A Non-Canonical Linearly Constrained Constant Modulus Algorithm for a Blind Multiuser Detector

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We investigate an alternative blind adaptive multiuser detection scheme based on a non-canonical linearly constrained constant modulus (LCCM) criterion and prove that, under the constrained condition, the non-canonical linearly constrained constant modulus algorithm (LCCMA) can completely remove multiple-access interference. We further demonstrate that the non-canonical LCCM criterion function is strictly convex in the noise-free state, and that under the constrained condition, it is also strictly convex even where small noise is present. We present a simple method for selecting the constant as well as a stochastic gradient algorithm for implementing our scheme. Numerical simulation results verify the scheme's efficiency.

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I. INTRODUCTION

Blind multiuser detection, which requires prior knowledge of only the signature waveform and timing of the desired user, has recently received considerable attention [1]. The main motivation for employing the blind scheme is to avoid the requirement of a training sequence, and thus offer better spectrum efficiency.

The purpose of blind equalization techniques in multiuser detection, such as the constant modulus algorithm (CMA) [2], is to remove multiple access interference (MAI). One study showed that the constant modulus receiver could perform almost as well as the non-blind receiver design if undesirable local minima could be avoided [3]. Another work discussed how the minimum mean square error (MMSE) receiver approximates the local minima of the CMA cost function [4]. The constrained versions of the CMA should be considered for avoiding the undesired local minima.

S. Verdu et al. proposed a method that insures global convergence of blind equalizers [5]. Oda and Sato attempted to apply blind equalization to a multiuser detection [6], but their paper did not satisfactorily address the issue of discriminating between local minima. Later on, a linearly constrained constant modulus (LCCM) algorithm for MAI suppression was developed in [7]. Next, Z. Tang et al. presented a closed-form analysis of the linearly constrained CMA-based blind multiuser detector [8]. However, the analysis in [8] was incorrect. In fact, according to their analysis, the global convergence of the LCCM detector cannot be obtained. In this paper, we further analyze the global convergence of the linearly constrained CMA-based blind multiuser detector and correct the mistakes in [8]. The mistakes in [8] along with their correction are shown in the appendix.

This paper considers a non-canonical linearly constrained constant modulus algorithm (LCCMA) for blind multiuser detection. We prove that under the constrained condition, the non-canonical LCCMA can completely remove multiple-access interference, and that the non-canonical LCCM criterion function is strictly convex in the noise-free state. When small noise is present, the non-canonical LCCM criterion function is also strictly convex under the constrained condition. We suggest a simple method for selecting the constant. Moreover, our paper presents an adaptive algorithm for implementing our scheme using stochastic gradient methods.

This paper is organized as follows. Section II introduces the system model. Section III investigates the non-canonical LCCM criterion and proposes three important propositions. Section IV presents our adaptive algorithm along with its numerical simulation. Section V draws our conclusions.

In this paper, boldface lower and upper case letters denote column vectors and matrices, respectively. The superscript T stands for transpose, and $\mathbf{I}_n$ denotes the $n\times n$ identity matrix. Bold zero '$\mathbf{0}$' denotes a zero matrix or a zero vector with corresponding dimensions.

II. THE SYSTEM MODEL

We consider a synchronous DS-CDMA system with K users. The received signal is given by

$$r(t)=\sum_{k=1}^{K} A_k b_k s_k(t)+n(t), \quad t\in[0,T], \tag{1}$$

where A_k, b_k, and $s_k(t)$ represent the received amplitude, transmitted symbol, and normalized signaling waveform of the k-th user, respectively, and $n(t)$ is the additive white Gaussian noise (AWGN) with a zero mean and variance σ^2. For simplicity, we assume that b_k is a binary phase-shift keying (BPSK) signal, that is, $b_k\in\{\pm 1\}$ is either of two independent equiprobable random variables, and that $s_k(t)$ is real and supported only on the interval $[0,T]$, which is of the form

$$s_k(t)=\sum_{j=0}^{N-1} c_j^k \phi(t-jT_c), \qquad t\in[0,T], \tag{2}$$

where N is the processing gain, $(c_0^k, c_1^k,\cdots,c_{N-1}^k)$ is a signature sequence of ± 1's assigned to the k-th user, and ϕ is a normalized chip waveform of duration T_c, where $NT_c=T$.

At the receiver, chip-matched filtering followed by chip rate sampling yields an N-vector of chip-matched filter output samples within a symbol interval T,

$$\mathbf{r}=\sum_{k=1}^{K} A_k b_k \mathbf{s}_k+\mathbf{n}, \tag{3}$$

where $\mathbf{s}_k=(1/\sqrt{N})[c_0^k,\cdots,c_{N-1}^k]^T$ is the normalized signature waveform vector of the k-th user, and $\mathbf{n}$ is an AWGN vector with mean $\mathbf{0}$ and covariance matrix $\sigma^2\mathbf{I}_N$. In this paper, we assume that the signal vectors $\{\mathbf{s}_k\}_{k=1}^K$ are independent, and user 1 is the desired user.

