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Teachers' Mathematical Beliefs and Teaching Practices

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I. Introduction

As the current reform movements give heavy emphasis to the need for improving teaching practice (NCTM, 2000, 1991), researchers generally agree that mathematics teachers should change their beliefs and dispositions toward mathematics and its teaching. Although during the last twenty years increasing research attention has been paid to mathematics teachers' beliefs, these findings take many directions. For instance, some perspectives (e.g., Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Carpenter & Fennema, 1992) suggest that teachers' beliefs should be necessarily changed in order to enhance the quality of teaching practice, while others (e. g., Silver, Smith, & Nelson, 1995) focus on changing teaching environments to change teachers' beliefs of mathematics and its teaching. This lack of agreement about the relationship between teachers' beliefs and teaching practice seems to be caused by the inherent nature of teachers' beliefs and suggests the need of modeling to explore the relationship more closely.

In this paper, the discussion starts by considering various definitions of beliefs and then proceeds to how teachers' beliefs can be distinguished from knowledge, both in education, in general, and in mathematics education, in particular. Then, the different models regarding the relationship between beliefs and teaching practice will be discussed, focusing on the issues of teacher learning and professional developmental programs.

II. Definitions of Beliefs

How is belief best defined? It seems difficult to clearly define what beliefs are, since they are complex in nature. Most definitions of beliefs to be discussed here are quite dependent upon subjective judgment.

Abelson(1979) defines belief as a process that manipulates knowledge for a particular purpose or under some circumstances. He calls a "stored body" of knowledge as a belief system (p. 356). Brown and Cooney (1982) characterize beliefs as "dispositions

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to act in certain way under certain circumstances· that are major underlying determinants of behavior and the environments that people create" (p. 14). Nespor (1987) focuses on emotional aspects in defining beliefs. That is, he views beliefs as affective and emotional components that influence retrieval and reconstruction of the elements in memory during recall. More recently, Pajares(1992) develops a more sophisticated conception of beliefs, drawn by reviewing numerous existing definitions of beliefs. According to Pajares' definition, beliefs are regarded as "an individual's judgment of the truth or falsity of a proposition, a judgment that can only be inferred from a collective understanding of what human beings say, intend, and do" (p. 316). From these four definitions of beliefs in general, we see clearly that beliefs are regarded as an obviously individual and subjective dimension. Now, we need to look into how teacher's beliefs in mathematics education are viewed.

Pajares(1992) points out that teachers' beliefs have been referred to as "teachers' attitudes about education-schooling, teaching, learning, and students" (p. 316). On the other hand, Goodman(1988) interprets teachers' beliefs in terms of teachers' disposition about education. In the area of mathematics education, many views of mathematics teachers' beliefs have been found.

Thompson(1985) takes beliefs as subdivision of the notion of conceptions or conceptual system. She describes mathematics teachers' conceptions as "complex organizations of beliefs, disbeliefs, and concepts of a given domain" (p. 282). In order to study the role of teachers' conceptions, Thompson(1985) classifies mathematics teachers' conceptions into the area of mathematics content knowledge and its teaching. However, the notion of conceptions seems to be used with variation, as she also described conceptions as beliefs, views, and preferences. Ernest(1989) explains different viewpoints of beliefs. He takes mathematics teachers' beliefs as larger notions than conceptions, which conflicts with Thompson's notion of beliefs. According to Ernest, beliefs consist of "the teachers' beliefs, conceptions, values and ideology also referred to elsewhere as the teachers' dispositions" (p. 20). Ernest, therefore, suggests that the key components of mathematics teachers' beliefs be composed of the nature of mathematics, its teaching and learning mathematics. While most definitions of beliefs described above are usually focused on teachers' beliefs about curriculum of subject-matter and its teaching and learning in general, Peterson, Fennema, Carpenter, and Loef(1989) adopt a definition of beliefs in the area of specific domains. For example, Peterson and his colleagues adopt "teachers' pedagogical content beliefs"

instead of general beliefs about mathematics (p. 3). Based on the assumption that teachers' beliefs in a specific domain will be better indicator to predict students' understanding and performance, Peterson, et al. explain teachers' pedagogical content beliefs as teachers' beliefs about teaching and learning in the area of a specific domain (e.g., addition and subtraction).

