Low-Complexity Symbol Timing Offset Estimation Schemes for OFDM Systems

So Ra Park, Young Ho Jung, and Soo In Lee

ABSTRACT— In this paper, we propose three symbol synchronization schemes for Orthogonal Frequency Division Multiplex (OFDM) systems. The cyclic extension preceding OFDM symbols is of decisive importance for these schemes. The first scheme uses the phase-differential coding of the received OFDM signal. The second and the third schemes use the length of the received OFDM signal. All three schemes make symbol synchronization possible, even though there is a frequency offset in the system. Simulation results show that these schemes can be used to synchronize an OFDM system over AWGN and multi-path fading channels.

I. INTRODUCTION

An Orthogonal Frequency Division Multiplex (OFDM) is a type of multi-carrier modulation. Recently, the OFDM technology has been applied to many digital transmission systems such as digital audio broadcasting (DAB), digital video broadcasting terrestrial TV (DVB-T), and wireless LANs [1], [2]. In the OFDM system, each sub-carrier is orthogonal to all other sub-carriers and modulated with fixed signal alphabets. A guard-interval is inserted between OFDM symbols to avoid inter-symbol interference (ISI) caused by multi-path distortion, which has shorter impulse responses than the cyclic prefix length. Symbol synchronization (S-sync) aligns the FFT window of the receiver with the useful data portion of the OFDM symbol. A false estimation leads to ISI, which may disturb the orthogonality of the system and causes essential degradation due to inter-channel interference (ICI) [3]. S-sync can be implemented in the time domain. These concepts have been widely discussed in [4]-[6]. Generally, S-sync is carried out above all other synchronization processes, so a reliable OFDM S-sync scheme is needed even though there are frequency offset, phase offset, sampling offset, and so on. When there are frequency offset and the other synchronization errors, some previous schemes have shown somewhat bad S-sync performance. Some of the previous schemes are difficult to implement because of multiplication operations, high computational complexity, a lot of bit allocation, and so on.

In this paper, we propose three OFDM S-sync schemes that are simple and reliable. Furthermore, they show better S-sync performance when effects that cause synchronization error are present. The first method uses differential coding of a quantized input signal phase, and the others use the length of the input signal. In order to get the length of the input signal, we propose two schemes that can take a simplified signal length.

II. OFDM SYSTEM

The transmitted OFDM signal $s_n(j)$ can be expressed as

$$s_n(j) = \sum_{n=-G}^{N-1} x_n(j) = \sum_{n=-G}^{-1}\sum_{k=0}^{N-1} X_k(j) e^{j2\pi k(N+n)/N} + \sum_{n=0}^{N-1}\sum_{k=0}^{N-1} X_k(j) e^{j2\pi kn/N} \tag{1}$$

where N is the IFFT size, $X_k(j)$ is the complex envelop of the k-th sub-carrier in j-th OFDM symbol and G is the guard-interval in the samples. The received OFDM symbol is

$$r_n(j) = \frac{1}{N}\sum_{k=0}^{N-1} X_k(j) H_k e^{j\{2\pi kn/N + \Phi_0 + 2\pi \Delta f T_{sym}\}} + n(jT_{sym} + nT_s) \tag{2}$$

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So Ra Park (phone: +82 42 860 5879, e-mail: parksora@etri.re.kr), Young Ho Jung (e-mail: yhcheong@etri.re.kr), and Soo In Lee (e-mail: silee@etri.re.kr) are with the Mobile Multimedia Broadcasting Research Team, ETRI, Daejeon, Korea.

where H_k is the transfer function of the channel at the *k*-th sub-carrier, Φ_0 is the phase offset, Δf is the frequency offset, and $n(t)$ is additive white Gaussian noise (AWGN). T_u, T_s and T_{sym} are the useful data interval, sampling period, and total OFDM symbol duration, respectively.

III. THREE OFDM S-SYNC SCHEMES

1. Differential Coding of Phase (Method 1)

In the following, we explain the process that simplifies the received signal $r(t)$ so that it is not affected by the frequency offset.

Using Table 1, we quantize the in-phase and quadrature components of $r(k)$ to form the integer sequence $p(k) = Q[r(k)]$, $k = 1, \ldots, 2N+G$ where $Q[.]$ denotes the integer quantizer. We generate the phase-differential coding data using (3). The sequence $d(k)$ denotes a differential coding of the quantized phase. The differential value is defined by counterclockwise rotation with 2 bits.

Table 1. Quantizer table.

Im[r(k)] \ Re[r(k)]	(+)	(–)
(+)	00	11
(–)	01	10

$$d(k) = \begin{cases} p(k) - p(k-1), & if \quad p(k) - p(k-1) \geq 0 \\ p(k) - p(k-1) + 4, & else \end{cases} \tag{3}$$

Figure 1 shows the basic idea of S-sync [6]. Assume that we observe $2N+G$ consecutive samples of $r(k)$, and these samples contain one complete OFDM symbol. The position of this symbol within the observed block of samples, however, is unknown. Define θ as the time index of the first sample of the *i*-th symbol, and the index sets $interv_1 = [\theta+N, \theta+N+G-1]$ and $interv_2 = [\theta, \theta+G-1]$ (Fig. 1).