III. LINEARLY CONSTRAINED CONSTANT MODULUS

1. Introduction of Cost Function

We consider an LCCMA receiver, which is given by the following constrained optimization problem:

$$\min_{\mathbf{w}^T\mathbf{s}_1=d} J(\mathbf{w})=E[(\mathbf{w}^T\mathbf{r})^2-1]^2, \tag{4}$$

where $d>0$ is a certain constant and $\mathbf{w}^T\mathbf{s}_1=d$ is called a non-canonical constraint. When $d=1$, the constraint is canonical as given in [9]. The canonical constant modulus is a special example of a non-canonical constant modulus. In the following, LCCM denotes a non-canonical LCCM.

We next analyze the ability of the LCCMA to remove MAI. We consider the situation in which the noise is negligible, i.e., $\sigma=0$. For convenience, we specify that $u_k=A_k(\mathbf{w}^T\mathbf{s}_k)$, and $\mathbf{u}=[u_1,\cdots,u_K]^T$. Since b_k denotes independent equiprobable ± 1 random variables, we have

$$E(\mathbf{w}^T\mathbf{r})^2=E[\sum_{k=1}^{K} A_k b_k(\mathbf{w}^T\mathbf{s}_k)]^2=E(\sum_{k=1}^{K} u_k b_k)^2=\sum_{k=1}^{K} u_k^2=\mathbf{u}^T\mathbf{u}, \tag{5}$$

and

$$\begin{aligned} E(\mathbf{w}^T\mathbf{r})^4 &= E[\sum_{k=1}^{K} A_k b_k(\mathbf{w}^T\mathbf{s}_k)]^4=E(\sum_{k=1}^{K} u_k b_k)^4 \\ &=\sum_{k=1}^{K} u_k^4+3\sum_{\substack{j,k=1\\ j\neq k}}^{K} u_j^2 u_k^2=3\sum_{j,k=1}^{K} u_j^2u_k^2-2\sum_{k=1}^{K} u_k^4 \\ &=3(\mathbf{u}^T\mathbf{u})^2-2\sum_{k=1}^{K} u_k^4. \end{aligned} \tag{6}$$

Moreover,

$$\begin{aligned} J(\mathbf{w}) &= E(\mathbf{w}^T\mathbf{r})^4-2E(\mathbf{w}^T\mathbf{r})^2+1 \\ &=3(\mathbf{u}^T\mathbf{u})^2-2\mathbf{u}^T\mathbf{u}-2\sum_{k=1}^{K} u_k^4+1. \end{aligned} \tag{7}$$

Since the linear constraint $\mathbf{w}^T\mathbf{s}_1 = d$ is equivalent to $u_1 = A_1 d$, optimization problem (4) reduces to

$$\min_{u_1 = A_1 d} \varphi(\mathbf{u}) = 3(\mathbf{u}^T\mathbf{u})^2 - 2\mathbf{u}^T\mathbf{u} - 2\sum_{k=1}^{K} u_k^4 + 1. \tag{8}$$

In the following, we consider the extrema of the function $\varphi(\mathbf{u})$ as $u_1 = A_1 d$. Define

$$\varphi(\bar{\mathbf{u}}) = \varphi(\mathbf{u}_d), \tag{9}$$

where $\bar{\mathbf{u}} = [u_2, \cdots, u_K]^T$ and $\mathbf{u}_d = [A_1 d, \bar{\mathbf{u}}^T]^T$. The first and second partial derivatives of function $\varphi(\bar{\mathbf{u}})$ are, respectively,

$$\begin{aligned}\frac{\partial \varphi(\bar{\mathbf{u}})}{\partial u_k} &= 4u_k[3(\mathbf{u}^T\mathbf{u}) - 1 - 2u_k^2] \\ &= 4u_k[(3A_1^2 d^2 - 1) + u_k^2 + 3\sum_{\substack{j=2 \\ j\neq k}}^{K} u_j^2]\end{aligned} \tag{10}$$

and

$$\frac{\partial^2 \varphi(\bar{\mathbf{u}})}{\partial u_k \partial u_l} = \begin{cases} 4[3(\mathbf{u}^T\mathbf{u}) - 1], & l = k \\ 24 u_k u_l, & l \neq k \end{cases}. \tag{11}$$

2. Analysis without Noise: Stationary Points and Propositions

When $3A_1^2 d^2 \geq 1$, it is obvious from (10) that $\varphi(\bar{\mathbf{u}})$ has the unique stationary point $\bar{\mathbf{u}}_0 = \mathbf{0}$. The Hessian matrix at the stationary point is

$$\mathbf{H}_0 = \left(\frac{\partial^2 \varphi(\bar{\mathbf{u}})}{\partial u_k \partial u_l}\right)_{2\leq k,l\leq K} = 4(3A_1^2 d^2 - 1)\mathbf{I}_{K-1}. \tag{12}$$

Therefore, $\bar{\mathrm{u}}_0 = 0$ is the minimum point since $\mathbf{H}_0 \geq \mathbf{0}$.

