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CORRELATION FUNCTIONS OF THE ABELL, APM, AND X-RAY CLUSTERS OF GALAXIES

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ABSTRACT

We have measured the correlation functions of the optically selected clusters of galaxies in the Abell and the APM catalogs, and of the X-ray clusters in the X-ray-Brightest Abell-type Clusters of galaxies (XBACs) catalog and the Brightest Clusters Sample (BCS). The same analysis method and the same method of characterizing the resulting correlation functions are applied to all observational samples. We have found that the amplitude of the correlation function of the APM clusters is much higher than what has been previously claimed, in particular for richer subsamples. The correlation length of the APM clusters with the richness $\mathcal{R} \geq 70$ (as defined by the APM team) is found to be $r_0 = 25.4^{+3.1}_{-3.0}\ h^{-1}$Mpc. The amplitude of correlation function is about 2.4 times higher than that of Croft et al. (1997). The correlation lengths of the Abell clusters with the richness class $RC \geq 0$ and 1 are measured to be $r_0 = 17.4^{+1.2}_{-1.1}$ and $21.0^{+2.8}_{-2.8}\ h^{-1}$Mpc, respectively, which is consistent with our results for the APM sample at the similar level of richness. The richness dependence of cluster correlations is found to be $r_0 = 0.40 d_c + 3.2$ where d_c is the mean intercluster separation. This is identical in slope with the Bahcall & West (1992)'s estimate, but is inconsistent with the weak dependence of Croft et al. (1997). The X-ray bright Abell clusters in the XBACs catalog and the X-ray selected clusters in the BCS catalog show strong clustering. The correlation length of the XBACs clusters with $L_x \geq 0.65 \times 10^{44}\ h^{-2}$erg s^{-1} is $30.3^{+8.2}_{-6.5}\ h^{-1}$Mpc, and that of the BCS clusters with $L_x \geq 0.70 \times 10^{44}\ h^{-2}$erg s^{-1} is $30.2^{+9.8}_{-8.9}\ h^{-1}$Mpc. The clustering strength of the X-ray clusters is much weaker than what is expected from the optical clusters.

Key words : cosmology: large-scale structure of the universe – galaxies: clusters: general

I. INTRODUCTION

Rich clusters of galaxies are used to study the large-scale structure of the universe on scales larger than about 10 h^{-1}Mpc (h = H$_0$ / 100 km s^{-1} Mpc^{-1}). To measure the clustering strength of clusters the spatial two-point correlation function (CF) of the Abell clusters (Abell 1958; Abell, Corwin, & Olowin 1989) has been estimated by many authors (Bahcall & Soneira 1983; Huchra et al. 1990; Postman, Huchra, & Geller 1992; Peacock & West 1992; Miller et al. 1999; see Bahcall 1988 for a review), and has been found to be consistent with the power law

$$\xi_{cc}(r) = \left(\frac{r}{r_0}\right)^{-\gamma} \quad (1)$$

with the correlation length $r_0 \simeq 20 \sim 26\ h^{-1}$Mpc and with the index $\gamma \simeq 2$. Recently new catalogs of clusters have been obtained by automated selection from the Edinburgh-Durham Southern Galaxy Catalog and the Automatic Plate Measuring (APM) Galaxy Survey (Lumsden et al. 1992; Dalton et al. 1997). Redshift data of these clusters (Collins et al. 1995; Dalton et al. 1994b) have been used to estimate the CF (Nichol et al. 1992; Dalton et al. 1992; Dalton et al. 1994a). The correlation lengths r_0 measured for these new cluster samples have been reported to be $14 \lesssim r_0 \lesssim 16\ h^{-1}$Mpc, which is much smaller than that of the Abell clusters. There are also more recent new catalogs such as Northern Sky Optical Cluster Survey based on Digitized Second Palomar Sky Survey (DPOSS) (Gal et al. 2000) and Sloan Digital Sky Survey Cut-and-Enhance (SDSS-CE) galaxy cluster catalog (Goto et al. 2002). They need, however, either more follow-up confirmations or spectroscopic observations before being able to be used to study the spatial clustering.