To be sure, there are more than one direction of defining beliefs. Some definitions are focused on individuals' emotional traits, others on area of interest, and some others on components of beliefs. These different directions of definitions for beliefs may be closely relevant to the distinctive nature of beliefs.

III. Distinctions between Beliefs and Knowledge

There is little doubt that teachers' beliefs and knowledge are the two major indicators in anticipating their instruction. These teacher characteristics need to be extensively examined with regard to their relationships with teaching practices. In the area of mathematics education, teachers' beliefs are usually divided into beliefs of the nature of mathematics and its teaching and learning (Thompson, 1984, 1992). Mathematics teachers' knowledge can be categorized in terms of mathematics content knowledge (including domain-specific content knowledge), pedagogical content knowledge (Carpenter, Fennema, Peterson, & Carey, 1988; Shulman, 1986), and epistemo logical mathematical knowledge (Steinbring, 1998). Few studies, however, provide an explicit discussion on how beliefs are fundamentally different from knowledge. There are some exceptional studies (e.g., Abelson, 1979; Nespor, 1987; Pajares, 1992), which discuss salient features of beliefs distinct from knowledge: (a) existential presumption, (b) affective and evaluative aspects, (c) episodic accumulation, and (d) nonconsensuality.

Existential Presumption

Nespor(1987), drawing from Abelson's (1979) efforts with artificial intelligence, suggests that beliefs are partly related with existence or nonexistence of entities. Beliefs in the existence of friendship, happiness, or abilities are included in this category. For instance, experienced teachers may think that skill-mastering is most effective teaching to increase students' learning outcomes, while novice teachers think that problem solving should be central to school mathematics in order to qualitatively improve students' learning mathematics.

On the other hand, mathematics educators may think that this inconsistency between novice and experienced teachers exists due to the lack of systemic support of teacher education programs. That is to say, from this example, experienced teachers' beliefs of how mathe- matics should be taught exist independently from the beliefs of novice teachers and mathematics educators, and further, they are non-interchangeable each other. Beliefs in this category are regarded as something that already exists regardless of ones own perception.

Affective and Evaluative Aspects

Another feature of beliefs is that beliefs have strong affective or evaluative aspects more than knowledge (Nespor, 1987; Abelson, 1979). Ernest(1989) explains that beliefs are an affective outcome, while knowledge is a cognitive outcome. In other words, affective outcomes are exemplified as feelings, moods, self-esteem, self-evaluation, all of which are dependent upon subjective evaluation. On the other hand, mathematics content knowledge is an example of cognitive outcomes. Knowledge of domain content knowledge is obviously distinguished from subjective perspectives to how such domain knowledge should be taught.

In more specific, an equation $3x+2=8$ can be solved in several ways, such as traditional rule-based method, graphical representations, modeling approach, and others. That a teacher knows all the approaches to this equation is clearly distinguished from a belief that he or she likes or dislikes skill-based approach. If a teacher likes rule-based problem solving for the equation, we assume that students in his class would probably think of mathematics as a boring and disciplined area of study. Belief that students in economically advantaged areas are better in mathematics achievement than students in economically disadvantaged areas is another example of this category.

Episodic Accumulation

Abelson(1979) suggests that beliefs are in part formed as a result of accumulated episodes from personal experiences, cultural environment, or folklore. Abelson argues that knowledge is relying on general principles and facts. Under this notion of beliefs, knowledge is thought of as structured body, but beliefs are accumulated with episodic stories (Nespor, 1987). teachers' beliefs of teaching mathematics are considerably influenced by both their teaching experiences as in-service teachers and by past experiences as students (Raymond, 1997). This implies that more critical and active episodes make a more significant effect on shaping teachers' beliefs. Abelson calls these kinds of beliefs "subjective proof of a belief" (p., 359). On the other hand, knowledge is semantically and cognitively structured.

Nonconsensuality

It is in their nonconsensual aspect that beliefs are most saliently distinguished from knowledge. Belief systems within groups do not require general agreement about beliefs. Likewise, individual belief systems are not necessarily internally consistent (Nespor, 1987; Pajares, 1992). This feature of beliefs implies that beliefs are characterized as disputable and individualistic. On the other hand, knowledge systems are based on reasoning and evidence. Nespor(1987, p.321) suggests that:

> Belief systems are less malleable or dynamic than knowledge systems. Knowledge accumulates and changes according to relatively well-established canons of argument. Beliefs, by contrast, are relatively static.... When beliefs change, it is more likely to be a matter of a conversion or gestalt shift than the result of argumentation.

