Combined Optimal Design of Robust Control System and Structure System for Truss Structure with Collocated Sensors and Actuators

Jung-Hyen Park

Dept. of Automotive Engineering, Silla University, Pusan, South Korea

ABSTRACT

A control-structure combined optimal design problem is discussed taking a 3-D truss structure as a design object. We use descriptor forms for a controlled object and a generalized plant because the structural parameters appear naturally in these forms. We consider not only minimum weight design problem for structure system, but also suppression problem of the effect of disturbances for control system as the purpose of the design. A numerical example shows the validity of combined optimal design of structure and control systems. We also consider the validity of sensor-actuator collocation for control system design in this paper.

Keywords: Combined optimal design, Structural control, H_∞ control, 3-D Truss structure, Sensor/Actuator collocation

1. Introduction

The combined design of structure and control systems is one of the powerful, dynamic design techniques to design a highly developed machine structure (the structure system and the control systems) simultaneously. Especially, it is thought an appropriate technique on the design of flexible space structure etc, for example, the satellite and the space station that are flexible, large-scale and difficult in control. That is, there are severe specifications of control system that is the vibration control, and of the structure system that it is necessary to become in lightness in the design of such a large-scale structure. However, if the structure is light, its stiffness becomes small, and a big vibration will be excited by a little external disturbance. And because the damping is small, the vibration caused once also has the defect that it is not easy to install. To secure the damping of vibration property for such a vibration, the cost to the control system design grows. It is necessary to give the stiffness of the structure to reduce the cost to the control system design oppositely, and the structure weight grows. It is necessary to design considering the structure system and the control system to be one system when there is a close relation between the structural design and the control system design like this [1].

It is done this various researches of the validity on combined optimal design of structure and control systems at present. Iwatsubo et al. minimized linear sum of a quadratic evaluation function of structure weight and linear regulator for the beam model by which a continuous body is imitated [2]. Salama et al. minimized linear sum of quadratic evaluation function of structure weight and linear regulator for 3 D.O.F beam model under the restriction of the natural frequency of closed-loop system [3]. Onoda et al. minimized linear sum of structure weight and the control energy under the restriction of the vibration energy [4]. Rao et al. did the combined design of structure weight and linear regulator by using game theory for the truss structure [5]. Knot et al. minimized the structure weight under the restriction of the natural frequency of closed-loop system or damping factor for the truss structure [6]. Grandhi minimized the structure weight or frobenius norm under the restriction of the natural frequency of closed-loop system or damping factor for the truss structure [7]. Kajiwara et al. minimized the quadratic evaluation functions of structure weight and linear regulator for 3 D.O.F system model [8]. Moreover, Tada *et al.* minimized the objective function of the control system, which shows the effect of the vibration by initial external force for three dimensional truss structure [9].

Generally, many of these problems have been taken up on the assumption that the design object is described by the state space equation. In this research, the descriptor form is used to express the design object. The descriptor system form is a description of a dynamic, static relation between variables which appear to the system as it is, and modeling by which the structure which derives from the arrangement of the bond between variables and a physical constant is shown in the positive can be done by using the descriptor form [10]-[12].

In this paper, we deal with the combined design of structure and control systems for 3-D truss structure modeled by the finite element method as design object. The structural objective function is the structure weight and the control objective function is H_∞ norm. The design variables are cross sectional areas of the truss members. The objective function of combined optimal design problem is the linear sum of the normalized structural objective function and control objective function. By minimizing this objective function, it is possible to make optimum design by which the balance of structure weight and control performance is taken. As the design variables when the combined optimal design problem is taken up, the cross sectional areas of the truss members are adopted, and the validity of our design method is shown through the numerical example. We also consider the validity of sensor-actuator collocation for control system design.

2. System Formulation and H_∞ Control Problem

In this paper, a material and structural arrangement make 3-D truss structure, which consists of n truss members, not changed a design object. Generally, flexible structures modeled by the finite element method (It is assumed that the axial displacement is a linear expression in the element, and assumes one truss member to be one element) are expressed by the following equation of motion.