The set $interv_1$ contains the indices of the data samples that are copied into the cyclic prefix, and the set $interv_2$ contains the indices of this prefix. A sample $d(k)$, $k=[interv_1]$ is correlated with $d(k-N)$, while all samples $d(k)$, $k \neq [interv_1]$ and $k \neq [interv_2]$, are independent. The probability for all $2N+G$ samples of $d(k)$ to be observed simultaneously, given a certain value of θ, can be separated into the marginal probabilities for the observed samples, except for those samples $d(k)$ within the correspondence region between $interv_1$ and $interv_2$, which are pair-wise correlated by using the look-up table in Table 2.

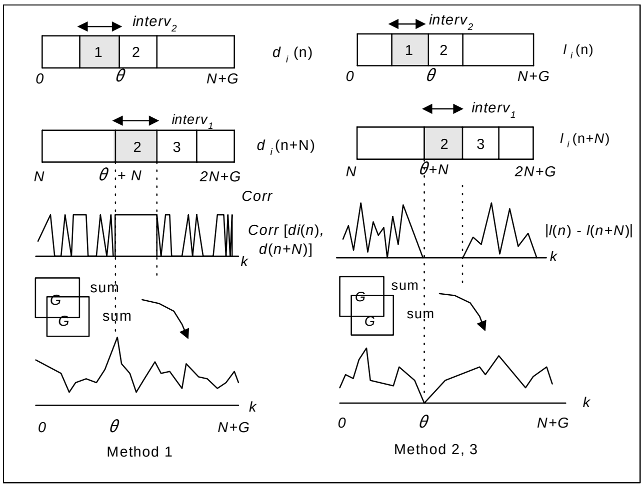

Fig. 1. Structure of the OFDM signal with cyclic extended symbols.

Table 2. Look-up table for correlation.

θ–N \ θ	0	1	2	3
0	1	0	–1	0
1	0	1	0	–1
2	–1	0	1	0
3	0	–1	0	1

Using (4), the estimator of θ given $d(k)$, $\hat{\theta}_c$, maximizes this function with respect to θ. $\hat{\theta}_a$ is the mean value of M estimated values. In the process of calculating the mean value, we exclude the minimum and maximum estimated mean values.

$$\hat{\theta}_c = \underset{\theta}{MAX}\left[\sum_{k=\theta}^{\theta+G-1} Corr\{d(n+k), d(n+k+N)\}\right], \qquad n = 0,1,\cdots,N+G-1 \tag{4}$$

2. Length of Received Signal (Method 2)

This method is based on the underlying property that the length of the input signal is not affected by the frequency offset. We used simple length information instead of the exact length of the input signal in this method. Eq. (5) shows the process of taking the brief length of the complex input signal, and (6) simplifies (5) to implement it easily. Eq. (7) shows a general S-sync method that is analogized in Fig. 1.

$$l(k) = |r(k)| = L + 0.4\,S$$

$$\text{where } L = |\mathrm{Re}(k)|,\ \ S = |\mathrm{Im}(k)|,\ \mathit{if}\ |\mathrm{Re}(k)| \geq |\mathrm{Im}(k)|, \qquad (5)$$

$$L = |\mathrm{Im}(k)|,\ \ S = |\mathrm{Re}(k)|,\ \mathit{else}$$

$$l(k) = |r(k)| = L + (0.5\,S) - (0.25\,S) + (0.125\,S) \qquad (6)$$

$$\hat{\theta}_c = \underset{\theta}{MIN}\left\{ \sum_{k=\theta}^{\theta+G-1} Sub[l(n+k), l(n+k+N)] \right\}, \qquad (7)$$

$$n = 0,1,\cdots,N+G-1.$$

Using (6), we calculate the length of $r(k)$ to form integer sequence $l(k) = L[r(k)]$, $k = 1, \ldots, 2N+G$ where $L[.]$ is the length-calculator. A sample $l(k)$, $k=[interv_1]$ is paired with $l(k-N)$, while all samples, $l(k)$, $k \neq [interv_1]$ and $k \neq [interv_2]$, are independent. The probability for all $2N+G$ samples of $l(k)$ to be observed simultaneously, given a certain value of θ, can be separated into the sufficient probabilities for the observed samples, except for those samples $l(k)$, within the correspondence region between $interv_1$ and $interv_2$, which are pair-wise subtracted. The estimator of θ given $l(k)$, $\hat{\theta}_c$, minimizes this function with respect to θ using (7). $\hat{\theta}_a$ is the mean value of M estimated values. Maximum and minimum values are excluded in calculating $\hat{\theta}_a$.