Summary

Beliefs and knowledge are very complex phenomena, and both are regarded as important indicators for teaching practices in terms of instructional decision-making. In some sense, there could be some area in which beliefs and knowledge are commonly shared, since they both are closely correlated each other (Peterson, Fennema, Carpenter, & Loef, 1989). For instance, teachers' pedagogical beliefs, which are accumulated through teaching experiences, might be referred to also as teachers' content knowledge if it is generally agreed. Based on the discussion for distinctive features of beliefs from knowledge, the nature of beliefs and knowledge is summarized as follows:

- beliefs---nonconsensual, disputable, illogical, static, episodic-based structured, inflexible, personal, individualistic, value-laden, evaluative, and affective
- knowledge---general, consensual, logical, dynamic, semantically organized, flexible, objective, well-defined, and valuefree

IV. Beliefs and Practice

Study of Consistency: Baseline Study

More than a decade ago, Thompson(1984, 1985) paved the way for further studies about mathematics teachers' beliefs. Her main question was to determine if there was a consistent relationship between three junior high school mathematics teachers' professed beliefs and their instructional practice. The finding in her exploratory case study is that teachers' beliefs of mathematics and its teaching play a significant role in forming their instructional patterns. For instance, Kay, who has a non-traditional view of mathematics with five years of teaching experience, was observed to frequently stress problem solving and stimulate her students with challenging problems. If there are incongruities between teachers' beliefs and teaching practice, these usually takes place due to several constraints (e.g., time, management, and different expectations on the part of teachers, students, principals, and parents) which happen in school settings (Thompson, 1984, 1985; Cooney, 1985; Raymond, 1997). Cooney(1985) demonstrates that the reality of the school setting significantly causes beginning teachers with a non-traditional view of teaching mathematics to develop dualistic beliefs, idealistic and practical. Fred, a novice teacher in his study, viewed mathematics as problem solving and dynamic as a preservice teacher, but in practice he realized that there are many constraints (e.g., time) that prevent him from teaching mathematics based on his beliefs. The school environment for teaching mathematics makes both novice and experienced teachers adjust their original beliefs of mathematics (Franke, Fennema, & Carpenter, 1997). Consistent with nonconsensual features of teachers' beliefs discussed in the previous section, many studies document that each teacher has a different perspective on the nature of mathematics and its teaching (e.g., Thompson, 1984; Raymond, 1997). Inconsistency of beliefs is sometimes found even within an individual' beliefs of mathematics and its teaching. However, these inconsistencies may be explained by preferences of beliefs (Nespor, 1987).

Abundant studies about teachers' beliefs are conducted by case studies. The weakness of the case study is that the findings are not easily understood as generalized knowledge. However, the findings from numerous case studies deserve consideration in that they can suggest other lines of research. Few studies designed quantitatively are found in the area of teachers' beliefs. Peterson, Fennema, Carpenter, & Loef(1989) examined the relationship between teachers' pedagogical content beliefs and their students' achievement in the area of computational skill and problem solving. One of the strengths of this belief study is that, while other previous studies (e.g., Cooney, 1985) were usually focused on general curriculum areas of interest, Peterson, et al. moved their interest into teachers' beliefs of the domain specific content area of addition and subtraction. Through comparing teachers' belief questionnaire and interview data, the researchers conclude that teachers' pedagogical content beliefs of addition and subtraction are highly correlated with teaching practice with correlation coefficient of $r = .88$, which, in turn becomes a highly significant predictor for students' achievement in the same area.

Despite these significant findings, a problem is the lack of certainty about whether teachers' beliefs affect changing teaching practice, or vice versa. Also, are teachers' beliefs of mathematics congruent with their students' beliefs? The relationship between teachers' beliefs of mathematics and its teaching is known to be complex phenomenon. For instance, several studies demonstrate that teachers' beliefs are intertwined with teachers' knowledge (Nespor, 1987; Ernest, 1989; Peterson, Fennema, Carpenter, & Loef, 1989). In particular, Peterson and his colleagues find that mathematics teachers' pedagogical content beliefs in addition and subtraction are highly correlated with their pedagogical content knowledge in the same area. This implies that mathematics teachers' beliefs seem to be influenced by their degree of content knowledge.