$$M_s\ddot{q} + D_s\dot{q} + K_s q = L_1 w + L_2 u \qquad (1)$$

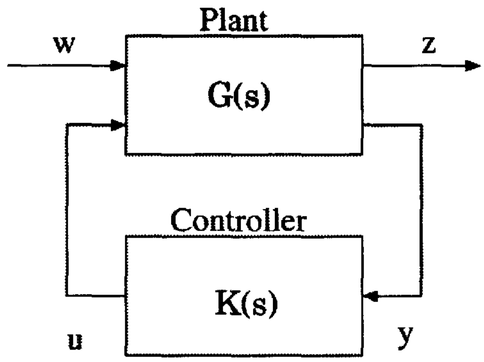

Fig. 1 H_∞ Control system

where M_s, D_s, and K_s are the mass, the damping, and the stiffness matrices. q, w and u are the displacement, the disturbance input and the control input and L_1 and L_2 are the disturbance and control input matrices. The descriptor system of the model becomes Eqs. (2), (3) and (4), where x, z and y are the descriptor variable, the controlled output and the measured output, C_1 and C_2 are the controlled and measured output matrices. D_{12} and D_{21} are the matrices which satisfy $D_{12}^T, D_{12} > 0$ and $D_{21}D_{21}^T > 0$ where super script T means its transposed matrix.

$$E\dot{x} = Ax + B_1 w + B_2 u \qquad (2)$$

$$z = C_1 x + D_{12} u \qquad (3)$$

$$y = C_2 x + D_{21} w \qquad (4)$$

$$E = \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -K_s & -D_s \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ L_1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ L_2 \end{bmatrix}, \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

In this paper, the control system is designed with the robust H_∞ control to suppress the effect of the disturbance. In Fig.1, H_∞ control problem is to find such a controller $K(s)$ that the closed-loop system is internally stable and the following H_∞ norm condition is satisfied

$$N = \|T_{zw}(s)\|_\infty < \gamma \qquad (5)$$

$$\|T_{zw}(s)\|_\infty = \sup_w \sigma_{\max}(T_{zw}(jw))$$

where $T_{zw}(s)$ is the transfer function from the disturbance input w to the controlled output z in the closed-loop system, γ is a prescribed positive number and $\sigma_{\max}(T_{zw})$ is the maximum singular value of T_{zw}. Eq.(5) which is expressed in frequency domain is equivalent to the following equation in time domain.

$$\int_0^{\infty} z^T(t)z(t)dt < \gamma^2 \int_0^{\infty} w^T(t)w(t)dt \qquad (6)$$

Because the right-hand side of Eq. (6) denotes the influence on the system by the disturbance, it is considered that H_{∞} norm, N, denotes the degree of disturbance suppression from Eqs. (5) and (6), that is, the smaller the value of N is, the more the influence of the vibration by the disturbance can be suppressed.

The necessary and sufficient conditions for the existence of H_{∞} controller are that there exist X and Y which satisfy the following symmetric Riccati equations[13]

$$(A - B_2 D_{12}^{\#} C_1)^T X + X^T (A - B_2 D_{12}^{\#} C_1)$$

$$-X^T(B_2 R_{12}^{-1} B_2^T - \frac{1}{\gamma^2} B_1 B_1^T)X \qquad (7)$$

$$+(D_{12}^{\perp} C_1)^T (D_{12}^{\perp} C_1) = 0$$

$$E^T X = X^T E \geq 0 \qquad (8)$$

$$(A - B_1 D_{21}^{\#} C_2)Y + Y^T (A - B_1 D_{21}^{\#} C_2)^T$$

$$-Y^T(C_2 R_{21}^{T} C_2^T - \frac{1}{\gamma^2} C_1 C_1^T)Y \qquad (9)$$

$$+B_1 C_{21}^{\perp} (B_1 D_{21}^{\perp})^T = 0$$

$$EY = Y^T E^T \geq 0 \qquad (10)$$

$$\det(I_n - \frac{1}{\gamma^2} YX) \neq 0 \qquad (11)$$

$$Z = (I_n - \frac{1}{\gamma^2} YX)^{-1} \geq 0, \qquad (12)$$

where R_{12}, R_{21}, $D_{12}^{\#}$, $D_{12}^{\perp}$, $D_{21}^{\#}$ and $D_{21}^{\perp}$ are

$$R_{12} = D_{12}^T D_{12} > 0 \ , \ R_{21} = D_{21} D_{21}^T > 0$$

$$D_{12}^{\#} = R_{12}^{-1} D_{12}^T \ , \ D_{12}^{\perp} = I - D_{12} D_{21}^{\#}$$

$$D_{21}^{\#} = D_{21}^T R_{21}^{-1} \ , \ D_{21}^{\perp} = I - D_{21}^{\#} D_{21}.$$