It has been argued that the counting radius r_c = 1.5 h^{-1}Mpc for the Abell clusters is so large that the catalog contains serious projection effects, which cause artificial line-of-sight correlations (Sutherland 1988; Efstathiou et al. 1992). It has also been pointed out that the intrinsically subjective nature of the Abell catalog can cause problems in homogeneity and statistical completeness (Nichol et al. 1992). However, Bahcall & West (1992) have claimed that the discrepancy between the Abell and the APM cluster CFs can be explained by the richness dependence of cluster correlation amplitudes $r_0 = 0.4 d_c$, where $d_c = n_c^{-1/3}$ is the mean intercluster separation and n_c is the mean space density of

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clusters. But Croft et al. (1997) have analyzed richness subsamples of the APM clusters and argued that there is only a weak dependence of correlation amplitudes on the cluster richness.

It is, therefore, necessary to understand why there have been disagreements in the cluster correlation length r_0 and on the relation between r_0 and d_c. In the previous studies individual observational samples have been analyzed by different authors who used different methods of analysis. In the current situation where the cluster clustering results from different studies do not agree, it is important to reanalyze and compare all preciously studied samples in an impartial way. In this paper we apply methods of calculating and characterizing the CFs to all cluster samples in a same way to remove any relative biases. It is hoped that in this objective and consistent way of analysis we could find the common characteristics of, and intrinsic differences in the spatial distributions of various cluster samples.

On the other hand, rich clusters of galaxies often have diffuse intracluster gas trapped in their potential wells. The thermal X-ray flux from the intracluster gas, which is heated to temperature of a few 10^7 K, is proportional to the square of the ion density, and thus is more confined to the center of the clusters than the projected galaxy distribution. Therefore, the X-ray selected clusters are expected to have negligible projection effects (Briel & Henry 1993; Ebeling et al. 1996). Ebeling et al. (1996) have cross-correlated the clusters in the ACO catalog (Abell et al. 1989) with the *ROSAT* X-ray sources (Trümper 1993) and created the X-ray-Brightest Abell-type Clusters of galaxies (XBACs) catalog. More recently, Ebeling et al. (1998) have published the Brightest Clusters Sample (BCS) which is a true X-ray selected sample from the *ROSAT* All-Sky Survey. It should be interesting to compare the clustering strengths of these X-ray cluster samples with those of optically selected samples to further extend our knowledge on the clustering properties of clusters of galaxies.

II. DATA

(a) The Abell Clusters

Postman et al. (1992) have published a complete redshift catalog of 351 Abell clusters with richness class $RC \geq 0$, declination $\delta \geq -27^{\circ}\!.5$, and tenth-ranked galaxy magnitude $m_{10} \leq 16.5$. We restrict our full sample to 233 clusters with $|b| \geq 40°$ to avoid serious incompleteness at low galactic latitude (Efstathiou et al. 1992). In this region with solid angle $\Omega = 3.48$ sr, clusters are nearly uniformly distributed over Galactic latitude. Two richness subsamples with $RC \geq 0$ and $RC \geq 1$ are adopted.

Assuming the Einstein-de Sitter universe, we transform redshifts into comoving distances by using the relation

$$d(z) = 6000(1 - 1/\sqrt{1+z})\ h^{-1}\text{Mpc}\ . \quad (2)$$

Since we use the same $d(z)$ relation to all cluster catalogs, the resulting CFs are not biased with respect to one another due to the choice of cosmology. Furthermore, most other authors have also adopted the standard Einstein-de Sitter model in their calculation.

To estimate the selection function of the survey in the radial direction, we first compute the absolute magnitude M_{10} corresponding to m_{10} of each cluster at a redshift z from the relation

$$m - M = 5\log[\,d(z)(1+z)\,] + 25 + K(z)\ . \quad (3)$$

The K-correction $K(z)$ is $1.122z$ for the red magnitudes of the northern Abell clusters and $4.14z - 0.44z^2$ for the visual magnitudes of the southern ACO clusters (Ebeling et al. 1996). Then the maximum distance $D_{max,i}$ to which each cluster could be observed for the survey magnitude limit $m_{10,lim} = 16.5$ can be calculated. And the selection function is given by

$$S(d) = \frac{3}{\Omega} \sum_{\substack{i;\\ D_{max,i} \geq d}} \frac{1}{D^3_{max,i}}\ . \quad (4)$$

Using the distribution of clusters smoothed in the redshift space as the radial selection function does not make much difference in our results. We limit the sample to the distance range from $d_{in} = 50\ h^{-1}$Mpc to $d_{out} = 500\ h^{-1}$Mpc, and the resulting sample includes 232 and 110 clusters for $RC \geq 0$ and $RC \geq 1$, respectively (see Table I). The region closer than $d_{in} = 50\ h^{-1}$Mpc is removed to improve the completeness of the Abell cluster sample and to avoid the possible local effects. Large variation of the locations of the inner and outer boundaries makes negligible effect on our results.

