy condition of the sand, and raining through water would not allow for the creation of unsaturated specimens with very low moisture contents. With these limitations in mind, direct compaction of the moist sand was selected, making the thick walls and generally stiff construction of the box critical to successful specimen creation. A donut hammer on a slide was dropped through a specified height, imparting energy to the sand through a triangular "foot" at the base of the compactor. The sand was compacted in four layers, and the number of blows per layer was varied according to the desired final density (Fig. 5). Typically, the top of each lift was roughened before placement of the next lift in order to minimize the effects of the lift interfaces.

Using the mixing and compaction procedures outlined above, it was possible to create repeatable specimens at consistent, predictable moisture contents and densities. Both moisture content and density were confirmed for each specimen by taking measurements of the water content, specimen mass and specimen dimensions. In all cases, measurements of specimen properties were made

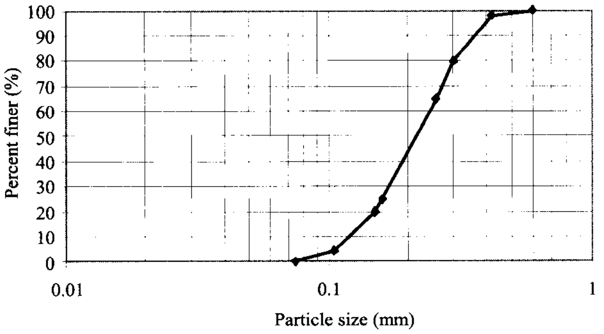


Fig. 4. Grain size distribution curve for F-75 sand

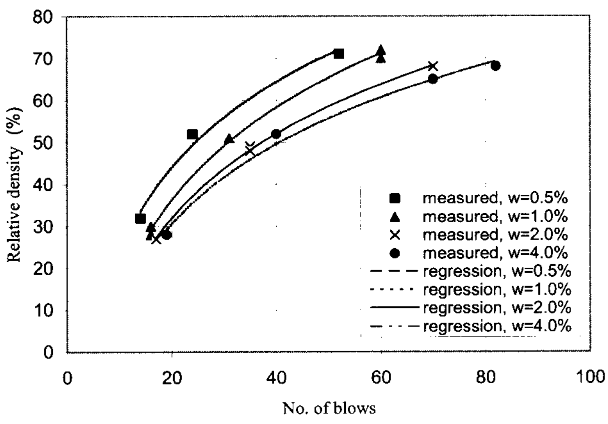


Fig. 5. Relationship between number of blows versus relative density for different water contents in F-75 moist sand

immediately following failure and used the entire specimen, rather than a representative sub-specimen, to improve accuracy.

3.3 Experimental Procedure

Prior to beginning each experiment, the two halves of the box were secured by taping them together with cellophane tape. Sand was then prepared at the desired moisture content and density as described previously. During this process, care was taken to use thin plastic wrap and a water-misting bottle to prevent drying of the sand specimen surface. The seam between the two halves of the box was watched carefully during the compaction process, and no separation was noted. In some cases, a load was then placed atop the sand specimen to induce an overconsolidation pressure. This load was typically left in place for less than one hour, and care was taken during this loading to ensure that moisture did not escape the specimen.

Immediately prior to loading the sand in tension, the cellophane tape was cut along the seam between the two halves of the box. A very sharp, thin blade was used to prevent disturbance of the specimen. The load was then slowly and steadily applied by introducing water to the loading container (front) at a rate of about 170 g/min or about 0.03 N/sec. This somewhat simple loading procedure provided excellent results, and proved to be highly repeatable. The load was steadily increased until failure occurred in the range of 661 (loose sand at w = 0.5%) to 1695 (dense sand at w = 4.0%) g of water, and the load was then measured by taking the mass of the water added to the loading container. The error in load-mass measurement was ± 0.01 g. The moment of failure was very apparent, as the two container halves parted rapidly, leaving a nearly vertical standing face of sand along the failure surface (Fig. 3).