Now, we have the following H_{∞} controller

$$E\dot{x}_k = A_k x_k + B_k y \qquad (13)$$

$$u = C_k x_k + D_k y \qquad (14)$$

where A_k, B_k, C_k, and D_k are

$$A_k = A - B_k C_2 + \gamma^{-2} Y^T C_1^T C_1$$

$$+(B_2 + \gamma^{-2} Y^T C_1^T D_{12}) C_k$$

$$B_k = (Y^T C_2^T + B_1 D_{21}^T) R_{21}^{-1}$$

$$C_k = R_{12}^{-1} \{ D_{12}^T C_1 + (B_2 + \frac{1}{\gamma^2} Y^T C_1^T D_{12})^T Z \}$$

$$D_k = 0$$

3. Optimum Design Problem

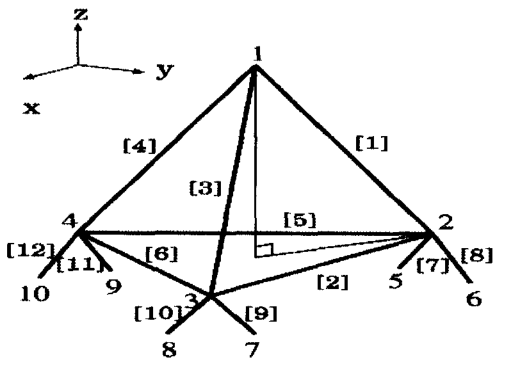

Fig. 2 3-D Truss structure

In this paper, we consider a minimum weight design problem for structural system and suppression problem of the effect of disturbances for control systems the purpose of the design. Taking a 3-D truss structure as an object, the mass, damping, and stiffness matrices of the system can be modeled as the function of the cross sectional areas of the truss members from FEM formulation [14][15]. The structural objective function is the structural weight W and the control objective function is N, that is, H_∞ norm of the transfer function from the disturbance input to the controlled output in closed-loop system.

We take a 3-D truss structure shown in Fig. 2 as design object. $1,\cdots,10$ are nodes and $[1],\cdots,[12]$ are truss members. Considering non-dimensional form, the length of long members is 10, short members $2\sqrt{2}$, density 1.0, and Young's modulus 10^4. The nodes from 5 to 10 are fixed. The damping matrix is assumed by

$$D_S = 0.001M_S + 0.001K_S \quad .$$

The structural objective function W is calculated by

$$W = \sum_{i=1}^{12} \rho_i l_i a_i \tag{15}$$

which ρ_i, l_i, and a_i are density, length and cross sectional area of the $i-th$ truss member. Control objective function N is the minimum value of γ in Eq. (5) that can be calculated by iteration method (γ iteration) [16][17] in the following interval.

$$0 < \gamma \le 10 \tag{16}$$

3.1 Sensor/Actuator Collocation

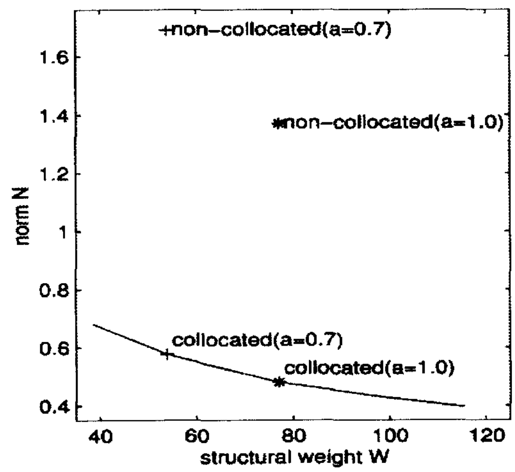

Fig. 3(a) Comparison between collocated case and non-collocated case

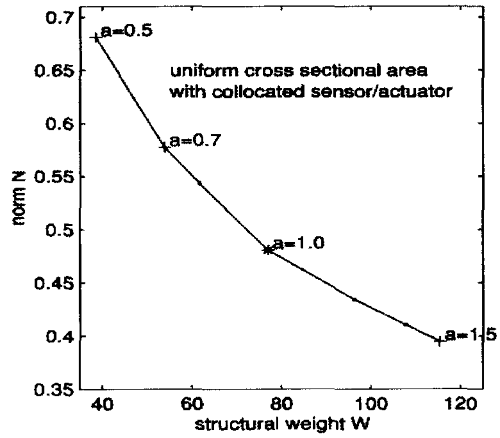

Fig. 3(b) Collocated case

Fig. 3 Relation between weight W and norm N (uniform cross sectional area)

The structures in which their sensors and actuators are located in the same directions and at the same positions (called collocation) are to be minimum phase system, i.e. real parts of invariant zero points are all negative. Minimum phase system is known as better than non-minimum phase system for control system design [18] In this section, we perform the control system design to minimize only the control objective function N under the condition that the structural system is given for several cases that all members of truss structure have uniform cross sectional areas. And we consider the positioning problem of sensors and actuators, which is important for the control system design. We compare the cases that sensors and actuators are collocated with non-collocated cases.

