Bit Allocation for Interframe Video Coding Systems

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In this work, we present a novel approach to the bit allocation problem that aims to minimize overall distortion subject to a bit rate constraint. The optimal solution can be found by the Lagrangian method with dynamic programming. However, the optimal bit allocation for block-based interframe coding is practically unattainable because of the interframe dependency of macroblocks caused by motion compensation. To reduce the computational burden while maintaining a result close to the optimum, i.e., near optimum, we propose an alternative method. First, we present a partitioned form of the bit allocation problem: a "frame-level problem" and "one-frame macroblock-level problems." We show that the solution to this new form is also the solution to the conventional bit allocation problem. Further, we propose a bit allocation algorithm using a "two-phase optimization technique" with an interframe dependency model and a rate-distortion model.

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I. INTRODUCTION

Bit allocation is one of the fundamental issues in lossy image and video coding. Bit allocation is generally represented as minimizing overall distortions within a given bit budget. A quantizer decision for block-based DCT coding and frame bit assignment for image sequence coding are typical examples of the bit allocation problem.

Studies on bit allocation can be classified into two types: one is for real-time applications that emphasize fast computation [1]-[6], and the other is for finding the optimal result for bit allocation [7]-[12]. The methods under the first type, which are used in practical video storage and transmission applications, mainly rely on assumptions that are based on empirical results. Though they are widely used for their computational efficiency, the theoretical basis of the methods is not fully supported. The methods under the second type regard bit allocation as a constrained optimization problem assuming noncausality (i.e., the whole input image sequence is known), whereas methods of the first type do not. These optimization-based approaches require a much higher computational burden than the first type and are not appropriate for real-time applications mainly because of the noncausality and computational expense. Nevertheless, they are worth being tackled because we can acquire valuable information about the performance of a video coding system. In addition to this, we can also use the optimal result as a benchmark for assessing performance and as a basis for developing new bit allocation methods.

For optimization-based bit allocation, Lagrangian optimization with the Viterbi algorithm (we call this the "Lagrangian-Viterbi method" in this paper) is widely used [7]-[9]. However, in general block-based interframe video coding systems, such as

H.261 [13] and H.263 [14], Lagrangian optimization becomes formidably complicated as the number of input frames increases. The final result of the bit allocation is a set of quantizers for macroblocks. For interframe video coding, there are too many possible branches in the trellis construction for the Viterbi algorithm. This is mainly due to the inter-dependency among macroblocks caused by motion estimation (ME) and motion compensation (MC). Because of this huge quantity, it is practically impossible to apply the Lagrangian-Viterbi method to the bit allocation of interframe coding, so previous bit allocation studies were restricted to cases of simplified video coding, such as intraframe only coding or one-quantizer for all macroblocks in a frame.

In this paper, we propose a new bit allocation method which is optimization-based and applicable to interframe video coding systems. The proposed method can obtain a result of bit allocation that is near optimum with a comparatively lower computational burden than the Lagrangian-Viterbi method. We decompose the conventional form of the bit allocation problem into two levels: a "frame-level problem" and "one-frame macroblock-level problems." For the macroblock-level problems, we use the conventional Lagrangian-Viterbi method, and for the frame-level problem, which possesses the main obstacle in conventional methods (the inter-dependency caused by ME and MC), we propose an algorithm that utilizes a "two-phase optimization technique" [18] with an interframe dependency model and a rate-distortion model.

This paper is organized as follows: In section II, we present a partitioned form of the bit allocation problem. In section III, we provide some basic concepts on the two-phase optimization technique. In section IV, we propose a bit allocation algorithm, and simulation results are provided in section V. Finally, conclusions are given in section VI.

II. PARTITIONED FORM OF THE BIT ALLOCATION PROBLEM

For an image sequence of N frames and M macroblocks in one frame, the bit allocation problem is defined as a constrained optimization problem that minimizes overall distortion subject to a bit-budget constraint:

$$\text{minimize} \quad \sum_{i=1}^{N}\sum_{j=1}^{M} d(i,j) \tag{1a}$$

$$\text{subject to} \quad \sum_{i=1}^{N}\sum_{j=1}^{M} r(i,j) \le R_{budget}\ , \tag{1b}$$

where $r(i,j)$ and $d(i,j)$ are the rate and distortion for the j-th macroblock of the i-th frame, respectively, and R_{budget} is the total bit budget. Under this expression, it is necessary to handle $N \times M$ unknown variables simultaneously in finding a bit allocation result.

Let's consider this conventional expression of the bit allocation problem from a different perspective. First, we define $R_i \ (i=1,\cdots,N)$ which satisfies (2a) and (2b):

$$\sum_{j=1}^{M} r(i,j) \le R_i \tag{2a}$$

$$\sum_{i=1}^{N} R_i \le R_{budget}\ . \tag{2b}$$

Note that R_i is the number of assigned bits for the *i*-th frame when the equality of (2a) holds, and R_i is only feasible when $\{r(i,j): i=1,\cdots,N;\ j=1,\cdots,M\}$ satisfies the bit budget constraint (1b).