In summary, a significant number of early studies about teachers' beliefs in relationship with their teaching practice found a positive correlation between the two areas of interest. Ernest(1989) suggests that, if two teachers have similar knowledge, then teachers' beliefs of the nature of mathematics and teaching mathematics will be a significant indicator to predict their teaching practice. These findings provide the current reform movements with empirical knowledge to support why researchers should focus on more deeply exploring teachers' beliefs and dispositions toward both the nature of mathematics, and teaching and learning mathematics for successful reform (NCTM, 1991).

Does beliefs influence teaching practice? (Does teaching practice influence teachers' beliefs?)

Thompson(1992) suggests that there is a need to study more closely the relationship between teachers' beliefs and their teaching practice. Although many studies discussed in the previous section contribute to determining the positive relationship between teachers' beliefs of mathematics and its teaching and their instructional practices, the issue of importance regarding how to improve teaching practice is whether teachers' beliefs have an effect on their teaching practice, or whether the environments for teaching practice make an effect on their belief change. This unsolved issue seems to determine fundamental assumptions in developing teacher education programs. That is to say, Cognitively Guided Instruction(CGI) is developed on the assumption that teachers' beliefs supported by knowledge will effect improvement of their teaching practice (e.g., Carpenter, Fennema, Peterson, & Carey, 1988; Carpenter & Fennema, 1992), while the QUASAR [Quantitative Understanding: Amplifying Student Achievement and Reasoning] project (e.g., Silver, Smith, & Nelson, 1995) assumes that if a learning environment is provided, beliefs will change. In considering this issue, Raymond(1997) deserves to be paid more attention.

In order to explore the relationship between mathematics teachers' beliefs and teaching practices through a multiple-case study, Raymond(1997) focuses on factors that influence beliefs and teaching practice. The major findings are that beginning mathematics teachers' beliefs of mathematics are most influenced by past school experience as students, while teaching experience and teacher education programs are most influential factors in shaping their beliefs of teaching practice. This finding seems inconsistent with previous beliefs argued by several studies (e.g., Thompson, 1985; Cooney, 1985; Ball, 1990), where teacher education programs have little influence on teachers' belief change. Raymond reports also that teachers' beliefs of mathematics content are more traditional than that of mathematics teaching and learning. However, teaching practices are found to be mostly influenced by teachers' beliefs, students' behaviors, and their personal traits. The critical implication here is that teaching practices are more influenced by teachers' beliefs of mathematics than of teaching and learning. In considering the affective aspects of teachers' beliefs in the previous section, teachers' beliefs of the nature of mathematics seem to be more strongly connected and evaluative than their views of teaching mathematics. The earlier acquired beliefs have a stronger connection to a belief system than beliefs newly acquired (Nespor, 1987).

In conclusion, teachers' beliefs of mathematics and its teaching are inextricably interconnected with other factors, such as teachers' knowledge, social factors, students' behaviors, prior education, and personal traits. The findings from reviewing these belief studies suggest that we need models to explore more closely the interrelationship between teachers' beliefs and their teaching practices from different perspectives.

V. Modeling the Relationship between Beliefs and Teaching Practice

In the earlier sections, the researcher discussed that there is a strong need for models to investigate the relationship between teachers' beliefs and teaching practice. Although numerous studies have been focused on how students learn mathematics, difficulties in studying teachers' learning and development have partly originated from the serious lack of theoretical studies about teachers' learning. Ernest(1989) compares the relationship between teachers' beliefs and teaching practice by categorizing teachers into three groups, based on teachers' beliefs, roles, and classroom activities: instrumental (traditional), explainer (transitional), and problem solving (reform-oriented).

However, it seems to make sense to consider also the theoretical background of students' learning in order to explore the relationship between teachers' beliefs and teaching practice, because teachers are supposed to learn similarly to students (Putnam & Borko, 1997). Thus, different models regarding teachers' beliefs and teaching practices will be focused on the level of individuals (cognitive perspective), classrooms (emergent perspective), and school contexts (situative perspective). As seen earlier, the basic principle is that change of teachers' beliefs does not easily occur with just logical argument. Note that the following three models were derived from reviewing both theoretical and practical studies underlying the dominant theories on teaching and learning in mathematics education.