We estimate the comoving space density of clusters in each sample using the method described in Section 2.3 of Efstathiou et al. (1992). The mean space density of the Abell cluster sample with $RC \geq 0$ is $1.74 \times 10^{-5}\ h^3\text{Mpc}^{-3}$ while Postman et al. (1992)'s estimate is $1.2 \times 10^{-5}\ h^3\text{Mpc}^{-3}$ and Efstathiou et al. (1992)'s is $1.9 \times 10^{-5}\ h^3\text{Mpc}^{-3}$. The density of our $RC \geq 1$ subsample, $n_c = 6.79 \times 10^{-6}\ h^3\text{Mpc}^{-3}$, agrees well with Bahcall & Soneira (1983)'s $6 \times 10^{-6}\ h^3\text{Mpc}^{-3}$ and with Efstathiou et al. (1992)'s $7 \times 10^{-6}\ h^3\text{Mpc}^{-3}$.

Figure 1 shows the distributions of clusters on the sky with measured redshifts in each catalog used in our CF analysis.

(b) The APM Clusters

Dalton et al. (1997) have published a catalog of APM clusters with richness range $\mathcal{R} \geq 40$. This is the catalog that Dalton et al. (1994a) have used to measure the APM cluster CF, which is consistent with the

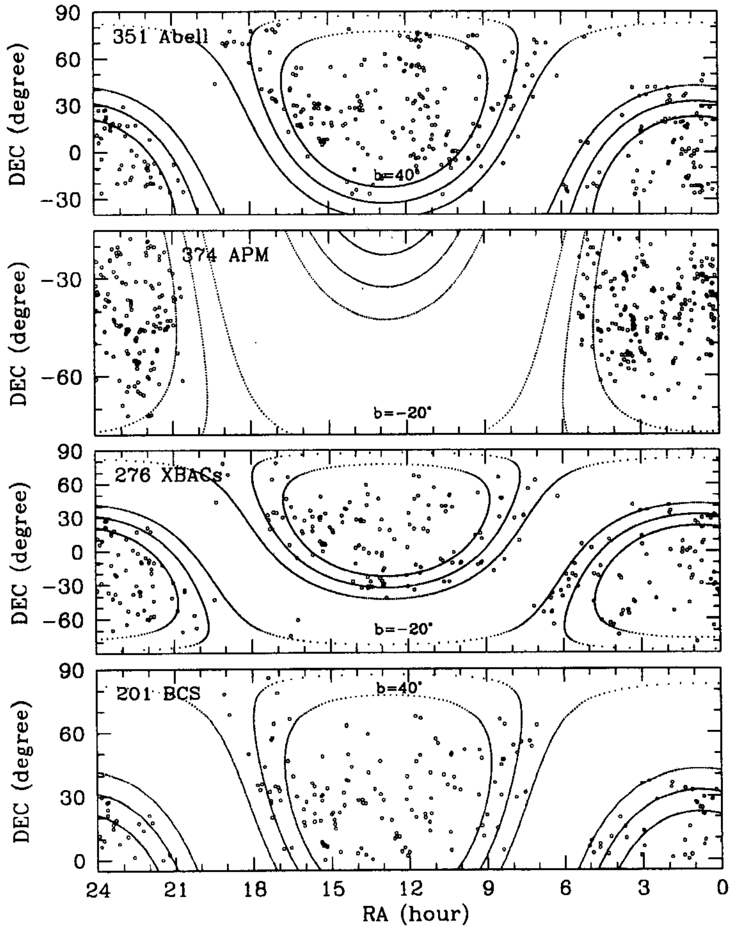

Fig. 1.— Distribution of clusters of galaxies on the sky with measured redshifts; 351 Abell clusters, 374 APM clusters, 276 XBACs clusters, and 201 BCS clusters. The dotted curves represent the galactic latitudes $b = \pm 40°$, $\pm 30°$, and $\pm 20°$.

results of Dalton et al. (1992) and Croft et al. (1997).

