Optimal Time Slot Assignment Algorithm for Combined Unicast and Multicast Packets

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ABSTRACT—This paper considers a packet-scheduling algorithm for a given combined traffic of unicast and multicast data packets and proposes a hybrid router with several dedicated buses for multicast traffic. Our objective is to develop a scheduling algorithm that minimizes schedule length for the given traffic in the hybrid router. We derive a lower bound and develop an optimal solution algorithm for the hybrid router.

I. INTRODUCTION

As the amount of telecommunication traffic increases, a router using an efficient switching algorithm is required for fast routing. The traffic may be composed of unicast and multicast traffic. A popular way to handle multicast traffic is to replicate it multiple times into unicast queues in order to treat the multicast traffic as unicast. However, this approach has two disadvantages: increasing the required memory and reducing the bandwidth available to other traffic at the same input [1], [2]. To overcome the disadvantages, we propose a hybrid router and develop a static packet-scheduling algorithm that minimizes schedule length without conflicts. The proposed hybrid router has an $N \times N$ crossbar switch fabric for unicast traffic and Y buses for multicast traffic (Fig. 1). Each input has N virtual output queues (VOQs) for unicast traffic and a single queue for multicast. An output port has two queues, one for unicast and the other for multicast traffic. The multicast traffic can be transmitted along one of the Y multicast buses if the connection does not lead to input or output conflict (blocking). Since the router has separate routes for multicast traffic, the router is almost surely faster in service, but slightly more difficult to implement than those in [1], [3].

Among the previous studies on static packet scheduling, [4] considered a satellite-switched time division multiple access system and [5] developed an optimal time slot assignment algorithm which minimizes schedule length. Recently, [6] and [7] developed heuristic algorithms for burst-based or packet mode scheduling. However, [4]-[7] treated multicast traffic as unicast without explicitly considering the multicast traffic. This paper describes our optimal scheduling algorithm on the hybrid router for the combined traffic of unicast and multicast data packets.

This paper is composed as follows. Section II defines the problem and section III derives a lower bound on the schedule length and develops the optimal solution algorithm. Finally, some concluding remarks and further studies are described in section IV.

II. PROBLEM DESCRIPTION

Consider the hybrid router shown in Fig. 1. Let $\mathbf{U}=(u_{ij})$ be the $N\times N$ unicast traffic matrix, where u_{ij} denotes the number of unicast packets at VOQ j of input port i waiting for the connection toward output port j. Let $\mathbf{M}=(m_i)$ be the $N\times 1$ multicast traffic matrix, where m_i denotes the number of multicast packets of input port i waiting for the connection toward any one of the Y multicast buses. Then, an $N\times(N+1)$ augmented traffic matrix $\mathbf{A}=(\mathbf{U}, \mathbf{M})=(a_{ij})$ is obtained by letting $a_{ij}=u_{ij}$, $j=1,2,...,N$ and $a_{i,N+1}=m_i$ for each $i=1,2,\ldots,N$. The total traffic to be serviced at the router is $\sum_{i=1}^{N}\sum_{j=1}^{N} u_{ij} + \sum_{i=1}^{N} m_i$ and denoted by T. For the given traffic $\mathbf{A}$, the switching can be performed according