Next, we define $D_i \ (i=1,\cdots,N)$ as

$$D_i = \min\left\{\sum_{j=1}^{M} d(i,j)\right\} \ \text{subject to} \ \sum_{j=1}^{M} r(i,j) \le R_i\ , \tag{3}$$

where $\min\{\cdot\}$ is a function that returns the minimum value of an argument. Note that D_i is the minimum sum-of-distortion of the *i*-th frame when the sum of assigned bits for the *i*-th frame is equal to or less than R_i.

Using the two newly defined variables, we propose a new form of the bit allocation problem, and we will show that the solution of the new form is also the solution of the conventional bit allocation problem.

Partitioned form of the bit allocation problem:

For a given video sequence and bit budget R_{budget}, find $\{r(i,j): i=1,\cdots,N;\ j=1,\cdots,M\}$ such that

$$\textit{minimize} \ \sum_{i=1}^{N} D_i \quad \textit{subject to} \ \sum_{i=1}^{N} R_i \le R_{budget} \tag{4a}$$

$$\textit{where} \ \ D_i = \min\left\{\sum_{j=1}^{M} d(i,j) \ \middle|\ \sum_{j=1}^{M} r(i,j) \le R_i\right\}. \tag{4b}$$

Note that $\min\{\cdot \mid C\}$ is a function that returns a minimum value on a certain condition C.

To show (1) and (4) are the same, we start from considering the constraints of (4a) and (4b). Combining $\sum_{i=1}^{N} R_i \le R_{budget}$ of (4a) and $\sum_{j=1}^{M} r(i,j) \le R_i \ (i=1,\cdots,N)$ of (4b), we obtain

$$\sum_{i=1}^{N}\left(\sum_{j=1}^{M} r(i,j)\right) \le \sum_{i=1}^{N} R_i \le R_{budget}\,. \tag{5}$$

From (5) we can derive

$$\sum_{i=1}^{N}\sum_{j=1}^{M} r(i,j) \le R_{budget}, \tag{6}$$

which is identical to (1b). Therefore, the constraint parts of (4) and (1) are the same.

Next, let's consider the objective parts. Assume that D_i' $(i=1,\cdots,N)$ is the minimum value of (4b) for arbitrary R_i' $(i=1,\cdots,N)$, and let $\{d'(i,j): i=1,\cdots,N;\, j=1,\cdots,M\}$ and $\{r'(i,j): i=1,\cdots,N;\, j=1,\cdots,M\}$ be the distortion and rate of each macroblock corresponding to D_i' and R_i', respectively. Then, D_i' and R_i' satisfy

$$D_i' = \min\left\{\sum_{j=1}^{M} d(i,j) \,\middle|\, \sum_{j=1}^{M} r(i,j) \le R_i'\right\} = \sum_{j=1}^{M} d'(i,j), \tag{7a}$$

$$\sum_{j=1}^{M} r'(i,j) \le R_i'. \tag{7b}$$

Substituting $d'(i,j)$ and $r'(i,j)$ into the objective part of (4a), we obtain

$$\begin{aligned}\sum_{i=1}^{N} D_i\Big|_{r'(i,j)} &= \sum_{i=1}^{N}\left[\min\left\{\sum_{j=1}^{M} d(i,j)\right\}\right]_{r'(i,j)} \\ &= \sum_{i=1}^{N} D_i' = \sum_{i=1}^{N}\sum_{j=1}^{M} d'(i,j).\end{aligned} \tag{8}$$

From (8), we can see that the minimization of $\sum_{i=1}^{N} D_i$ equals the minimization of $\sum_{i=1}^{N}\sum_{j=1}^{M} d'(i,j)$, and (8) is also the same as (1a). Therefore, it is sure that (4) is an alternative form of the conventional bit allocation problem, and the solution of (4) is also the solution of (1).

In the above, we provided a partitioned form of the bit allocation problem and showed that the solution of the proposed form (4) is the same as the solution of the conventional bit allocation problem (1). While the conventional form is expressed at a macroblock-level, (4) is a partitioned representation: the frame-level allocation problem (4a) and the one-frame macroblock-level problem (4b). The benefit from considering the bit allocation problem as (4) is the reduced number of unknown variables. Handling bit allocation in the form of (1), it is necessary to handle $N \times M$ unknown variables at a time. However, regarding the problem in the form of (4), the number of unknowns becomes N for (4a) and M for (4b). This allows us to achieve a significant computational economy.

III. TWO-PHASE OPTIMIZATION METHOD

Before we elaborate on the proposed algorithm, in this section, we provide a brief introduction to the two-phase optimization method that is used in the proposed algorithm.

The constrained optimization problem [17] is defined as follows:

Minimize $f(\vec{x})$ *subject to constraints*

$$g_1(\vec{x}) \le 0, \cdots, g_r(\vec{x}) \le 0$$

$$h_1(\vec{x}) = 0, \cdots, h_m(\vec{x}) = 0\,, \tag{9}$$

where f and g_i are functions of $\Re^n$ and the h_j's are functions of $\Re^n$ for $m \le n$. To find the solution to the problem, Maa and Shanblatt [18] introduced an optimization method. The strength of their method was that it can resolve the infeasibility that is inherent in the conventional gradient method. Their approach consisted of two phases:

1) First Phase (for $0 \le t < t_1$):

$$(\dot{\vec{x}}) = -\nabla f(\vec{x}) - s\left[\sum_{i=1}^{r} g_i^+(\vec{x})\nabla g_i(\vec{x}) + \sum_{j=1}^{m} h_j(\vec{x})\nabla h_j(\vec{x})\right], \tag{10}$$

where t_1 is a predetermined switching time, s is a sufficiently large positive real number, and $g_i^+ = \max(0, g_i)$.