When $3A_1^2 d^2 < 1$, $\varphi(\bar{\mathbf{u}})$ has other nonzero stationary points besides $\bar{\mathbf{u}}_0 = \mathbf{0}$. Let $\bar{\mathbf{u}}_m$ be these nonzero stationary points in which there are m nonzero elements, which are indicated by u_{k_i}, where $i = 1, \cdots, m$, and $k_i \in \{2, 3, \cdots, K\}$. From (10), we have

$$2u_{k_i}^2 = 3(\mathbf{u}^T\mathbf{u}) - 1, \tag{13}$$

where $\mathbf{u}^T = [A_1 d, \bar{\mathbf{u}}_m^T]$. Therefore, the squares of the nonzero elements in the stationary point $\bar{\mathbf{u}}_m$ are equal, and

$$2u_{k_i}^2 = 3A_1^2 d^2 - 1 + 3m u_{k_i}^2. \tag{14}$$

Hence

$$u_{k_i}^2 = \frac{1 - 3A_1^2 d^2}{3m - 2}. \tag{15}$$

Thus, when $3A_1^2 d^2 < 1$, $\varphi(\bar{\mathbf{u}})$ has the stationary points $\bar{\mathbf{u}}_0 = \mathbf{0}$ and

$$\bar{\mathbf{u}}_m = [u_{m,2} \quad u_{m,3} \quad \cdots \quad u_{m,K}]^T, \tag{16}$$

where

$$u_{m,k} = \begin{cases} \pm\sqrt{\dfrac{1 - 3A_1^2 d^2}{3m - 2}}, & k = k_i, \quad i = 1, \cdots, m \\ 0, & otherwise \end{cases}, \tag{17}$$

in which $k_i \in \{2, 3, \cdots, K\}$ and m denote the subscript and number of nonzero elements, respectively, in the stationary point $\bar{\mathbf{u}}_m$. We next examine the property of the above stationary points.

Since the Hessian matrix at the stationary point $\bar{\mathbf{u}}_0$ is $\mathbf{H}_0 = 4(3A_1^2 d^2 - 1)\mathbf{I}_{K-1}$ and $3A_1^2 d^2 < 1$, we have $\mathbf{H}_0 < \mathbf{0}$. Therefore, $\bar{\mathbf{u}}_0$ is the maximum point. With the symmetrical property of $\varphi(\bar{\mathbf{u}})$, we know that the stationary points $\bar{\mathbf{u}}_m$ for given m possess the same property. So let's only consider the positive situation without a loss of generality.

According to (11), the Hessian matrix at the stationary point $\bar{\mathbf{u}}_m$ is

$$\begin{aligned}\mathbf{H}_m &= \left(\frac{\partial^2 \varphi(\bar{\mathbf{u}})}{\partial u_k \partial u_l}\right)_{2\leq k,l\leq K} \\ &= \frac{8(1 - 3A_1^2 d^2)}{3m - 2}\begin{bmatrix} \begin{matrix} 1 & 3 & \cdots & 3 \\ 3 & 1 & \cdots & 3 \\ \vdots & \vdots & \ddots & \vdots \\ 3 & 3 & \cdots & 1 \end{matrix} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{K-1-m} \end{bmatrix}.\end{aligned} \tag{18}$$

Therefore, $\mathbf{H}_m = 8(1 - 3A_1^2 d^2)\mathbf{I}_{K-1} > \mathbf{0}$ when $m = 1$, and $\mathbf{H}_m$ is indefinite when $m > 1$. Consequently, $\bar{\mathbf{u}}_1$ is the minimum point, and $\bar{\mathbf{u}}_m$ is the saddle point when $m > 1$, which is not an extreme point.