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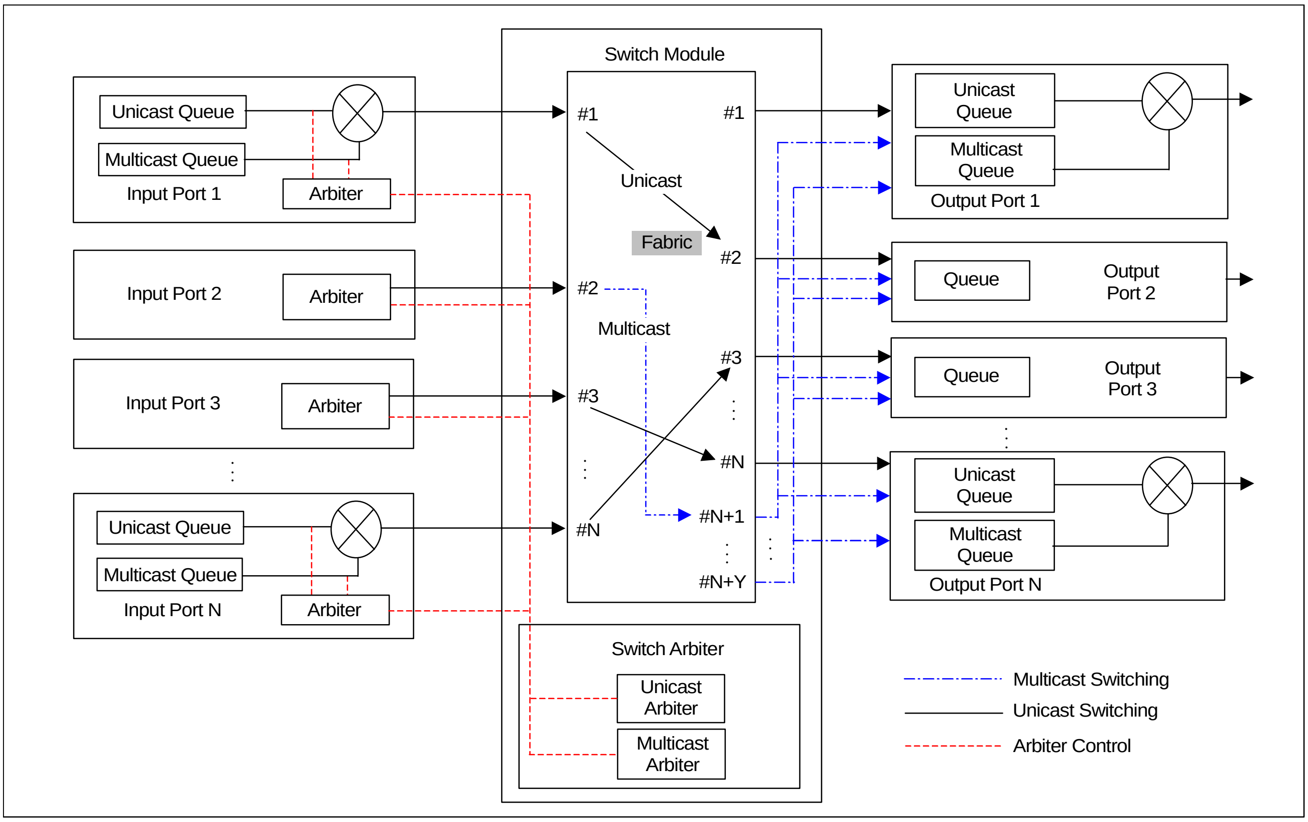

Fig. 1. Architecture of the proposed hybrid router.

to the switching matrix, denoted by $\mathbf{S}_t=(s_{ijt})$, where the switching matrix is the t-th $N\times(N+1)$ matrix and each row i and column j, $i,j=1,2,...,N$, has at most one positive element, and column $N+1$ has at most Y positive elements. Assume that there are both unicast and multicast traffic (packets) at the starting time of scheduling. The scheduling problem of [4] is the special case of this paper with $Y=1$. The remaining objective of the paper is to find an optimal time slot assignment algorithm on the proposed router with Y dedicated multicast buses so as to minimize the total schedule length.

III. SCHEDULING ALGORITHM

Property 1. For a given augmented $N\times(N+1)$ traffic matrix **A**, the optimal schedule length has the lower bound B, B = $\max\left\{\left\lceil\sum_{i=1}^{N}\sum_{j=1}^{N}u_{ij}/N\right\rceil, \left\lceil\sum_{i=1}^{N}m_i/Y\right\rceil, \max_{1\le j\le N}\{\sum_{j=1}^{N}u_{ij}+m_i\}, \max_{1\le j\le N}\{\sum_{i=1}^{N}u_{ij}\}\right\}$, where $\lceil x\rceil$ denotes the smallest integer larger than or equal to x.