Fig. 3 (a) results show that the structures with collocated sensors and actuators are better for control system design than those with which they are not collocated. Therefore, from now in this paper, we consider the optimum design problem for truss structure as showed in Fig.2 on the assumption that both the sensors and actuators are located at the node 1 in x, y and z directions. In Fig.3 (b), the relation between the structural weight, W, and H_∞ norm, N, is shown for several cases that all members of truss structure have uniform cross sectional areas. We recognize that two objective functions are on competitive terms.

3.2 Combined Optimal Design

In this section, we consider a combined optimal design problem of structural and control systems for flexible structure. The objective function in this approach is the linear sum of the normalized structural objective function and control objective function as follows

$$J(a) = w_W \frac{W(a)}{W_0} + w_N \frac{N(a)}{N_0}$$

where w_W and w_N are the weighting factors for structural weight, W, and H_∞ norm, N, and $W_0(=76.971)$ and $N_0(=0.485)$ are the values of the structural weight and of H_∞ norm for the initial structure in which all members have uniform cross sectional areas $a_i = 1(1,\cdots,12)$. We formulate the combined optimal design problem such as to find cross sectional areas of truss members for the minimization of the objective function, J, as follows:

$$\min_a J(a) = w_W \frac{W(a)}{W_0} + w_N \frac{N(a)}{N_0} \tag{17}$$

$$\text{subject to} \begin{cases} w_W + w_N = 1 \\ \text{There exists } X \text{ in Eq.(7).} \\ \text{There exists } Y \text{ in Eq.(9).} \\ \text{Eqs.(8),(10),(11) and (12)} \\ \text{are satisfied.} \\ a^{\min} \le a \le a^{\max} \end{cases} \tag{18}$$

where a is the set of cross sectional areas of the truss members, $a^{\min}$ and $a^{\max}$ are lower and upper limits of the cross sectional area. By minimizing the objective function, J, it is possible to make optimum design by which the balance of structural weight and control performance is taken. The simplex method [19][20] is used to solve the optimization problem above. We take into consideration the following constraint for cross sectional areas as design variables.

$$0.5 \le a_i \le 1.5 \quad (i = 1,\cdots,12) \tag{19}$$

First, we perform the combined optimal design in the case of the set of weighting factor for the structural and control objective functions $(w_W, w_N) = (0.5,0.5)$. In this case (case1), from the result by the minimization of J, the structural weight, W, is 71.383 and the H_∞ norm, N, is 0.473. We get 7.3% lighter structural weight, W, and 9.9% smaller value of H_∞ norm, N, than their initial values, W_0 and N_0.

Then, in the case of the set of weighting factor for $(w_W, w_N) = (0.6,0.4)$, we perform the combined optimization. In this case (case2), W is 64.945 and N is 0.460. We get 15.6% lighter weight and 5.2% smaller value of the norm than those of initial structure. In both cases, we get lighter structural weight, W, and smaller H_∞ norm, N, than the initial structure, i.e. the reduction of the cost for structural system design and the improvement of the suppression for the effect of disturbances for control system by the combined optimal design are obtained.

Next, we perform the combined optimal design of control and structure systems in the case of the set of weighting factor for $(w_W, w_N) = (0.7,0.3)$. In this case (case3), the structural weight, W, is 54.693 and the H_∞ norm, N, is 0.519. We get 28.9% lighter structural weight, W, than the initial weight W_0. But, the value of H_∞ norm, N, is 7.0% increased than N_0, that is, the suppression problem of the effect of disturbances got worse than the initial structure. And in the case of $(w_W, w_N) = (0.3,0.7)$, W is 85.308 and N is 0.419. In this case (case4), we get 13.6% smaller norm, N, than N_0, but the structural weight, W, is 10.8% increased than W_0. These results show the effect of weighting factors for structural and control objective functions in our optimum design method. Fig. 4 shows the distribution of the optimum cross sectional areas for these cases.