Immediately following failure, the mass, and thereby the density, of the specimen were determined by placing the entire apparatus on a large scale. The majority of the specimen was then quickly dug out of the box and immediately weighed for the determination of moisture content. The tensile strength of the sand was computed by dividing the applied load by the cross-sectional area of the failure surface.

4. Results and Repeatability

The direct tension experiments were conducted on specimens having three different relative densities, D_r (30, 50, 70%) and four different water contents in the range of 0.5 ~ 4.0%. Sets of duplicate experiments (two times at the selected condition randomly) were conducted to demonstrate the repeatability of the testing technique developed in this study. Results for clean F-75 sand are presented in Table 1 and summarized in Fig. 6. The data shows that the tensile strengths on the order of 500 Pa are possible in moist sands at moisture contents, about 0.5%. These results clearly show two key points:

- The data show clear trends and repeatability at different densities and moisture contents. The coherent variations in the gathered data indicate that the phenomenon observed is real rather than an artifact of the experimental process, while the repeatability gives confirmation that we can compare data gathered at different densities and moisture contents to one

Table 1. Direct tension test results for F-75 sand wetted at $0.5 < w < 4.0\%$

Sample	w (%)	S (%)	D_r (%)	σ_t (Pa)
Loose 1	0.46	1.73	32	409.68
Loose 2	1.01	3.77	30	580.67
Loose 3	1.07	3.96	28	586.11
Loose 4	2.13	7.85	27	704.93
Loose 5	4.04	14.83	26	873.03
Loose 6	4.02	14.89	28	850.64
Medium 1	0.46	1.91	52	473.35
Medium 2	1.01	4.17	51	623.86
Medium 3	2.05	8.37	49	886.48
Medium 4	2.08	8.46	48	856.53
Medium 5	4.11	17.04	52	1073.41
Dense 1	0.47	2.15	71	498.52
Dense 2	1.02	4.70	72	730.45
Dense 3	1.04	4.74	70	732.94
Dense 4	2.05	9.24	68	981.97
Dense 5	3.89	17.53	68	1164.45
Dense 6	4.06	18.00	65	1150.84

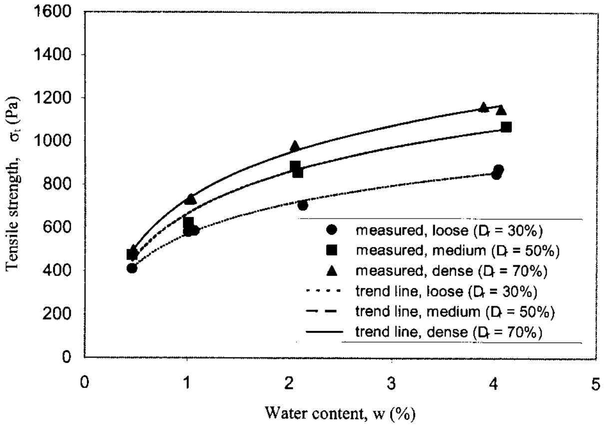


Fig. 6. Representative results for F-75 moist sand

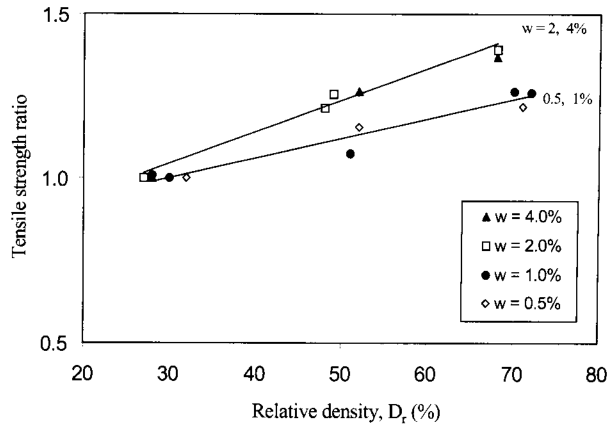


Fig. 7. Relationship between tensile strength ratio versus relative density for different water contents in F-75 moist sand

another.