From the above analysis, we get the following:

i) When $3A_1^2 d^2 \geq 1$ and $\sigma = 0$, the LCCM cost function $\varphi(\bar{\mathbf{u}})$ has the global minima $\bar{\mathbf{u}}_0 = \mathbf{0}$, which satisfy a decorrelating condition:

$$\begin{aligned} &\mathbf{w}^T\mathbf{s}_1 = 1 \\ &\mathbf{w}^T\mathbf{s}_k = 0, \qquad k = 2, \cdots, K. \end{aligned} \tag{19}$$

ii) When $3A_1^2 d^2 < 1$ and $\sigma = 0$, $\varphi(\bar{\mathbf{u}})$ has a local maximum point $\bar{\mathbf{u}}_0 = \mathbf{0}$ and $2K - 2$ local minimum points

$\bar{\mathbf{u}}_1 = [0,\cdots,0,u_{k_1},0,\cdots,0]^T$, where $u_{k_1} = \pm\sqrt{1-3A_1^2d^2}$, $k_1 \in \{2,3,\cdots,K\}$. The LCCM cost function has the local minima, which satisfy

$$\begin{aligned} &\mathbf{w}^T\mathbf{s}_1 = 1 \\ &\mathbf{w}^T\mathbf{s}_{k_1} = \mu_{k_1} \\ &\mathbf{w}^T\mathbf{s}_k = 0, \quad k \neq k_1, k \in \{2,\cdots,K\}. \end{aligned} \tag{20}$$

Therefore, we have the following proposition.

Proposition 1: i) When $3A_1^2d^2 \geq 1$, the LCCMA receiver possesses the ability to remove MAI. ii) When $3A_1^2d^2 < 1$, the LCCMA receiver cannot completely remove MAI. In this case the performance is relatively poor.

When $3A_1^2d^2 \geq 1$ and $\sigma^2 = 0$, the LCCM cost function $\varphi(\bar{\mathbf{u}})$ has the unique stationary point $\bar{\mathbf{u}}_0 = \mathbf{0}$, which is the global minimum point. It also means the function $\varphi(\bar{\mathbf{u}})$ is convex. From this, we get the following proposition.

Proposition 2: When $3A_1^2d^2 \geq 1$ and $\sigma^2 = 0$, the function $J(\mathbf{w})$ with the constraint $\mathbf{w}^T\mathbf{s}_1 = d$ is convex.

In the following, we will consider the convexity of the function $J(\mathbf{w})$ with the constraint $\mathbf{w}^T\mathbf{s}_1 = d$ when noise is present.

3. Analysis with Noise: Proposition and Example

Denote $\mathbf{u} = \mathbf{A}\mathbf{S}^T\mathbf{w}$, where $\mathbf{S} = [\mathbf{s}_1,\cdots,\mathbf{s}_K]$ and $\mathbf{A} = diag(A_1,\cdots,A_K)$.

By analogue to the derivation given in [4] and [7], we can calculate

$$J(\mathbf{w}) = E(\mathbf{w}^T\mathbf{r})^4 - 2E(\mathbf{w}^T\mathbf{r})^2 + 1 = J_0(\mathbf{u}) + \sigma^2 J_1(\mathbf{w}), \tag{21}$$

where

$$J_0(\mathbf{u}) = 3(\mathbf{u}^T\mathbf{u})^2 - 2\mathbf{u}^T\mathbf{u} - 2\sum_{k=1}^{K} u_k^4 + 1, \tag{22}$$

$$J_1(\mathbf{w}) = (6\mathbf{u}^T\mathbf{u} - 2 + 3\sigma^2\mathbf{w}^T\mathbf{w})(\mathbf{w}^T\mathbf{w}). \tag{23}$$

From proposition 2, we know when $3A_1^2d^2 \geq 1$ and $\sigma^2 = 0$, $J(\mathbf{w})$ with the constraint $\mathbf{w}^T\mathbf{s}_1 = d$ is convex. Since $J(\mathbf{w})$ is continuous in terms of σ^2, we can assume that the extrema of the function in the noisy case can be deduced for a small σ^2 by a slight perturbation of the noise-free extrema. Hence, for small σ^2, it also follows that $J(\mathbf{w})$ with the constraint $\mathbf{w}^T\mathbf{s}_1 = d$ is convex when $3A_1^2d^2 \geq 1$.

Proposition 3: When $3A_1^2d^2 \geq 1$ and $\sigma^2 \neq 0$, for a small σ^2, the function $J(\mathbf{w})$ with the constraint $\mathbf{w}^T\mathbf{s}_1 = d$ is convex.