The survey region is shown by Maddox et al. (1990)* and contains 957 clusters with $m_x \le 19.4$ (m_x is the apparent magnitude of x-th bright member galaxy where $x = \mathcal{R}/2.1$; this is analogous to Abell cluster's m_{10}). The number of clusters in the APM sample shows a rather strong dependence on declination (Park & Lee 1998). In addition to the APM survey limits we have restricted our APM samples to the declination range of $-25° \ge \delta \ge -65°$ to reduce the potential effects of this declination dependence. But this change of declination boundaries turned out to make an insignificant difference in the CF.

Since the APM galaxy catalog is complete over the magnitude range $17.0 \le b_J \le 20.5$ (Maddox et al. 1990) and the cluster richness counting slice is $[m_x - 0.5, m_x + 1.0]$, the APM cluster catalog is complete over magnitude range $17.5 \le m_x \le 19.4$. This complete sample contains 927 clusters. But, in this sample, the number of clusters with measured redshifts is only 350. For clusters with $\mathcal{R} \ge 50$ and $17.5 \le m_x \le 19.2$, however, the completeness becomes 83% and increases for higher richness limits. We therefore make four magnitude-limited ($17.5 \le m_x \le 19.2$) richness subsamples with $\mathcal{R} \ge 50$, 60 (completeness 93%), 70 (96%), and 80 (100%)(see Table I).

To calculate the selection function of the APM clus-

*The exact APM survey field definition can be found in the Stromlo-APM redshift catalog (Loveday et al. 1996) served by the Astronomical Data Center (http://adc.gsfc.nasa.gov/).

Table 1. Richness subsamples of the Abell and the APM clusters

Richness	N_c	n_c (h^3Mpc^{-3})	r_0 (h^{-1}Mpc)	γ
		Abell		
$RC \geq 0$	232	1.74×10^{-5}	$17.4^{+1.2}_{-1.1}$	$1.89^{+0.15}_{-0.16}$
$RC \geq 1$	110	0.68×10^{-5}	$21.0^{+2.8}_{-2.8}$	$1.91^{+0.34}_{-0.36}$
		APM		
$\mathcal{R} \geq 50$	218	2.62×10^{-5}	$15.4^{+0.8}_{-0.8}$	$2.49^{+0.17}_{-0.16}$
$\mathcal{R} \geq 60$	139	1.66×10^{-5}	$22.3^{+1.6}_{-1.6}$	$1.96^{+0.16}_{-0.17}$
$\mathcal{R} \geq 70$	73	0.93×10^{-5}	$25.4^{+3.1}_{-3.0}$	$2.01^{+0.29}_{-0.30}$
$\mathcal{R} \geq 80$	42	0.57×10^{-5}	$26.8^{+7.0}_{-5.7}$	$2.03^{+0.58}_{-0.67}$

ters, the bright magnitude limit $m_{x,low} = 17.5$ should be taken into account as well as the faint limit $m_{x,upp} = 19.2$. The absolute magnitude M_x corresponding to m_x of each cluster is computed by equation (3), where $K(z) = 3z$ is adopted (Dalton et al. 1997). Then the selection function of the APM survey is given by

$$S(d) = \frac{3}{\Omega} \sum_{\substack{i; \\ D_{max,i} \geq d \\ D_{min,i} \leq d}} \frac{1}{D^3_{max,i} - D^3_{min,i}} \quad (5)$$

where $D_{max,i}$ is the maximum distance to which an i-th cluster can be observed by the faint end of the survey magnitude $m_{x,upp} = 19.2$, $D_{min,i}$ is the minimum distance to which the cluster could be included in the sample of $m_{x,low} = 17.5$, and $\Omega = 1.02$ sr is the solid angle of our survey region of the APM sample. Again, using the distribution of clusters smoothed in redshift space as the selection function makes little change in our results. The sample is limited to the distance interval from $d_{in} = 50\ h^{-1}$Mpc to $d_{out} = 500\ h^{-1}$Mpc.

The mean space densities of our APM $\mathcal{R} \geq 70$ and 80 subsamples agree well with those estimated by Croft et al. (1997). But the density of our $\mathcal{R} \geq 50$ sample is somewhat lower than $3.4 \times 10^{-5}\ h^3$Mpc^{-3} of Croft et al. (1997) because of lower completeness of clusters with published redshifts. We adopt Croft et al. (1997)'s value in the study of the richness dependence of correlation amplitudes in § IV.