2) Second Phase (for $t \ge t_1$):

$$\begin{aligned}(\dot{\vec{x}}) = -\nabla f(\vec{x}) - \Bigg[&\sum_{i=1}^{r}\nabla g_i(\vec{x})\big(s g_i^+(\vec{x}) + \lambda_i\big) \\ &+ \sum_{j=1}^{m}\nabla h_j(\vec{x})\big(s h_j(\vec{x}) + \mu_j\big)\Bigg],\end{aligned} \tag{11}$$

where $\dot{\lambda}_i = \varepsilon \cdot s g_i^+$, $\dot{\mu}_j = \varepsilon \cdot s h_j$, and ε is a small positive constant.

According to the penalty function theorem [17], the solution of (10) is not equivalent to the minimum of $f(\vec{x})$ unless the penalty parameter s goes to infinity. Thus, the use of the second phase optimization is required for any finite value of s. The penalty function theorem is illustrated for an informative purpose as follows:

Penalty Function Theorem: *Let* $\{s_k\}_1^\infty$ *be a non-negative, strictly increasing sequence tending to infinity. Define the function* $L(s,\vec{x})$ *as*

$$L(s,\vec{x}) = f(\vec{x}) + \frac{s}{2}\left[\sum_{i=1}^{r}\left(g_i^+(\vec{x})\right)^2 + \sum_{j=1}^{m}\left(h_j(\vec{x})\right)^2\right]. \quad (12)$$

Let the minimizer of $L(s,\vec{x})$ *be* $\vec{x}_k$. *Then any limit point of the sequence* $\{\vec{x}_k\}_1^\infty$ *is an optimal solution of* (10). *Furthermore, if* $\vec{x}_k \to \bar{x}$ *and* $\bar{x}$ *is a regular point, then* $s_k g_i^+(\vec{x}_k) \to \lambda_i$ *and* $s_k h_j^+(\vec{x}_k) \to \mu_j$ *are the Lagrange multipliers associated with* g_i *and* h_j, *respectively.*

The system (12) is in equilibrium when $g_i^+(\vec{x}) = 0$, $h_j(\vec{x}) = 0$, $\lambda > 0$, and $\nabla f + \sum_i \nabla g_i \lambda_i + \sum_j \nabla h_j \mu_j = 0$. This satisfies the optimality conditions of the Kuhn-Tucker theorem [17], and an equilibrium point of the two-phase network is the precise global minimizer to a convex program.

IV. PROPOSED BIT ALLOCATION ALGORITHM

The conventional bit allocation problem (1) is an integer programming problem with a nonlinear cost function and nonlinear constraints. One approach that has been proven to be useful is the Lagrangian-Viterbi method [7]-[9]. However, as previously mentioned, the Viterbi algorithm is practically inapplicable to the bit allocation of interframe video coding systems, mainly because the number of required nodes for the Viterbi algorithm increases drastically as the number of input frames increases. In this section, we propose a novel method based on the partitioned form of the bit allocation problem (4).

1. One Frame Macroblock-Level Bit Allocation: (4b)

Once R_i is determined, (4b) finds the rate of each macroblock $\{r(i,j) : j = 1,\cdots,M\}$ in a frame, which minimizes D_i. Eq. (4b) can be rewritten as

$$\text{minimize } \sum_{j=1}^{M} d(i,j) \text{ subject to } \sum_{j=1}^{M} r(i,j) \le R_i. \quad (13)$$

This is also a bit allocation problem. For standardized video coding systems such as H.263, there are 31 possible quantizers for each macroblock. In applying the Lagrangian-Viterbi method, it is necessary to construct a trellis diagram whose stage number is the total number of macroblocks in a frame, and each stage should have 31 nodes. Fortunately, the computational capacity of modern computers is powerful enough to accommodate this complexity. Therefore, we also use the Lagrangian-Viterbi method in solving the one frame macroblock-level problem (4b).

2. Frame-Level Bit Allocation: (4a)

The frame-level bit allocation problem, (4a), is a constrained optimization problem when

$$\begin{gathered}
\vec{x} = [R_1, R_2, \cdots, R_N]^T \\
f(\vec{x}) = \frac{1}{N}\sum_{i=1}^{N} D_i \\
g_1(\vec{x}) = \frac{1}{N}\left(\sum_{i=1}^{N} R_i - R_{budget}\right) \\
g_2(\vec{x}) = R_1 - R_1^{upper},\ g_3(\vec{x}) = R_1^{lower} - R_1 \\
\vdots \\
g_{2r}(\vec{x}) = R_r - R_r^{upper},\ g_{2r+1}(\vec{x}) = R_r^{lower} - R_r \\
\vdots \\
g_{2N}(\vec{x}) = R_N - R_N^{upper},\ g_{2N+1}(\vec{x}) = R_N^{lower} - R_N, \quad (14)
\end{gathered}$$

where R_i^{lower} and R_i^{upper} are the lower and upper limits of R_i; for instance, R_i can't be a negative value and can't be larger than R_{budget}. Hence, we can use the two-phase optimization method, which was provided in section III, in finding optimal $\vec{x} = [R_1, R_2, \cdots, R_N]^T$.