Model 1: Knowledge-driven approach to beliefs and teaching (Constructive perspective)

Description

When thinking from a cognitive constructivist perspective, heavy attention is focused on the need for teachers' knowledge of content and of pedagogy. This approach to teachers' beliefs and teaching practice assumes that increased knowledge of mathematics will motivate teachers to focus on teaching mathematics more conceptually. The CGI project (e.g., Fennema, Carpenter, Franke, & Carey, 1993 ; Fennema, Franke, Carpenter, & Carey, 1993), for example, provides in-service teachers with research-based knowledge during a four week summer workshop, assuming that, if teachers have increased knowledge of how students think, then their teaching practice and beliefs will change to a reform-oriented approach. Thus, research-based knowledge of content plays a role as motivating CGI teachers to better understand what students express and how they are thinking. As they understand students learning deeper and deeper, CGI teachers' teaching practices change to a students-centered practice supported by the current reform movements.

With respect to the relationship between teachers' beliefs and teaching practice, the CGI suggests that, without teachers' belief changes, it is impossible to become a "self-sustaining" CGI teacher (Franke, Fennema, & Carpenter, 1997, p. 277). Franke, et al. divide the process of teacher change into four levels, regarding beliefs, knowledge, and use of students thinking in practice. The CGI reports that teachers' beliefs change is significantly related to their teaching practices.

Issues for teacher learning

The cognitive perspective of teacher learning assumes that teachers, like students, make sense of their new beliefs and teaching practices on the basis of their existing beliefs and teaching practices (Putnam & Borko, 1997). In this model, teachers are initially motivated by increasing their knowledge about students' thinking. Thus, the issue to be considered is how to sustain teachers' teaching practices and beliefs. It was reported that some of CGI teachers have never used CGI as more than a supplement, and others used CGI as the primary teaching practice (Knapp & Peterson, 1995). What makes this difference happen? Regarding this issue, CGI(Franke, Fennema, & Carpenter, 1997) emphasizes the importance of "self-generated, self-sustained changes that enable teachers to create opportunities in the classroom that reflect the needs and understanding of their students" (p. 277).

Another important feature of CGI is found in the ongoing support system for in-service teachers. Differently from short period of usual workshops, the CGI provides a combination of support to teachers through summer workshops and onsite support from CGI staff members (e.g., Putnam & Borko, 2000; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998).

Implications for designing professional development for conducting teacher change

With respect to designing a teacher development program, we may need to focus on the design of the CGI. One of the reasons for the CGI success may be found in their design for support of in-service teachers. In order to support teacher change, the CGI staff used both summer workshops for proving knowledge and onsite consultation for helping its transfer to classroom contexts.

As known earlier, teachersteachers' knowledge of mathematics is closely related with their beliefs of mathematics and its teaching. Knowledge is a necessary condition for teaching mathematics, but teachers' beliefs are a sufficient condition for better teaching and a necessary condition to become fully CGI teachers. Many studies (e.g., Ma, 1999) demonstrate that currently elementary teachers seriously lack in their knowledge of mathematics content.

Model 2: Belief change in contexts (Situative perspective)

Description

Over the last decades, increasing attention has been given to the effects of social aspects on teachers' professional development. This approach to teacher change in both beliefs and teaching practices is based on the assumption that teachers become professional through the process of participation in the community of practices (e.g., Lave & Wenger, 1991; Bereiter, 1996; Greeno, 1997). The second model is greatly different from the first in the sense that teacher change in the former is heavily dependent upon the social interactions in the community, while in the latter, the changes of teachers' beliefs and teaching practice are attributed to individual dimensions.

The QUASAR project seems to be a good example regarding the process of teachers' professional development by participating in the community of practice. Stein and Brown (1997) reports that teachers in QUASAR sites improve the quality of teaching practice through collaborative interactions with other members in the community. The community of practice is defined as "a group of individuals who share understandings concerning what they are doing and what that means in their lives and for their communities" (Lave & Wenger, 1991, p. 98). Therefore, school sites are regarded as the community of practice for teachers to develop teaching practice and to change beliefs through interactions with other members. In this sense, Stein and Brown (1997) view schools as the "authentic workplace" to learn (p. 161).