(c) The XBACs

The XBACs catalog (Ebeling et al. 1996) is a complete, all-sky, X-ray flux-limited ($f_x \geq 5.0 \times 10^{-12}$ erg cm^{-2} s^{-1} in the 0.1 ~ 2.4 keV band) sample of 276 Abell clusters of galaxies compiled from the *ROSAT* All-Sky Survey. We constrain our sample to $|b| \geq 30°$ to reduce the effects of the galactic obscuration. After the LMC and the SMC regions are removed, the remaining survey area covers 6.27 sr of the sky. The completeness of the XBACs catalog is about 80% due to various sources of incompleteness which are very difficult

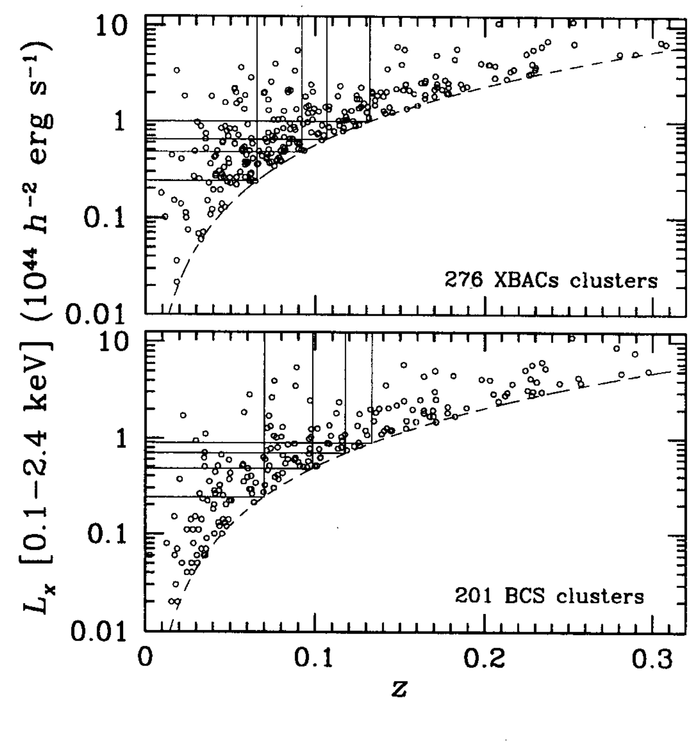

Fig. 2.— X-ray luminosity L_x versus redshift z distribution of 276 XBACs clusters and 201 BCS clusters. The dashed lines represent the L_x limit corresponding to the survey flux limit; 5.0×10^{-12} erg cm^{-2} s^{-1} for the XBACs and 4.4×10^{-12} erg cm^{-2} s^{-1} for the BCS, respectively. The solid lines show the L_x limits and distance limits of volume-limited subsamples.

to remove (Ebeling et al. 1996). We have further removed the X-ray sources contaminated by point sources or without measured redshifts from the XBACs catalog. The six double clusters in the XBACs catalog are considered as single systems which have undergone merging. The resulting final sample includes 186 clusters with the flux-limit of $f_x \geq 5.0 \times 10^{-12}$ erg cm^{-2} s^{-1} and with $|b| \geq 30°$ in the redshift range $0 < z < 0.2$. This is 91% of clusters in the catalog with the same flux, galactic latitude, and redshift limits.

Assuming the Einstein-de Sitter universe we calculate the X-ray luminosity L_x of clusters from the observed flux f_x and redshift z. Four volume-limited subsamples with $L_x \geq 0.24$, 0.48, 0.65 and 1.00 $\times 10^{44}\ h^{-2}$ erg s^{-1} and with the corresponding redshift limits of 0.066, 0.092, 0.107, and 0.132 are made (see Fig. 2). These samples contain 49, 67, 59, and 51 clusters, respectively. Our volume-limited XBACs subsamples contain many rich Abell clusters with $RC > 1$.

(d) The BCS

Recently, the *ROSAT* BCS catalog has been published (Ebeling et al. 1998). Unlike the XBACs catalog which is based on the Abell cluster catalog, the BCS clusters are selected purely by their X-ray properties. The BCS contains 201 X-ray brightest clusters of galaxies with measured redshifts in the northern hemisphere ($\delta \geq 0°$) and at $|b| \geq 20°$, and is 90 % complete at the X-ray flux limit of 4.4×10^{-12} erg cm^{-2} s^{-1} in the $0.1 \sim 2.4$ keV band. Six sources, identified as three double clusters in the Abell catalog, are considered as six independent X-ray clusters since the sample should be based only on the X-ray information. Adding these clusters and excluding three clusters with fluxes lower than the flux limit, we have 201 X-ray clusters.