3. Length of Received Signal (Method 3)

The concept of Methods 3 and 2 are analogous. We used the absolute value of the complex input signal. This process once more reduces the complexity of the calculation of the simple length of the input signal. Eq. (8) shows the process of taking the brief length of the complex input signal. We calculate the length of $r(k)$ to form the integer sequence $l(k) = A[r(k)]$, $k = 1, \ldots, 2N+G$ where $A[.]$ denotes the length calculator using (8). The estimator of θ given $l(k)$, $\hat{\theta}_c$ minimizes this function with respect to θ using (7). $\hat{\theta}_a$ is the mean value of M estimated values. Maximum and minimum values are excluded in calculating $\hat{\theta}_a$.

$$l(k) = A[r(k)] = |\mathrm{Re}(k)| + |\mathrm{Im}(k)| \qquad (8)$$

IV. EXPERIMENT RESULTS

N = 512, G=32, T_u= 1.061 ms, T_{sym} =1.094 ms, and $X_k(j)$ is the complex 16-QAM symbol which is associated with the carrier k of the OFDM symbol j in (1) and (2). The number of frames used is 100,000 for simulation results of the methods in 10dB E_b/N_0. The performances of the estimators are presented for the AWGN channel and fading channels of multi-path profiles with nine rays. EIA (Electronic Industries Association) have elected to use the more conservative indirect signal path Rayleigh fading model in all analyses of FM performance [7]. These are urban slow (US), urban fast (UF), rural fast (RF), and terrain-obstructed fast (TO).

Figure 2 shows the cumulative probability of correct detection according to the values of $|\theta - \hat{\theta}_a|$ for the coarse acquisition, where the frequency offset is 0.15 of the OFDM sub-

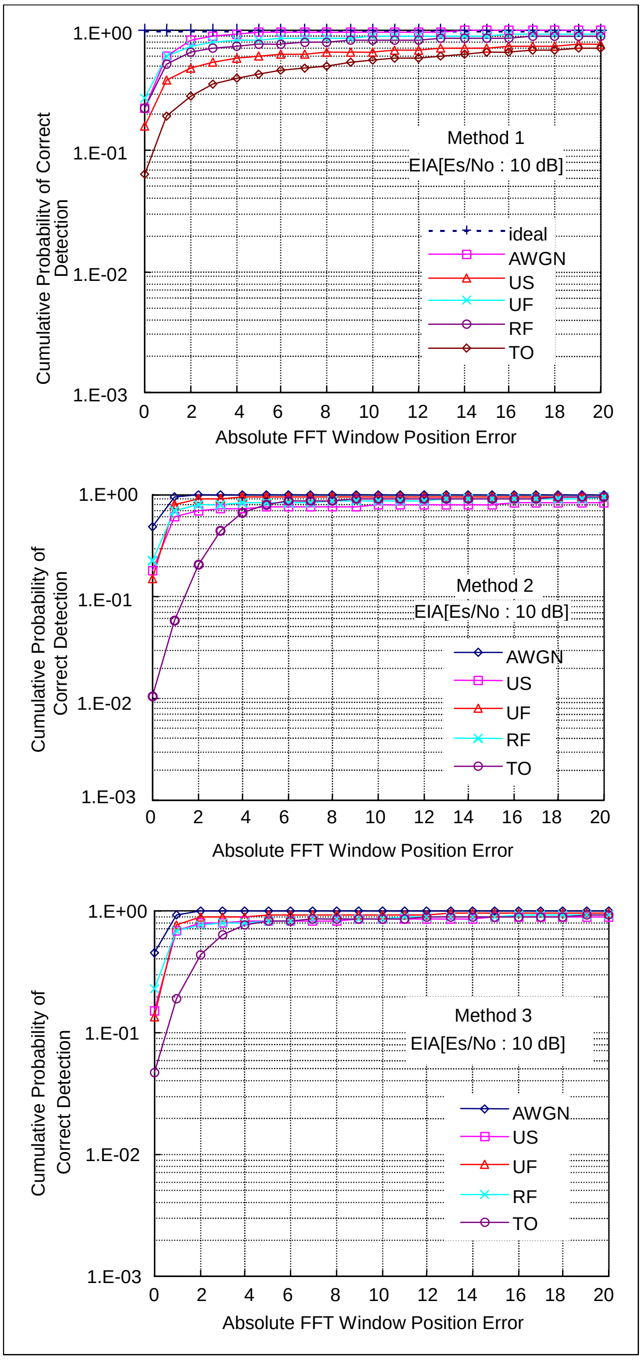

Fig. 2. Coarse S-sync Performance [10 dB, 0.15 frequency offset] of Method 1, Method 2 and Method 3.

carrier space. Simulation results indicate that the estimation error decreases to within approximately ± 8 samples. We could not show the probability curves of the previous methods that are closely related to frequency offset in Fig. 2 because of their large estimation error. The implementation of all of these proposed methods are easier than those of the previous methods (Method 3 is the easiest).

V. CONCLUSION

In this paper, we have shown how the cyclic extension of OFDM symbols can be used for OFDM S-sync at the receiver. The Method 1 applies phase-differential coding of the received OFDM signal to the S-sync. Methods 2 and 3 apply the length of the received OFDM signal to the S-sync. These schemes are applied to coarse S-sync, which are not affected by frequency offset. All three schemes are useful and good for OFDM S-sync even when there exist the frequency offset, common phase error, or other severe noise. The Method 3 demonstrated improved performance over Methods 1 and 2. It is also the easiest method to implement.

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