In applying the two-phase optimization method, i.e., to search for a solution by (10) and (11), we need the gradient of $f(\vec{x})$ and $g(\vec{x})$. Based on the definition of $g(\vec{x})$, $\nabla g_i(\vec{x})$ becomes

$$\begin{aligned}
\nabla g_1(\vec{x}) &= 1/N\,[1,\cdots,1]^T \\
\nabla g_{2r}(\vec{x}) &= [0,\cdots,0,\ 1_{(r-\mathrm{th})},0,\cdots,0]^T \\
\nabla g_{2r+1}(\vec{x}) &= [0,\cdots,0,-1_{(r-\mathrm{th})},0,\cdots,0]^T\ (r = 1,\cdots,N).
\end{aligned} \quad (15)$$

Because D_i is a function of $R_1, R_2, \cdots, R_i$, $\nabla f(\vec{x})$ is determined as

$$\nabla f(\vec{x}) = \frac{1}{N}\begin{bmatrix} \sum_{i=1}^{N} \partial D_i / \partial R_1 \\ \sum_{i=1}^{N} \partial D_i / \partial R_2 \\ \sum_{i=1}^{N} \partial D_i / \partial R_3 \\ \vdots \\ \sum_{i=1}^{N} \partial D_i / \partial R_N \end{bmatrix} = \frac{1}{N}\begin{bmatrix} \frac{\partial D_1}{\partial R_1} + \frac{\partial D_2}{\partial R_1} + \cdots + \frac{\partial D_{N-1}}{\partial R_1} + \frac{\partial D_N}{\partial R_1} \\ \frac{\partial D_2}{\partial R_2} + \frac{\partial D_3}{\partial R_2} + \cdots + \frac{\partial D_N}{\partial R_2} \\ \frac{\partial D_3}{\partial R_3} + \cdots + \frac{\partial D_N}{\partial R_3} \\ \vdots \\ \frac{\partial D_N}{\partial R_N} \end{bmatrix}. \quad (16)$$

The gradient of $g(\vec{x})$ is a constant vector. However, $\nabla f(\vec{x})$ is not feasible mathematically because D_i is not differentiable by R_i (i.e., R_i is an integer value, so D_i is not a continuous function of R_i).

We defined D_i as the distortion of the *i*-th frame when the sum of assigned bits for the *i*-th frame is equal to or less than R_i. On that account, we use a distortion-rate model based on the information theory [19] without loss of generality. We assume that D_i has an exponential relation with R_i as

$$D_i = \hat{\sigma}_i{}^2 \cdot H_i(R_i) \tag{17a}$$

$$H_i(R_i) = a_i \exp(-b_i R_i), \tag{17b}$$

where a_i and b_i are arbitrary constants and $\hat{\sigma}_i^2$ is the mean pixel variance of a motion compensated error between the *i*-th original frame and the previous reconstructed frame. In addition to this, as a way of taking into account the interframe dependency caused by MC and ME in the relation of D_i and R_i, Cheng and et al. [20] proposed that $\hat{\sigma}_i^2$ can be represented as a linear combination of motion compensated errors as

$$\hat{\sigma}_i^2 = \sigma_i^2 + \alpha_i \cdot D_{i-1} + \text{error}, \tag{18a}$$

where σ_i^2 is the mean pixel variance of a motion compensated error between the current frame and the previous original frame, and α_i is a scaling factor. Introducing $\tilde{\sigma}^2 \left(= \sigma^2 + \text{error}\right)$, we can rewrite (17) as

$$D_i = (\tilde{\sigma}_i^2 + \alpha_i \cdot D_{i-1}) \cdot H_i(R_i). \tag{18b}$$

Based on (18b), we can express $\dfrac{\partial D_j}{\partial R_i}$ $(N \geq j \geq i \geq 1)$ as

$$\begin{aligned}
\frac{\partial D_i}{\partial R_i} &= \hat{\sigma}_i^2 \cdot \frac{\partial H_i(R_i)}{\partial R_i} \\
&= -\hat{\sigma}_i^2 \cdot a_i \cdot b_i \cdot \exp[-b_i R_i] \\
\frac{\partial D_{i+1}}{\partial R_i} &= \frac{\partial}{\partial R_i}\left[\hat{\sigma}_{i+1}^2 \cdot H_{i+1}(R_{i+1})\right] \\
&= \frac{\partial}{\partial R_i}\left[\left(\tilde{\sigma}_{i+1}^2 + \alpha_{i+1} \cdot D_i\right) \cdot H_{i+1}(R_{i+1})\right] \left(\because \hat{\sigma}_{i+1}^2 = \tilde{\sigma}_{i+1}^2 + \alpha_{i+1} \cdot D_i\right) \\
&= \frac{\partial}{\partial R_i}\left[\sigma_{i+1}^2 \cdot H_{i+1}(R_{i+1}) + \alpha_{i+1} \cdot H_{i+1}(R_{i+1}) \cdot D_i\right] \\
&= \alpha_{i+1} \cdot H_{i+1}(R_{i+1}) \cdot \frac{\partial D_i}{\partial R_i} \quad \left(\because \frac{\partial H_{i+1}(R_{i+1})}{\partial R_i} = 0\right) \\
&\vdots \\
\frac{\partial D_j}{\partial R_i} &= \alpha_j \cdot H_j(R_j) \cdot \frac{\partial D_{j-1}}{\partial R_i}, \text{ where } j > i.
\end{aligned} \tag{19}$$

Therefore, with suitable a_i, b_i and α_i $(i = 1, \cdots, N)$, which are unique for an input frame, we can determine $\nabla f(\vec{x})$.