We have used the BCS to make four volume-limited subsamples with $L_x \geq 0.24$, 0.48, 0.70, and 0.90 $\times 10^{44}$ h^{-2}erg s^{-1} and with the redshift limits of 0.070, 0.099, 0.118, and 0.134 (see Fig. 2). These subsamples contain 33, 49, 40, and 34 clusters, respectively. Our high L_x-limited subsamples of the XBACs and the BCS clusters contain very rare objects (see Table (c)).

III. CORRELATION FUNCTIONS

The correlation function is an estimate of excess clustering over the random Poisson distribution. We estimate the CFs using the conventional estimator DD/DR. We have also used the Hamilton (1993)'s estimator

$$\xi_{cc}(r) = \frac{\mathrm{DD} \cdot \mathrm{RR}}{\mathrm{DR}^2} \frac{4N_c N_r}{(N_c - 1)(N_r - 1)} - 1, \quad (6)$$

which is less affected by uncertainties in the selection function for $\xi_{cc} < 1$ compared to the DD/DR-method. We find that the differences between two estimators are insignificant in our analysis. Here DD is the number of pairs at separation r in the sample with N_c clusters, RR is the number of pairs in a random sample with N_r random points, occupying the same survey volume and with the same selection function as the real cluster sample, and DR is the cluster-random pair count. We compute the uncertainties of ξ_{cc} using the formula $\delta\xi_{cc} = (1 + \xi_{cc})/\sqrt{\mathrm{DD}}$. Table (a) lists the CF data we have calculated.

(a) Optically Selected Clusters

Least-square fits of a power law equation (1) to the best estimates of CF are presented in Table I for the richness subsamples of the Abell and the APM clusters. We determine the 68.3% confidence limits of each parameter individually assuming that the distribution for the uncertainties of ξ_{cc} is Gaussian and different ξ_{cc} bins are independent of each other, which is a popular, but strictly speaking, imprecise assumption.

Figure 3a shows the CFs of the Abell clusters with $RC \geq 0$ (circle) and $RC \geq 1$ (square). The dashed line is Postman et al. (1992)'s power law fit with $r_0 = 20.6\ h^{-1}$Mpc and $\gamma = 1.86$ estimated for the Abell $RC \geq 0$ sample. The CFs in Figure 3b are those of the APM clusters with $\mathcal{R} \geq 50$ (filled circle), $\mathcal{R} \geq 60$ (triangle), $\mathcal{R} \geq 70$ (square), and $\mathcal{R} \geq 80$ (open circle). The dotted line is a power law with $r_0 = 14.3\ h^{-1}$Mpc and $\gamma = 2.05$ ($\mathcal{R} \geq 50$) and the dashed line is with $r_0 = 16.6\ h^{-1}$Mpc and $\gamma = 2.10$ ($\mathcal{R} \geq 70$) both determined by Croft et al. (1997). It can be noted that the amplitude of CF steadily increases as the richness increases.

Dalton et al. (1992) have compared the richness of the ACO clusters with that of the APM clusters (hereafter APM92) and found that the mean difference in richness is $\langle R_{\mathrm{ACO}} - \mathcal{R}_{\mathrm{APM92}} \rangle = 10.1$. And Dalton et al. (1994a) have compared a new version of the APM cluster catalog (which is the catalog we use) with their old version catalog, and found a relation $\mathcal{R}_{\mathrm{APM}} = 1.2\mathcal{R}_{\mathrm{APM92}} + 24.8$ with a scatter of 16 in $\mathcal{R}_{\mathrm{APM}}$. We then get a rough relation $\mathcal{R}_{\mathrm{APM}} = 1.2R_{\mathrm{ACO}} + 12.7$. Therefore, the counts of the Abell clusters with $R = 30$ ($RC = 0$) and 50 ($RC = 1$) correspond to the richnesses of the APM clusters with roughly $\mathcal{R} = 49$ and 73, respectively. In Table I we find that the correlation lengths of the APM clusters are generally similar to those of the Abell clusters at similar levels of richness. Contrary to Croft et al. (1997), the APM subsamples with higher richness limits show clustering as strong as or rather stronger than the Abell cluster samples. It should be noted that there are large uncertainties both in the richness relation and in the richness itself (the uncertainties of R_{ACO} and $\mathcal{R}_{\mathrm{APM}}$ is about 18 and 5, respectively; Dalton et al. 1997), and that the Abell $RC \geq 0$ sample is an incomplete sample with fewer poorer clusters.