- The magnitudes of the measured tensile strength are significantly different from zero. This is a key finding, and indicates that the observed differences between computed and observed soil behavior (bearing capacity, earth pressures, etc) may be strongly influenced by the assumption that the tensile strength of moist sands is equal to zero.

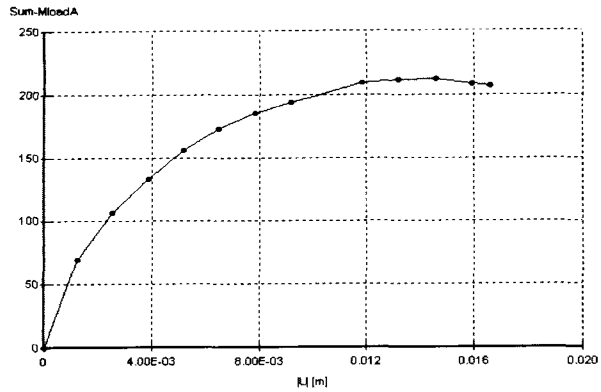
Figure 6 also indicates that the tensile strength tends to increase as the moisture content increases. This can be explained by considering the capillary bonding forces induced by moisture. At low moisture levels, water-bridges form at the particle-particle contact points. This results in capillary bonding forces between the particles, which lead not only to cohesion, but also certain amount of tensile strength in the soil. As the moisture level increases, the water-bridges become more developed in the contact geometries, and the tensile strength increases. Higher relative densities also lead to more contacts between soil particles, thus increased number of water-bridges, and this causes higher measured tensile strengths, and this phenomenon becomes more pronounced as the moisture content increases. However, the influence of relative density on the tensile strength is dependent on the moisture content. This is clearly shown in Fig. 7, which describes tensile strength ratio versus relative density. The data points were obtained by dividing the tensile strengths by the tensile strength of the loose specimen for each of different water contents. Trend lines for the data are also shown, and their slopes indicate increasing levels of tensile strength as the relative density increases. The slopes are relatively higher for water contents in the range of 2.0 to 4.0% compared to the slopes for low water content levels, below 1.0%. This means that the influence of relative density on the tensile strength is diminished at low moisture levels.

5. Example

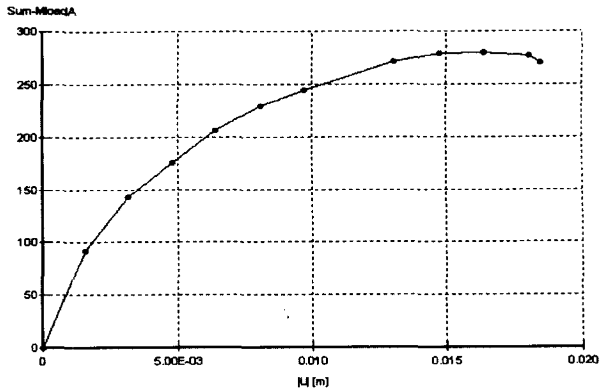
A rigid circular footing with a radius 0.1 m is placed on a sand layer of 4.0 m thickness. The material is modeled by an elasto-plastic Mohr-Coulomb model (Table 2). PLAXIS program developed by the Technical University of Delft was used for this analysis, because this program can consider both cohesion and tension. An