The solution to optimization problem (4) lies in the space spanned by the columns of the signature waveform matrix $\mathbf{S}$. Let $\mathbf{w} = \mathbf{S}\mathbf{v}$. We have $\mathbf{u} = \mathbf{A}\mathbf{S}^T\mathbf{w} = \mathbf{A}\mathbf{S}^T\mathbf{S}\mathbf{v}$ or $\mathbf{v} = (\mathbf{S}^T\mathbf{S})^{-1}\mathbf{A}^{-1}\mathbf{u}$. Thus, the solution to this optimization problem can be

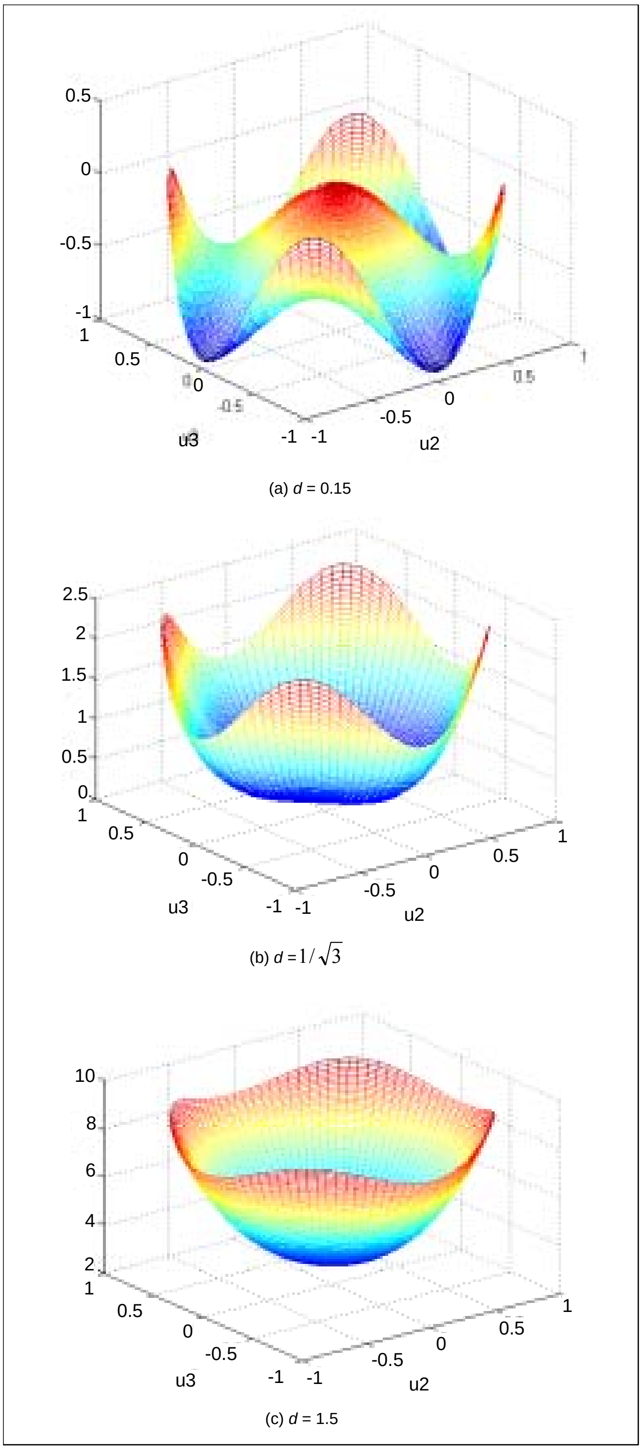

Fig. 1. Surfaces of the function $\varphi(\bar{\mathbf{u}})$.

expressed with $\mathbf{w}=\mathbf{S}(\mathbf{S}^T\mathbf{S})^{-1}\mathbf{A}^{-1}\mathbf{u}\overset{\text{def}}{=}\Phi\mathbf{u}$. According to (21), we define

$$\varphi(\overline{\mathbf{u}})=J_0(\mathbf{u}_d)+\sigma^2 J_1(\mathbf{w}_d), \tag{24}$$

where $\mathbf{u}_d=[A_1 d,\overline{\mathbf{u}}^T]^T$ and $\mathbf{w}_d=\Phi\mathbf{u}_d$.

An example demonstrates the exactness of the above proposition. The surfaces of function $\varphi(\overline{\mathbf{u}})$ are plotted in Figs. 1(a)-(c) for three different values, $d=0.15$, $1/\sqrt{3}$, and 1.5, where the number of users is $K=3$ and the processing gain is $N=7$, $A_1=A_2=A_3=1$, and $\sigma^2=0.01$.

Figure 1(a) shows the case of $3A_1^2d^2<1$, where the function $\varphi(\overline{\mathbf{u}})$ is a saddle. Figs. 1(b) and (c) show the case of $3A_1^2d^2\geq 1$, where the function $\varphi(\overline{\mathbf{u}})$ is strictly convex in a small noise state.