Proof. The total number of required time slots (traffic) for input port i and output port j is at least $\sum_{j=1}^{N}u_{ij}+m_i$ and $\sum_{i=1}^{N}u_{ij}$, respectively. The required number of time slots is at least $\left\lceil\sum_{i=1}^{N}\sum_{j=1}^{N}u_{ij}/N\right\rceil$ for all the unicast traffic $\sum_{i=1}^{N}\sum_{j=1}^{N}u_{ij}$ due to input conflict. Furthermore, there are Y multicast buses for all the multicast traffic $\sum_{i=1}^{N}m_i$. Thus, at least $\left\lceil\sum_{i=1}^{N}m_i/Y\right\rceil$ time slots are required for the multicast traffic due to output conflict. This completes the proof. □

To find an optimal schedule with a schedule length of the lower bound of Property 1, let us define four terms. The $N\times(N+1)$ augmented traffic matrix **A** with total traffic T is *N-complete with respect to the lower bound B* if the entire line sum in **A** is less than or equal to B and $T=N\times B$. Then, it is always possible to add appropriate nonnegative integers to the entry of **A** so as to obtain a new $N\times(N+1)$ traffic matrix **A'** which is N-complete with respect to B, where $\mathbf{A}'=(a'_{ij})$, $u'_{ij}=u_{ij}+q_{ij}$, $m'_i=m_i+q_{i,N+1}$, and $q_{ij}=\min\left\{B-(\sum_{j=1}^{N}u_{ij}+m_i), B-\sum_{i=1}^{N}u_{ij}, N\times B-\sum_{i=1}^{N}(\sum_{j=1}^{N}u_{ij}+m_i)\right\}$. The addition process terminates when the new total traffic $\sum_{i=1}^{N}\sum_{j=1}^{N}(u'_{ij}+m'_i)$ equals $N\times B$.

A line in **A** is a *critical line* if its sum equals B, and a set of N

positive entries in **A** is called *System of N Representatives* (*SNR*) if they are distinct, that is, if there is at most one such entry in each row of **A**, there is at most one entry in each column of **U** part, and there are at most Y entries in vector **M**. An SNR in **A** is a *critical SNR* (*CSNR*) if each critical line in **A** contains one member of the SNR. We now develop the following algorithm.

Step 1. Construct an augmented matrix **A** for the given traffic U and M.
Step 2. Transform, if necessary, the given traffic matrix **A** into **A'** so as to be N-complete with respect to B. Let i =1.
Step 3. Find a CSNR $\{x_1, x_2, \cdots, x_N\}$ in **A'** by using the max-min matching algorithm [8].
Step 4. Construct a switching matrix $\mathbf{S}_i$ with N positive elements of value $b = \min\{B - L, x_1, x_2, \cdots, x_N\}$ instead of $\{x_j\}$, where L = max {row sum, column sum for **U'** part, $\left\lceil \sum_{i=1}^{N} m_i / Y \right\rceil$ for **M'** part }, for all the lines not covered by the CSNR. Let $B = B - b$.
Step 5. Let $\mathbf{A}' = \mathbf{A}' - \mathbf{S}_i$. Go to Step 2 unless **A'**=0 by letting $i = i + 1$. Otherwise, stop.

Property 2. The above algorithm guarantees an optimal solution with computational complexity $O(N^4)$.
Proof. The schedule length has no effect on the addition of dummy traffic at the critical line since the addition never affects the current schedule length B in Step 2, where the transformed quantity a'_{ij} does not compose a critical line. Step 3 always constructs a CSNR by using the max-min matching algorithm with computational complexity $O(N^2(N+1)^2)$ [8]. The augmented matrix **A'** has at least one CSNR in Step 3 since all the rows remained as critical lines in Steps 3 and 4 of the algorithm. Furthermore, the matrix **A'** with lower bound B can always be decomposed as sum of $\mathbf{S}_i$ and **A''** in Steps 4 and 5, where $\mathbf{S}_i$ is the switching matrix having b-valued N elements and **A''** has the lower bound B'', $B'' = B - b$. Therefore, the algorithm guarantees the optimal solution. The computational complexity of the algorithm is calculated as the sum of $O(N)$ in Step 2, $O(N^2(N+1)^2)$ in Step 3, and $O(2N+1)$ in Step 4. Therefore, the overall computational complexity is $O(N^4)$. This completes the proof. □