Table 2. Material properties of a sand layer

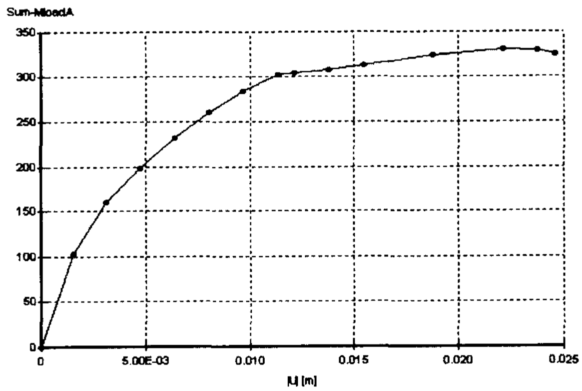
Parameters	Value	Unit
Material model	Mohr−Coulomb	−
Type of material behavior	Drained	−
Dry soil weight	17.0	kN/m^3
Wet soil weight	20.0	kN/m^3
Permeability in horizontal direction	1.0	m/day
Permeability in vertical direction	1.0	m/day
Young's modulus	13,000	kN/m^2
Poisson's ratio	0.3	−
Friction angle	31.0	°
Dilatancy angle	0.0	°



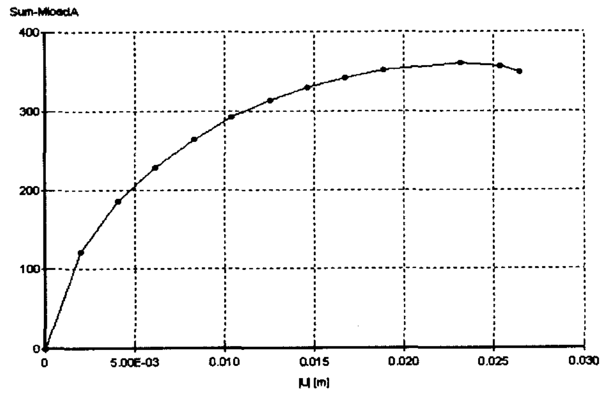
(a) Case 1: c = 0.001 and σ_t = 0 kPa



(b) Case 2: c = 0.5 and σ_t = 0 kPa



(c) Case 3: c = 1.0 and σ_t = 0 kPa



(d) Case 4: c = 1.0 and σ_t = 1 kPa

Fig. 8. Results of load-displacement curves of a circular footing (radius 0.1 m)

axisymmetric analysis with a 15-noded element was carried out, and thus only one symmetric half of the total geometry was modeled.

Figure 8 shows the results of the final calculation steps for four different values of cohesion and tensile strength considered to demonstrate the influence of a small amount of cohesion and tensile strength on this example. Applied load (Sum-MloadA in Fig. 8) gives a total load that is approximately equal to the ultimate footing force. For instance, if Sum-MloadA is 360, the ultimate footing force is calculated as,

$$360 \times 1.0\ (\mathrm{kN/m^2}) \times \pi \times (0.1\ \mathrm{m})^2 = 11.3\ \mathrm{kN}.$$

Table 3. Ultimate footing force for different cohesions and tensile strengths

	Cohesion (kPa)	Tensile strength (kPa)	Footing force (kN)	Difference (%)
Case 1	0.001	0	6.7	-
Case 2	0.5	0	8.8	31.3
Case 3	1.0	0	10.3	53.7
Case 4	1.0	1	11.3	68.7

Table 3 shows the ultimate footing forces of four different cases. The result clearly shows that the cohesion and the tensile strength are one of the most critical factors controlling the geotechnical systems.

6. Conclusions

The tensile strength of moist sands is significantly different from zero. Measuring these tensile strengths, while challenging, is possible using a fairly straightforward direct-tension device. Repeatable, coherent data were gathered for F-75 sand, and the procedures and methods used are suitable for a wide variety of granular materials, from clean sands to relatively fine silts. The simplicity and rapidity of the equipment and methods should make it possible for those wishing to include tensile strength in their soil behavior models to do so in a coherent way, linking model parameters to laboratory data. Examples indicate that small amount of tensile strength and cohesion in sandy soils can affect the stability of earth-structures and structure-soil systems.

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