Park & Lee (1998) have calculated the CFs of the APM clusters using various CF calculation methods and sample corrections. They have used different selection functions, varied sample boundaries both in angle

Table 2. Volume-limited subsamples of the XBACs and the BCS clusters

L_X ($10^{44}\ h^{-2}$erg s^{-1})	N_c	n_c (h^3Mpc^{-3})	r_0 (h^{-1}Mpc)	γ
		XBACs		
$L_x \geq 0.24$	49	3.53×10^{-6}	$25.7^{+3.7}_{-3.8}$	$2.55^{+0.51}_{-0.43}$
$L_x \geq 0.48$	67	1.84×10^{-6}	$25.2^{+4.1}_{-4.3}$	$2.48^{+0.55}_{-0.54}$
$L_x \geq 0.65$	59	1.07×10^{-6}	$30.3^{+8.2}_{-6.5}$	$2.21^{+0.72}_{-1.27}$
$L_x \geq 1.00$	51	0.53×10^{-6}	$45.8^{+26.4}_{-12.5}$	$1.84^{+0.75}_{-0.99}$
		BCS		
$L_x \geq 0.24$	33	3.02×10^{-6}	$33.0^{+6.2}_{-5.9}$	$1.82^{+0.49}_{-0.50}$
$L_x \geq 0.48$	49	1.70×10^{-6}	$24.9^{+5.4}_{-5.6}$	$2.68^{+0.91}_{-0.77}$
$L_x \geq 0.70$	40	0.84×10^{-6}	$30.2^{+9.8}_{-8.9}$	$2.22^{+0.94}_{-1.26}$
$L_x \geq 0.90$	34	0.51×10^{-6}	$36.3^{+21.8}_{-13.8}$	$2.05^{+1.24}_{-1.76}$

Table 3. Two-point correlation functions of clusters of galaxies

Abell				APM							
$RC \geq 0$		$RC \geq 1$		$\mathcal{R} \geq 50$		$\mathcal{R} \geq 60$		$\mathcal{R} \geq 70$		$\mathcal{R} \geq 80$	
r[a]	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$
3.6	19 ± 7	5.3	12 ± 8	3.6	42 ± 10	3.9	28 ± 12	5.3	21 ± 12	6.3	15 ± 14
5.3	10 ± 3	9.5	4.7 ± 2.2	5.3	12 ± 4	5.8	12 ± 5	9.5	7.5 ± 2.9	12.5	7.1 ± 3.1
7.7	3.8 ± 1.5	17.0	1.8 ± 0.6	7.7	4.8 ± 1.2	8.8	6.5 ± 1.7	17.0	2.4 ± 0.8	24.7	0.83 ± 0.59
11.1	2.3 ± 0.7	30.5	0.42 ± 0.20	11.1	3.0 ± 0.6	13.2	3.2 ± 0.8	30.5	0.66 ± 0.24	48.8	$0.64 \pm 0\ 50$
16.1	1.5 ± 0.3	...	...	16.1	0.77 ± 0.26	19.9	1.2 ± 0.3	54.8	0.19 ± 0.28	...	...
23.4	0.61 ± 0.16	...	...	23.4	0.44 ± 0.13	30.0	0.61 ± 0.15	...	...	...	...
33.9	0.23 ± 0.08	...	...	33.9	0.12 ± 0.07	45.2	0.13 ± 0.16	...	...	...	...
...	...	...	...	49.2	0.050 ± 0.064	...	...	...	...	...	...