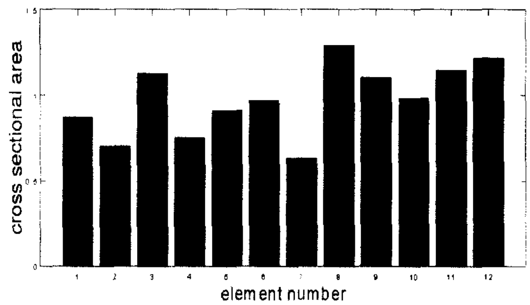

Fig. 4(a) Case 1: $(w_W, w_N) = (0.5,0.5)$

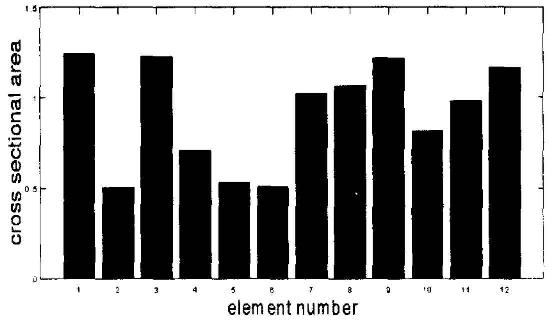

Fig. 4(b) Case 2: $(w_W, w_N) = (0.6, 0.4)$

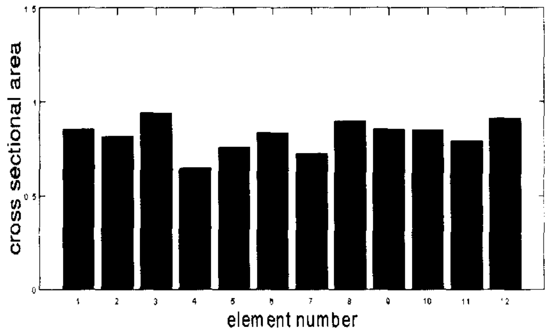

Fig. 4(c) Case 3: $(w_W, w_N) = (0.7, 0.3)$

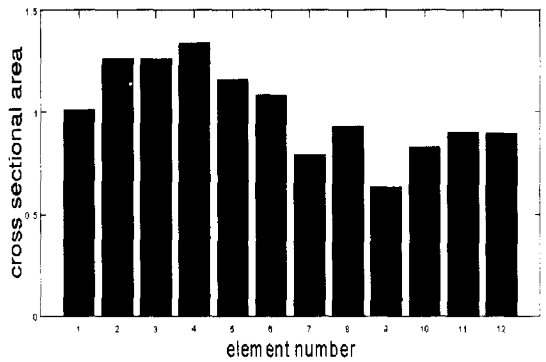

Fig. 4(d) Case 4: $(w_W, w_N) = (0.3, 0.7)$

Fig. 4 Distribution of cross sectional areas

In Fig.5, the set of structural weight and H_∞ norm, (W, N), corresponding to the minimum state of J in Eq. (17) is shown for several sets of (w_W, w_N).

4. Conclusion

In this paper, we formulated a combined optimal design problem of structure and control systems represented by descriptor system forms and suggested a design method for 3-D truss structure as the design plant.

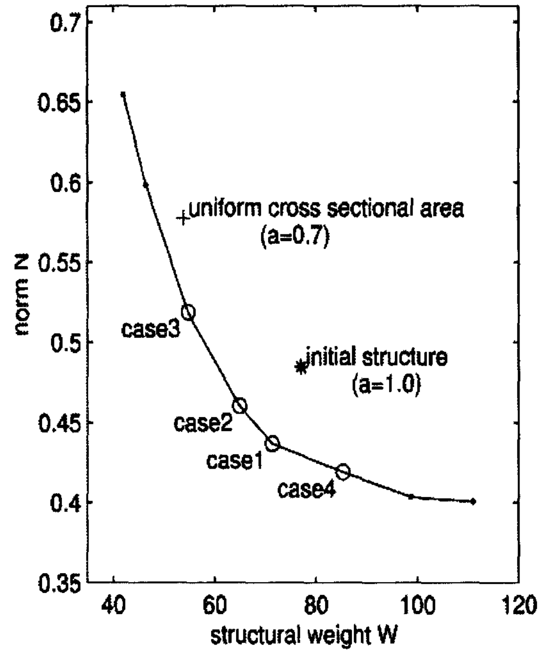

Fig. 5 Pareto Optimality for Weight and Norm

The structural objective function of the structure weight and the control objective function of H_∞ norm, and the design variables of cross sectional areas of the truss members were used in our method. The objective function of combined optimal design problem was the linear sum of the normalized structural objective function and control objective function. By our combined design method, we obtained the reduction of the cost for structure system design and the improvement of the suppression for the effect of disturbances for control system compared with the design that considers only the control system. We also showed the validity of sensor/actuator collocation for control system design and that of weighting method for combined optimization of structural and control objective functions. We recognized the relation of competition between two objective functions and effect of weightings for two objective functions. And it was possible to reduce the time of simulation by use of descriptor system forms, because inverse matrices of mass matrices are not calculated in optimization process that contains a lot of iterative calculation processes.

Acknowledgement

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