For a numerical example, let us consider a router with two multicast ports for an augmented matrix $\mathbf{A}=\begin{pmatrix} 6 & 0 & 0 & 3 \\ 0 & 2 & 5 & 15 \\ 0 & 6 & 4 & 5 \end{pmatrix}$. In Step 2, the lower bound is calculated as B =22 by Property 1. Therefore, the matrix **A** requires modification to be 3-complete such that $u'_{11} = u_{11}+13=19$, $u'_{31}=3$, $u'_{32}=10$ and the others are the same as those of **A**. In Step 3, a CSNR is selected as (u_{11}, u_{31}, m_2). Then $b = \min\{B - L, x_1, x_2, \cdots, x_N\} = \min\{22 - 9, 19, 10, 15\} = 10$ in Step 4. Therefore, we find **A'** as $\mathbf{A}'=\mathbf{A}'-\mathbf{S}_1=\begin{pmatrix} 19 & 0 & 0 & 3 \\ 0 & 2 & 5 & 15 \\ 3 & 10 & 4 & 5 \end{pmatrix}-\begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 \\ 0 & 10 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 9 & 0 & 0 & 3 \\ 0 & 2 & 5 & 5 \\ 3 & 0 & 4 & 5 \end{pmatrix}$ in Step 5.
Similarly, the remaining switching matrices are obtained as $S_2 = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 \end{pmatrix}$, $S_3 = \begin{pmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 3 & 0 \\ 3 & 0 & 0 & 0 \end{pmatrix}$, $S_4 = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$, $S_5 = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$, and $S_6 = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{pmatrix}$.
Therefore, an optimal time slot assignment is obtained as the switching matrix $\{\mathbf{S}_i\}$ with minimal schedule length 22, 22=10+3+3+ 2+2+2.

Notice that the optimal schedule does not depend on the sequence of the resulting switching matrix and the dummy traffic $u'_{11} - u_{11} = 13$, $u'_{31} - u_{31} = 3$, and $u'_{32} - u_{32} = 4$ of $\{\mathbf{S}_i\}$ needs to be deleted in service. We should note that the deletion method for the dummy traffic might have an effect on the number of required reconfigurations.

IV. CONCLUSIONS

An increasing amount of telecommunication traffic necessitates a high-speed router that can efficiently handle given data packets. This paper proposed an $N \times N$ hybrid router with an $N \times (N+Y)$ switch fabric, where each input port has $(N+1)$ VOQs for unicast and multicast traffic, where multicast traffic is serviced through one of the Y multicast buses. We also found an optimal time slot assignment schedule to minimize schedule length. For the combined traffic of unicast and multicast traffic, an optimal scheduling algorithm with polynomial complexity was developed based upon a derived lower bound.

We found that there are several multiple optima with respect to the schedule length; further analysis of the schedule with respect to secondary criteria, such as the required number of reconfigurations of the router, would be valuable.

REFERENCES

[1] Cisco GSR 12000 Technical Documents, "Fast Switched Backplane for a Gigabit Switched Router," http://www.cisco.com/warp/public/cc/pd/rt/12000/tech/fasts_wp.pdf.
[2] M.H. Guo and R.S. Chang, "A Simple Multicast ATM Switches Based on Broadcast Buses," *IEEE GLOBECOM'98*, vol. 4, 1998, pp.2369-2374.
[3] M.K. Shin, Y.J. Kim, K.S. Park, and S.G. Kim, "Explicit Multicast Extension(Xcast+)for Efficient Multicast Packet Delivery," *ETRI J.*, vol. 23, no. 4, Dec. 2001, pp.202-204.

[4] G. Bongiovanni, D. Coppersmith, and C.K. Wong, "An Optimum Time Slot Assignment Algorithm for an SS/TDMA System with Variable Number of Transponders," *IEEE Trans. on Comm.*, vol. COM-29, no. 5, 1981, pp.721-726.

[5] I.S. Gopal and C.K. Wong, "Minimizing the Number of Switchings in an SS/TDMA System," *IEEE Trans. on Comm.*, vol. COM-33, no. 6, 1985, pp. 497-501.

[6] G. Nong and M. Hamdi, "Burst-Based Scheduling Algorithms for Non-Blocking ATM Switches with Multiple Input Queues," *IEEE Comm. Lett.*, vol. 4, no. 6, 2000, pp.202-204.

[7] M. Ajmone Marsan, A. Bianco, P. Giaccone, E. Leonardi, and F. Neri, "Packet Scheduling in Input-Queued Cell-Based Switches," *IEEE INFOCOM2001*, 2001, pp.1085-1094.

[8] E.L. Lawler, *Combinatorial Optimization: Networks and Matroids*, Holt, Rinehart and Winston, 1976.