IV. ADAPTIVE ALGORITHM AND SIMULATION

Using a stochastic gradient algorithm, we derive an adaptive algorithm to solve optimization problem (4) in this section. Since $\mathbf{w}^T\mathbf{s}_1=d$ is equivalent to $\mathbf{w}=d\mathbf{s}_1+\mathbf{Bx}$, ($\mathbf{x}\in R^N$), where $\mathbf{B}=\mathbf{I}_N-\mathbf{s}_1\mathbf{s}_1^T$ whose columns span the orthogonal complement of vector $\mathbf{s}_1$. Therefore, constrained optimization problem (4) can be converted into an unconstrained form:

$$\min_{\mathbf{x}\in R^{N-1}}\eta(\mathbf{x})=J(d\mathbf{s}_1+\mathbf{Bx}). \tag{25}$$

Using stochastic gradient methods, we can solve optimization problem (25) and derive the following adaptive algorithm.

Algorithm:

$$\mathbf{x}_n=\mathbf{x}_{n-1}-\mu\mathbf{B}^T[(\mathbf{w}_{n-1}{}^T\mathbf{r}_n)^2-1](\mathbf{w}_{n-1}{}^T\mathbf{r}_n)\mathbf{r}_n, \tag{26}$$

$$\mathbf{w}_n=d\mathbf{s}_1+\mathbf{Bx}_n, \tag{27}$$

and

$$\mathbf{w}_0=d\mathbf{s}_1, \tag{28}$$

where μ is the step size and $d\geq 1/\sqrt{3}A_1$.

The output signal-to-interference ratio (SIR) is

$$SIR(n)=\frac{E^2\{\mathbf{w}_n^T\mathbf{r}_n\}}{Var\{\mathbf{w}_n^T\mathbf{r}_n\}}=\frac{A_1^2[\mathbf{w}_n^T\mathbf{s}_1]^2}{\sum_{k=2}^{K}A_k^2[\mathbf{w}_n^T\mathbf{s}_k]^2+\sigma^2\mathbf{w}_n^T\mathbf{w}_n}. \tag{29}$$

We consider a synchronous CDMA system with the processing gain $N=14$ and the number of users $K=5$. The spreading sequences $\{c_j^k, j=0,\cdots,N-1\}$ are randomly generated. The desired user is user 1. The received amplitude of user 1 is $A_1=0.1$. There are four 20 dB multiple-access interferers, i.e., $A_k^2/A_1^2=100$ $(k=2,\cdots,5)$. The signal-to-noise ratio is 10 dB and μ =0.007.

Figure 2 shows the output signal-to-interference ratio of the LCCMA receiver versus the number of iterations for three different constants d. Figure 2 reveals that the performance of the LCCMA receiver is very poor when $d=2$. Here $d=2$ is the case of $3A_1^2d^2<1$. On the other hand, we know from Fig. 2 that the LCCMA receiver exhibits a good performance when $d=8$ and 40. In this case, d satisfies the condition of $3A_1^2d^2\geq 1$.

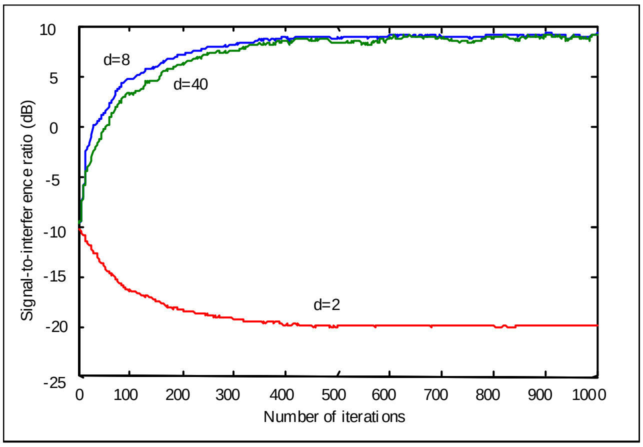

Fig. 2. Signal-to-interference ratio of the LCCMA receiver for d=2, 8 and 40, respectively.

Later on, we consider the effect from step size μ. From simulation experiments we know that the LCCMA with a step size that is too small has a slower convergence speed. However, if μ is too big, the algorithm converges to undesired points. It is better to select a smaller value in practical systems. Through a large number of simulation experiments, we concluded that the range of the step size is 0< μ <0.05. This conclusion corresponds to the result of [11].

Figure 3 shows the output signal-to-interference ratio of the LCCMA receiver ($d=8$) and the minimum output energy (MOE) receiver [9]. This figure shows that the modified CMA receiver outperforms the MOE receiver when the adaptive algorithms of the two receivers reach convergence.