The main advantage to this situative approach to teacher change is that teachers can use multiple resources for learning. For instance, teachers bring episodes about teaching experiences into the community to share with other members, which helps improve their teaching practices and extend their beliefs. According to Lortie (1975), school teachers rarely find opportunities to observe how other teachers teach in their own classrooms. Therefore, teacher change in light of beliefs and teaching practice will be highly affected by the quality of the community they participate in, such as an expertly provided support system for teachers.

Issues for teacher learning

In considering teacher learning from the situative perspective, the biggest issue is about how to situate the learning environment for teachers. Teachers do not usually have opportunities to experience diverse ways of teaching, since workplaces for teachers are characterized as isolated (Lortie, 1975). In considering the principle of learning from the situative perspective, it is natural that a school setting is the most appropriate learning environment for teachers.

Attention should also be given to the role of motivation. Simply put, the perspective of the second model stresses teachers' internal motivation, by which Lave and Wenger(1991) refer to "use value" of learning against its "exchange value" (p. 112). As teachers learn, they should develop a feeling of professionalism as they are engaged in teaching rather than for the sake of making something. According to Lave and Wenger, teacher learning is well compared with students learning in a sense that use value (true value of mathematics; e.g., reasoning, problem solving) should be focused on rather than on exchange value (e.g., mathematics as means to pass an exam).

The fundamental issue regarding the meaning of learning from the situative standpoint is the possibility of the transfer of knowledge. With regard to students' learning, several studies have demonstrated that school mathematical knowledge is noticeably absent in out-of-school settings (e.g., Nunes, Schliemann, & Carraher, 1993). Similar problems regarding teacher learning can be raised. That is to say, it might be questionable that what teachers learn through the participation in the community of practices (out of classroom settings) will be effective in their classroom teaching practices away from learning places.

Stein and Brown (1997) point out that school sites should focus on how to assist teachers in learning and improving teaching practice rather than on how to direct and assess the teachers. As noted earlier, teachers' beliefs and teaching practices are not assumed to change easily, but this needs long supportive efforts from researchers, school contexts, peers, or districts.

Implications for designing professional development for conducting teacher change

First of all, the situative perspective made important contributions to the mathematics education community, as they demonstrate that the social plays a large role in teacher learning. A decade before, understanding of students' learning was approached mostly from the individual psychological dimension. Currently, such understanding necessarily requires deep consideration of social and cultural dimensions.

In the social contexts for learning, each teacher is supposed to bring individual experiences (e.g., problems in teaching for understanding) and stories (e.g., episodes) into the community of practices to share with other members. In doing so, teachers can together create new models of teaching practices to improve students' understanding (Stein & Brown, 1997). In designing a new professional development program, this approach implies that active peer collaborations with willingness to change will be important to consider.

Model 3: Dilemma-driven belief change in classroom contexts (Emergent perspective)

Description

We usually agree that mathematical understanding is heavily influenced by classroom social practices that structure the opportunities for learning. There is a frequent debate about whether learning is a product of individuals' cognitive construction or a product of social practices (e.g., Anderson, Reder, & Simon, 1996, 1997; Greeno, 1997; Sfard, 1998). While model 1 heavily emphasizes individuals' ability in construction mathematical knowledge and model 2 the roles of social practices, the third model proposes that learning emerges from classroom contexts through the process of negotiating meanings.

The fundamental assumption in this perspective is that teachers' learning emerges from classroom interactions just as students learning. teachers' beliefs are not assumed to change easily, but are supposed to be embedded in the process of teacher change, for example, from the traditional to reform-oriented perspectives. So, in order to understand the relationship between teachers' beliefs and teaching practice, it is necessary to understand the process of teacher change.

The fundamental principle of teacher learning is the process of resolving dilemmas created due to the gap between their traditional perspectives and their emergent perspectives. An example of dilemmas teachers should solve is whether to encourage students resolve wrong answers by themselves or to provide them with correct answers. Just as teaching practices are assumed to improve while resolving instructional dilemmas, teachers' beliefs are assumed to change as they reflectively interact with situations. In other words, the classroom should be regarded as learning environment for teachers.