3. Proposed Algorithm

We have explained the underlying ideas and the details of how we can use the two-phase optimization algorithm for our purpose. Now we present the proposed algorithm for the bit allocation of interframe video coding.

Step 1: Encode the first frame of the input sequence as I-frame (since we are only concerned about the bit allocation of P-frames, we don't present any coding strategy for the first frame). Determine the bit budget R_{budget} of the remaining P-frames.

Step 2: Initialization:

(a) Set $t \leftarrow 0$.

(b) Initialize the values of R_i $(i = 1, \cdots, N)$. The initial R_i $(i = 1, \cdots, N)$ must satisfy (2b) with the given bit budget R_{budget} and be a feasible value within the boundary of the minimum and maximum of R_i.

Step 3: From the first P-frame ($i = 1$) to the last frame ($i = N$), determine the parameters below:

(a) α_i and $\tilde{\sigma}_i^2$: For an arbitrary value of R_{i-1}, we can obtain the corresponding D_{i-1} and $\hat{\sigma}_i^2$ by applying the Lagrangian-Viterbi method. If we vary R_{i-1}, we can obtain a set of data D_{i-1} and $\hat{\sigma}_i^2$. Based on these data, α_i and $\tilde{\sigma}_i^2$ can be determined by applying a line-fitting method to (18a).

(b) R_i^{lower} and R_i^{upper}: These values are the possible minimum and maximum values of R_i. The values should be the boundary of a reasonable range where the modeling error of (18b) is less than a certain value.

(c) a_i and b_i: Varying R_i between R_i^{lower} and R_i^{upper}, we can obtain the pairs of data R_i and the corresponding D_i. With the pairs of data, a_i and b_i can be determined by applying a curve fitting method to (17a).

Step 4: Determine the gradient of $f(\vec{x}[t])$ based on (16) and (19).

Step 5: Update R_i $(i = 1, \cdots, N)$ by integrating (10).

Step 6: Set $t \leftarrow t+1$ and calculate $f(\vec{x}[t+1])$. If t exceeds a predetermined switching time t_1, or the value of $|f(\vec{x}[t]) - f(\vec{x}[t+1])|$ is smaller than a pre-determined threshold value T_{1st} for successive N_{1st} times, the procedure proceeds to the next step. Otherwise, go to step 4).

Step 7: Initialization for the second phase search: set $\lambda_i \leftarrow 0$, $i = 1, \cdots, N$.

Step 8: Set scaling factors[1] c_i $(i = 1, \cdots, N)$ as

[1] The scaling factors c_i are for updating λ_i in the next step. In applying the two-phase optimization algorithm to multivariable heavily constrained optimization problems, an optimization process is often dominated by one constraint function if a problem is ill-conditioned – i.e., a constraint function is of a different magnitude or changes more rapidly than the other constraint functions or the objective function. In order not to be dominated by a single constraint and to make the result less sensitive to the initial choice of parameters, scaling each constraint is generally used.

$\|\nabla f(\vec{x})\| = c_i \|\nabla g_i(\vec{x})\|$.

Step 9: Update λ_i $(i = 1, \cdots, N)$ as $\lambda_i[t+1] \leftarrow \lambda_i[t] + \varepsilon \cdot sg_i^+ \cdot c_i$.

Step 10: By integrating (12), update R_i $(i = 1, \cdots, N)$.

Step 11: Set $t \leftarrow t+1$ and examine the variation of $f(\vec{x})$ as we did in the first phase. If the variation is smaller than a predetermined threshold value T_{2nd} for successive N_{2nd} iterations, proceed to the next procedure; otherwise, go to step 8).

Step 12: Considering the result obtained by the above steps as a new initial value of R_i $(i = 1, \cdots, N)$, iterate the procedures from step 2) until $f(\vec{x})$ converges to within the user-defined limits.

V. SIMULATIONS

We claimed that the proposed algorithm could obtain results which are close to the optimum. If the models provided the exact relation between D_i and R_i, the proposed algorithm could guarantee the optimality. However, because there are certain errors in practical situations, there is a gap between the optimum and the obtained results. In this section, we first show by simulations how the proposed algorithm can find a solution close to the optimum. In addition, we provide bit allocation results obtained by applying the proposed algorithm to an interframe video coding.

There are several parameters in the algorithm. We set them in our simulations as $s = 200$, $\varepsilon = 0.1$, $dt = 0.0005$, $t_1 = 1000$, $T_{1st} = T_{2nd} = 10^{-5}$, and $N_{1st} = N_{2nd} = 100$. These parameters can be assigned other values, but they are typical values for the two-phase optimization. For the values of $\hat{\sigma}_i^2$ and σ_i^2, we use the mean squared error of the motion compensated error. To find α_i and $\tilde{\sigma}_i^2$ in the step 3-a), we use the Chi-square fitting [22] method upon the data, which are obtained by the same procedure as in [20]—i.e., apply all possible quantizers (e.g., 31 quantizers for H.263 [14]) to the (*i*-1)-th frame for obtaining the data pairs of D_{i-1} and $\hat{\sigma}_i^2$. The Chi-square fitting is also used for a_i and b_i in step 3-c). For the integration in steps 5) and 10), we use the 4th order Runge-Kutta method [22]. Lastly, for the iteration in step 12), we iterate all the steps until the mean PSNR of the reconstructed images converges to within 0.01 dB for five successive times.