In the above sections we defined the best choice of this constant d which is $3A_1^2d^2\geq 1$. In the above simulation we directly give the value of d according to A_1. However, we should point out that we don't know A_1 a priori because it depends on channel features. The simplest choice of d

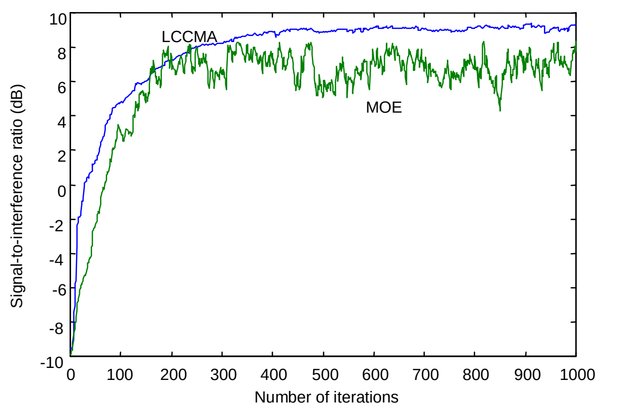

Fig. 3. Signal-to-interference ratio of the LCCMA receiver and the MOE receiver.

which satisfies the condition for the convexity of the cost function is

$$d = 1/\sqrt{3} sqrt\{E[|\mathbf{s}_1{}^T \mathbf{r}|^2]\}. \tag{30}$$

V. CONCLUSIONS

In this paper, we presented a non-canonical LCCM-based blind adaptive multiuser detection scheme. When $3A_1^2 d^2 \geq 1$, the LCCMA receiver possesses the ability to remove MAI; when $3A_1^2 d^2 < 1$, the LCCMA receiver cannot completely remove MAI. When $3A_1^2 d^2 \geq 1$ and $\sigma^2 = 0$, the function $J(\mathbf{w})$ with the constraint $\mathbf{w}^T \mathbf{s}_1 = d$ is convex; when $3A_1^2 d^2 \geq 1$ and $\sigma^2 \neq 0$, for a small σ^2, the function $J(\mathbf{w})$ with the constraint $\mathbf{w}^T \mathbf{s}_1 = d$ is also convex. Moreover, we proposed an adaptive algorithm for implementing the blind scheme using stochastic gradient methods and a simple method for selecting the constant d. Simulation examples demonstrate the efficiency of the propositions and the algorithm for our scheme. In addition, we identified the deficiencies in the analysis and conclusion of [8].

For simplicity our propositions are deduced on the basis of a BPSK signal and real signaling. In fact, the extension to the MPSK signal and complex signaling is straightforward.

APPENDIX

Mistakes in [8] and correction

Assume user 1 to be the desired user whose signature code $\mathbf{s}_1$ and power A_1^2 are known. The LCCM criterion is used for updating $\mathbf{w}$ following the optimization problem in [8]:

$$\mathbf{w}_{opt} = \min_{\mathbf{w}} J(\mathbf{w}) = \min_{\mathbf{w}} \{E[(y^2 - \alpha^2)^2]\}$$
$$\text{subject to} \quad \mathbf{w}^T \mathbf{s}_1 = 1, \alpha^2 \leq A_1^2. \tag{31}$$

Let $\mathbf{Q} = \mathbf{SAA}^T\mathbf{S}^T$ and $\mathbf{R} = \mathbf{Q} + \sigma^2 \mathbf{I}_N$, and the following equation is derived in [8]:

$$\begin{aligned} J(\mathbf{w}) &= (\mathbf{w}^T\mathbf{R}\mathbf{w} - \alpha^2) + 4\sum_{k=1}^{K} A_k^2 (\mathbf{w}^T\mathbf{s}_k)^2 \sigma^2 \mathbf{w}^T\mathbf{w} \\ &\quad + 2\sigma^4(\mathbf{w}^T\mathbf{w})^2 + 2\sum_{k=1}^{K}\sum_{\substack{l=1 \\ l\neq k}}^{K} A_k^2 (\mathbf{w}^T\mathbf{s}_k)^2 A_l^2 (\mathbf{w}^T\mathbf{s}_l)^2 \\ &\overset{def}{=} J_1(\mathbf{w}) + J_2(\mathbf{w}) + J_3(\mathbf{w}) + J_4(\mathbf{w}). \end{aligned} \tag{32}$$

In order to analyze the global convergence of the linearly constrained CMA-based blind multiuser detector, the authors in [8] calculated the Hessian matrix of $J(\mathbf{w})$. However, the calculation of the Hessian matrices of $J_2(\mathbf{w})$ and $J_4(\mathbf{w})$ are incorrect in [8]. In [8],

$$\begin{aligned} J_2''(\mathbf{w}) &= 4\sum_{k=1}^{K}\{2\sigma^2 A_k^2 [(\mathbf{w}^T\mathbf{s}_k)^2 \mathbf{I}_N + (\mathbf{w}^T\mathbf{w})\mathbf{s}_k\mathbf{s}_k^T \\ &\quad + 2(\mathbf{w}^T\mathbf{s}_k)\mathbf{s}_k\mathbf{w}^T]\} \\ &\overset{def}{=} 4\sum_{k=1}^{K} 2\sigma^2 A_k^2 \mathbf{D}_k, \end{aligned} \tag{33}$$