Issues for teacher learning

First of all, an important issue to consider is how to motivate practical teachers to realize their teaching practice as problematic (process of motivation). Teachers are not viewed as knowledge transmitters, but are seen as co-learners who encourage students' conceptual understanding and create a classroom learning environment with students. Videotaping target teachers' classroom teaching and letting them see their own teaching may be one of a variety of ways for motivating (Wood, Cobb, & Yackel, 1990).

Another important issue is raised in light of providing a support system for teachers. Dilemmas may occur whenever they are teaching, but they tend to easily give up emergent-oriented (reform-oriented) teaching (Wood, 1995). An on-going support system seems to be necessary for implementing this model.

Thirdly, the task of creating a learning environment for teachers should be considered. Since both teachers and students are viewed as co-learners who construct shared knowledge of mathematics through interactions in classroom contexts, initial support for practical teachers should be provided to help create such learning environments.

Implications for designing professional development for conducting teacher change

An emergent perspective on teacher learning yields several implications with respect to designing a teacher development program. First of all, while the other two models focus on individuals' abilities or social activities as learning environments, model 3 focuses our attention on classroom contexts for teacher learning, which is the teachers' actual workplace. Furthermore, this context will change more effectively teachers' traditional teaching practices and beliefs, in contrast with the first two models which place heavy emphasis on individuals' psychological construction of mathematical knowledge or school contexts as learning environments.

However, in order to follow the third model, teaching models to guide teachers need to be developed, since numerous dilemmas at the initial phase of teacher change will confront the teachers. Therefore, teaching models will be useful in order to help overcome such dilemmas.

VI. Reflections

teachers' beliefs should not be assumed to change without opportunities to experience how reform-oriented teaching should be implemented. As discussed earlier, beliefs are characterized as individualistic, unnconsensual, static, inflexible, but affective. In addition, we generally agree that teachers' beliefs are strong indicators of their teaching practices, so that, without belief change, their reform-oriented teaching practices (the core of educational reform) might be difficult to achieve.

In order to understand the relationship between beliefs and practices, the researcher discussed three different types of models, basically implied by the current dominant theories of students' learning. The assumption of model 1 (cognitive perspective) is that teachers' knowledge of content and pedagogy of mathematics motivates teachers to realize the need to change beliefs and teaching practices. Model 2 (situative perspective) has the assumption that teacher learning occurs while they participate in the community of practices, so that beliefs and teaching practices change as they are more engaged in a community. In contrast, model 3 (emergent perspective) focuses on dilemmas of teaching which occur due to the gap between the familiar traditional ways of teaching and reform-oriented teaching practices.

Although all three models approach the problem of interest from different perspectives, that is, individual, social (school contexts), and classroom (individual and social contexts), they all agree that beliefs should be regarded in conjunction with teaching practices. The major differences lie in the choice of contexts where teacher change occurs and the ways which teaching practices and beliefs improve and change.

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교사들의 수학적 신념과 수업 관행의 관계

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본 연구의 목적은 교사들이 갖고 있는 수학 및 수학을 어떻게 가르칠 것인가에 대한 신념과 수업 관행과의 관계를 문헌적 고찰을 통하여 교사 변화를 위한 모델을 제시하는데 있다. 이를 위하여 먼 저 신념에 대한 정의, 신념과 지식의 차이점, 그리고 신념이 교사들의 수업 관행과 어떻게 관련이 있는지를 논의하였다. 신념과 수업 관행과의 상호 관계를 통하여 본 연구에서는 수업 개선 프로그램의 개발을 위한 모델을 개인적 수준, 학급 수준, 및 학교 수준의 세 시각에서 논의하였다. 이들 모델들은 결국 교사의 학습도 학생들의 학습 방법과 유사한 형태를 띄고 있다는 점에서 현재의 주요한 수학 학습 이론들에 근거를 두고 있다. 결국, 교사들의 수업 관행에 큰 영향을 끼치는 것으로 알려진 교사들의 수학적 신념은 위에 논의된 세 요소의 측면에서 수업 개선 프로그램들이 운영될 때 수업 관행과 함께 변화한다는 것을 본 이론 연구에서는 암시해 주고 있다.