In our first simulation, we compared the bit allocation results obtained by the Lagrangian-Viterbi method with the results by the proposed method under a condition where there was no interframe dependency. If there is no interframe dependency, the Lagrangian-Viterbi method can find the optimal result. The purpose of this simulation was to show how close to the optimum the results by our proposed method would be. The less the difference between two results, the more we can trust the proposed method. The simulation results are summarized in Tables 1 and 2. For the QCIF "Stefan" and "Mobile" sequence,

Table 1. The mean PSNR difference between the Lagrangian-Viterbi method and the proposed method: the case of "Stefan" sequence.

Bit budget [bpp]	Mean PSNR by the Lagragian-Viterbi method (psnr1) [dB]	Mean PSNR by the proposed method (psnr2) [dB]	PSNR difference of the two results (psnr1-psnr2) [dB]
0.70	26.339071	26.315758	0.023314
0.75	26.761526	26.751970	0.009556
0.80	27.165047	27.149363	0.015684
0.85	27.557985	27.557985	0.011982
0.90	27.942375	27.927332	0.015043
0.95	28.313541	28.292801	0.020741
1.00	28.674675	28.664642	0.010033
1.05	29.022474	29.009039	0.013435
1.10	29.362230	29.341034	0.021196
1.15	29.698000	29.676435	0.021564
1.20	30.024433	30.005772	0.018661
1.25	30.343206	30.334949	0.008257
1.30	30.657316	30.641047	0.016270
1.35	30.965660	30.946745	0.018915

Table 2. The mean PSNR difference between the Lagrangian-Viterbi method and the proposed method: the case of "Mobile" sequence.

Bit budget [bpp]	Mean PSNR by the Lagragian-Viterbi method (psnr1) [dB]	Mean PSNR by the proposed method (psnr2) [dB]	PSNR difference of the two results (psnr1-psnr2) [dB]
0.70	23.323662	23.320374	0.003288
0.75	23.686844	23.678068	0.008776
0.80	24.008427	23.997320	0.011106
0.85	24.304201	24.290415	0.013786
0.90	24.587582	24.574041	0.013540
0.95	24.860392	24.848860	0.011532
1.00	25.129765	25.119028	0.010736
1.05	25.391403	25.378090	0.013313
1.10	25.646585	25.638153	0.008432
1.15	25.898767	25.888924	0.009844
1.20	26.145823	26.135771	0.010052
1.25	26.384661	26.380175	0.004486
1.30	26.623035	26.610067	0.012968
1.35	26.864092	26.855268	0.008823

from frames 1 to 10, we obtained the mean PSNR of the reconstructed pictures assuming MPEG-1 Intra picture coding[2)]. We varied the bit budget from 0.7 bpp (bit-per-pel) to 1.35 bpp. The mean PSNR of the Lagrangian-Viterbi method was always bigger than the proposed method because it was the optimum result. The maximum differences between the two results were 0.021196 dB at the bit budget 1.10 bpp for the "Stefan" sequence and 0.013786 dB at the bit budget 0.85 bpp for the "Mobile" sequence. As these results show, the proposed method can find bit allocation results that are close (within 0.03 dB difference in our simulations) to the optimum results.

Next, we conducted a simulation to examine the fidelity of the two models, the exponential rate-distortion model (17) and the interframe motion dependency model (18b). Once $R_i \ (i = 1, \cdots, N)$ is determined, we can obtain D_i corresponding to R_i by the Lagrangian-Viterbi method. We call this D_i an "obtained" distortion with a given R_i. On the other hand, the value of D_i can be estimated using (18b)[3)]. We call this D_i a "calculated" distortion. If the models show the characteristic of R_i and D_i properly, the obtained value and the calculated value should be the same (or very close to each other at least). The purpose of the second simulation was to see how close the two values were. We assumed the case of H.263 [14] coding with the basic frame structure (i.e., I-P-P-P-…), and the transmission rate and frame rate were set to 64 kbps and 10 frame/s, respectively. The QCIF "Foreman" and "Coast Guard" sequences were used as the input sequence, and we let R_i be 6400 (= 64000/10) bits. Figure 1 shows the results of the comparison between the obtained PSNR (solid lines) and the calculated PSNR (dotted lines). As the figure shows, the two lines move similarly and they are close to each other. Therefore, we can certify that the models can effectively estimate the relation of R_i and D_i.

The above two simulations were mainly for demonstrating the appropriateness of the proposed method by experiments. In this final simulation, we provide bit allocation results by the proposed method in H.263 video coding with the same frame structure as above (i.e., I-P-P-P-…) and show how much improvement we can expect from the proposed method compared to conventional methods. In this simulation, we compared three results: the method in TMN6 [21], a method we call "TMN6+Lagrangian," and the proposed method.