and

$$\begin{aligned} J_4''(\mathbf{w}) &= 4\sum_{k=1}^{K}\sum_{\substack{l=1 \\ l\neq k}}^{K} A_k^2 A_l^2 [(\mathbf{w}^T\mathbf{s}_k)^2(\mathbf{s}_l\mathbf{s}_l^T) + (\mathbf{w}^T\mathbf{s}_l)^2(\mathbf{s}_k\mathbf{s}_k^T) \\ &\quad + 2(\mathbf{w}^T\mathbf{s}_k)(\mathbf{w}^T\mathbf{s}_l)\mathbf{s}_k\mathbf{s}_l^T] \\ &\overset{def}{=} 4\sum_{k=1}^{K}\sum_{\substack{l=1 \\ l\neq k}}^{K} A_k^2 A_l^2 \mathbf{G}_{kl}. \end{aligned} \tag{34}$$

We can easily check items $\mathbf{D}_k$ and $\mathbf{G}_{kl}$ and see that they are incorrect in (33) and (34) by showing that $\mathbf{D}_k$ and $\mathbf{G}_{kl}$ are not positive definite. For simplicity, we consider two users, and $N = 2$, $\mathbf{s}_1 = [1 \quad 0]$, and $\mathbf{s}_2 = [0 \quad 1]$. Then we have

$$\mathbf{D}_k = \begin{bmatrix} 6w_1^2 + w_2^2 & 2w_1 w_2 \\ 2w_1 w_2 & w_1^2 \end{bmatrix}, \tag{35}$$

$$\mathbf{G}_{kl} = \begin{bmatrix} w_2^2 & 2w_1 w_2 \\ 2w_1 w_2 & w_1^2 \end{bmatrix}. \tag{36}$$

It is obvious from (35) and (36) that matrices $\mathbf{D}_k$ and $\mathbf{G}_{kl}$ are non-definite. Therefore, the proof that $J''(\mathbf{w}) > 0$ is incorrect in [8].

In fact, $\mathbf{D}_k$ and $\mathbf{G}_{kl}$ should be

$$\begin{aligned}\mathbf{D}_k &= (\mathbf{w}^T\mathbf{s}_k)^2\mathbf{I}_N + (\mathbf{w}^T\mathbf{w})\mathbf{s}_k\mathbf{s}_k^T + 2(\mathbf{w}^T\mathbf{s}_k)\mathbf{s}_k\mathbf{w}^T \\ &\quad + 2(\mathbf{w}^T\mathbf{s}_k)\mathbf{w}\mathbf{s}_k^T,\end{aligned} \tag{37}$$

$$\begin{aligned}\mathbf{G}_{kl} &= (\mathbf{w}^T\mathbf{s}_k)^2(\mathbf{s}_l\mathbf{s}_l^T) + (\mathbf{w}^T\mathbf{s}_l)^2(\mathbf{s}_k\mathbf{s}_k^T) \\ &\quad + 2(\mathbf{w}^T\mathbf{s}_k)(\mathbf{w}^T\mathbf{s}_l)\mathbf{s}_k\mathbf{s}_l^T + 2(\mathbf{w}^T\mathbf{s}_k)(\mathbf{w}^T\mathbf{s}_l)\mathbf{s}_l\mathbf{s}_k^T.\end{aligned} \tag{38}$$

Moreover, by using the transformation of $\mathbf{w} \Rightarrow \alpha\mathbf{w}, \alpha > 0$, we can see that the following optimization problem

$$\begin{aligned}&\min_{\mathbf{w}} J(\mathbf{w}) = \min_{\mathbf{w}}\{E[(y^2-1)^2]\} \\ &\text{subject to } \mathbf{w}^T\mathbf{s}_1 = 1/\alpha,\end{aligned} \tag{39}$$

and optimization problem (31) are equivalent.

Note that criterion (39) is the same as criterion (4) when $d = 1/\alpha$. Then according to the conclusions presented in section III, the constrained condition on the power A_1^2 in optimization problem (4) can be reduced to

$$3A_1^2 d^2 \geq 1. \tag{40}$$

From (40) and $d = 1/\alpha$, we can easily get the constrained condition on the power A_1^2 in optimization problems (31) and (39), which is

$$A_1^2 \geq \frac{1}{3}\alpha^2. \tag{41}$$

Under the constrained condition, the LCCM detector possesses the property of global convergence, which refutes the conclusion in [8] that the global convergence of the LCCM detector cannot be obtained.

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