[2)] The MPEG-1 Intra coding scheme can assign a different quantizer for each macroblock, and this allows the use of the bit budget in a fine unit. Any other Intra-picture coding scheme could be used for the simulation.

[3)] At the instance of caring D_i, D_{i-1} in (18b) is already a determined value. We can also determine the parameters $\tilde{\sigma}_i^2$, α_i as step 3-a), and the parameters a_i, b_i as step 3-c), respectively. Hence, using (18b) and with the determined parameters, we can calculate D_i for a given value of R_i.

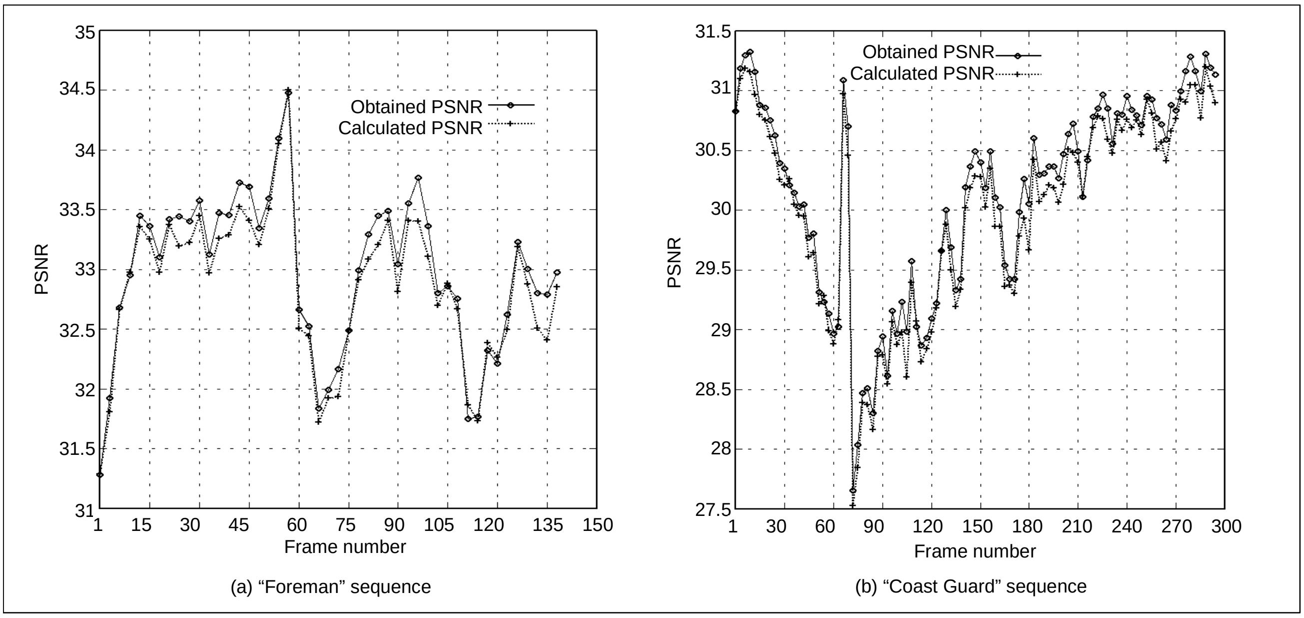


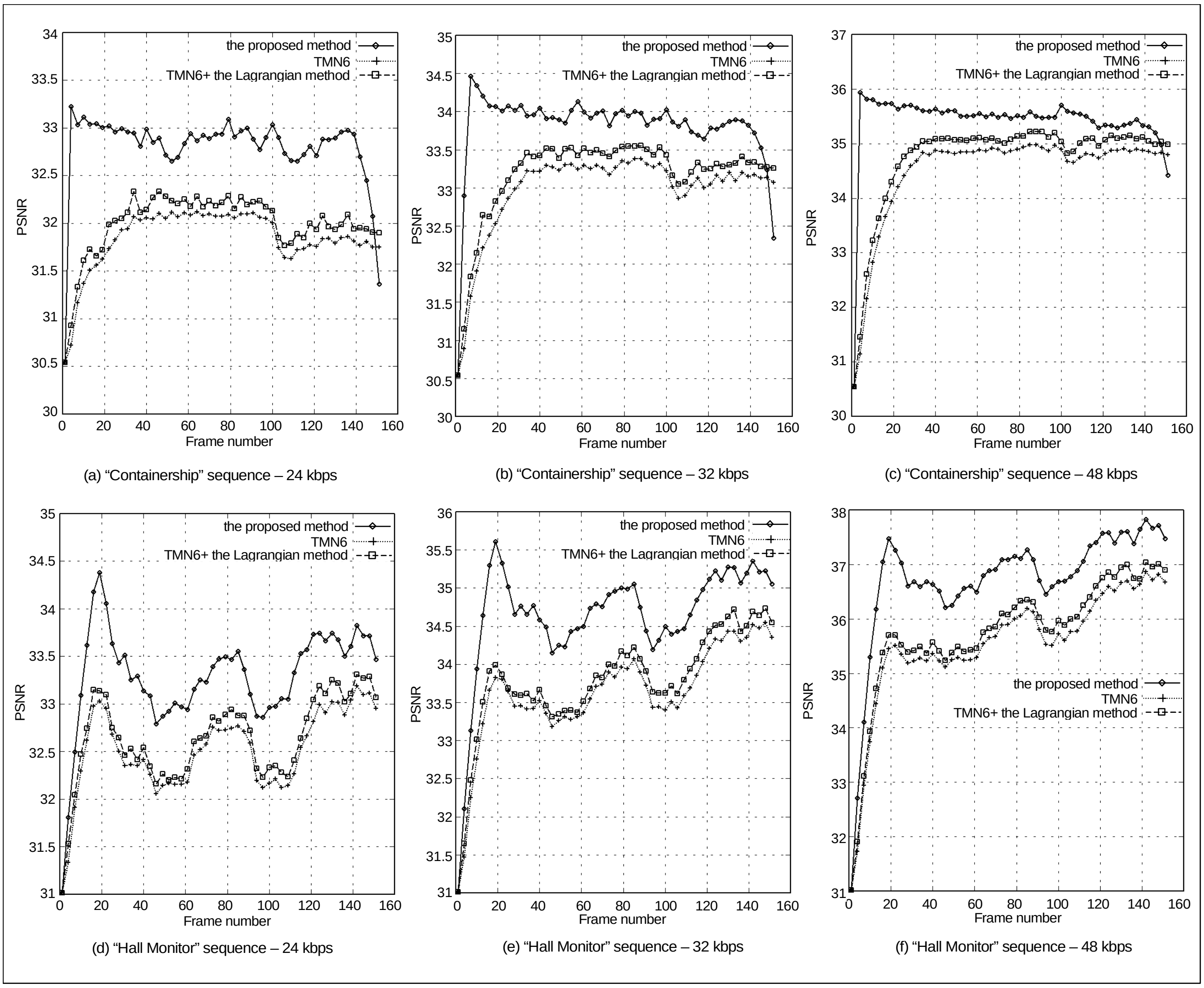
Fig. 1. Comparison between the obtained and calculated PSNR. Solid lines represent the obtained PSNR, and dotted lines represent the calculated PSNR using the models.

Table 3. Comparison of the mean PSNR values obtained by each bit allocation method.

Test image sequence	Bit allocation method	Mean PSNR under a transmission bit rate		
		24 kbps	32 kbps	48 kbps
Containership	TMN6	31.8647 [dB]	33.0184 [dB]	34.5804 [dB]
	TMN6+Lagrangian method	32.0223 [dB]	33.2280 [dB]	34.8285 [dB]
	Proposed method	32.8433 [dB]	33.8708 [dB]	35.5031 [dB]
Hall monitor	TMN6	32.5394 [dB]	33.6780 [dB]	35.6311 [dB]
	TMN6+Lagrangian method	32.6750 [dB]	33.8481 [dB]	35.8459 [dB]
	Proposed method	33.3325 [dB]	34.7059 [dB]	36.8164 [dB]

In the TMN6, a bit allocation method (rate control method) is suggested, and this method is commonly used as a benchmark. The method in TMN6 is composed of two stages: the first is for determining the amount of bits for a frame, and the second is for assigning quantizers for each macroblock in a frame. Once the amount of bits for a frame is determined, because the remaining steps can be regarded as the one-frame macroblock-level bit allocation, we can apply the Lagrangian-Viterbi method in assigning quantizers for macroblocks. Thus, the TMN6+Lagrangian method is composed of the method in TMN6 for determining the amount of frame bits and the Lagrangian-Viterbi method for quantizer assignment in each macroblock.

Using the QCIF "Containership" and "Hall monitor" sequences and varying the transmission bit rate to 24 kbps, 32 kbps, and 48 kbps for a frame rate of 10 frames/sec, we compared the PSNR values of the reconstructed images. Table 3 shows the results. Between the TMN6+Lagrangian method and the TMN6 method, we see slight improvement – about 0.2 dB. However, the proposed method produced about a 1 dB improvement of the mean PSNR only by efficient bit allocation. Figure 2 shows the comparison of the PSNR in a frame unit. The solid lines represent the proposed method, the dotted lines represent the TMN6 method, and the dashed lines represent the TMN6+Lagrangian method. The proposed method shows the best performance. We expect that the results, such as the assigned bits for each frame and the assigned quantizers for macroblocks, obtained from the proposed method will be



(a) "Containership" sequence – 24 kbps
(b) "Containership" sequence – 32 kbps
(c) "Containership" sequence – 48 kbps
(d) "Hall Monitor" sequence – 24 kbps
(e) "Hall Monitor" sequence – 32 kbps
(f) "Hall Monitor" sequence – 48 kbps

Fig. 2. The PSNR comparison of the results obtained by the proposed method (solid line), TMN6 method (dotted line), and the TMN6+Lagrangian method (dashed line).

valuable information for the development of better bit allocation methods.

VI. CONCLUSION

In this paper, we proposed a novel optimal bit allocation method. Though the Lagrangian-Viterbi method is not suitable for motion compensated interframe coding, the proposed method can find a bit allocation result that is close to the optimum. We presented a new form of the bit allocation problem that is partitioned into a frame-level problem and macroblock-level problems, and we proposed an algorithm using a two-phase optimization algorithm for the frame-level problem. Optimal bit allocation results can be used in various areas [23], [24] for the improvement of video coding performance. Therefore, we expect that the proposed method can be used effectively in many related studies.

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