XBACs						BCS					
$L_x \geq 0.24$[b]		$L_x \geq 0.48$		$L_x \geq 1.00$		$L_x \geq 0.24$		$L_x \geq 0.48$		$L_x \geq 0.90$	
r	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$	r	$\xi_{cc}(r)$
6.3	46 ± 19	5.3	57 ± 32	8.1	37 ± 27	5.3	79 ± 41	6.3	52 ± 29	6.3	63 ± 70
12.5	4.5 ± 2.4	9.5	15 ± 7	18.4	4.0 ± 3.0	9.5	6.8 ± 6.0	12.5	5.1 ± 3.4	12.5	8.4 ± 9.3
24.7	1.5 ± 0.6	17.0	1.3 ± 1.2	41.8	1.3 ± 0.6	17.0	2.6 ± 1.8	24.7	0.87 ± 0.73	24.7	1.5 ± 1.8
48.8	0.19 ± 0.16	30.5	1.0 ± 0.5	...	...	30.5	1.3 ± 0.6	48.8	0.32 ± 0.24	48.8	0.77 ± 0.59
...	...	54.8	0.20 ± 0.17	...	...	54.8	0.44 ± 0.23	...	...	...	...

[a] Separation in units of h^{-1}Mpc.
[b] X-ray luminosity in units of $10^{44}\ h^{-2}$erg s^{-1}.

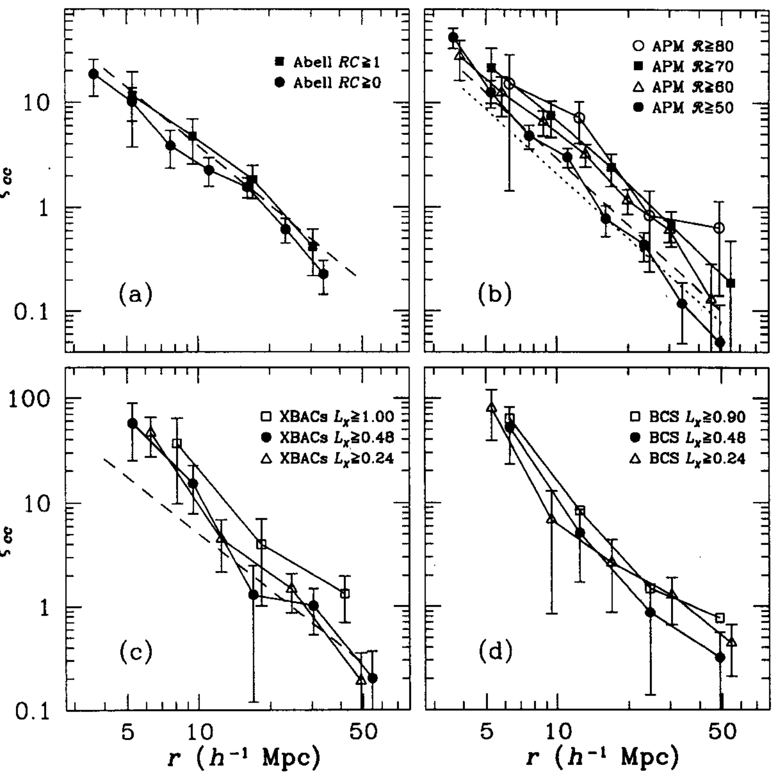

Fig. 3.— Correlation functions of clusters of galaxies. (*a*) Magnitude-limited samples of the Abell clusters with $RC \geq 0$ and 1. The dashed line is $\xi(r) = (r/20.6\ h^{-1}\mathrm{Mpc})^{-1.86}$ of Postman et al. (1992) for $RC \geq 0$ Abell clusters. (*b*) Magnitude-limited samples of the APM clusters with $\mathcal{R} \geq 50$, 60, 70, and 80. The dotted line is $\xi(r) = (r/14.3\ h^{-1}\mathrm{Mpc})^{-2.05}$ of $\mathcal{R} \geq 50$ sample of Croft et al. (1997) and the dashed line is $\xi(r) = (r/16.6\ h^{-1}\mathrm{Mpc})^{-2.10}$ of their $\mathcal{R} \geq 70$ sample. (*c*) Volume-limited samples of the XBACs clusters with $L_\mathrm{x} \geq 0.24$, 0.48, and $1.00 \times 10^{44}\ h^{-2}\mathrm{erg\ s}^{-1}$. The dashed line is $\xi(r) = (r/24.6\ h^{-1}\mathrm{Mpc})^{-1.80}$ of Abadi et al. (1998) for clusters with $L_\mathrm{x} \geq 0.54 \times 10^{44}\ h^{-2}\mathrm{erg\ s}^{-1}$. (*d*) Volume-limited samples of the BCS clusters with $L_\mathrm{x} \geq 0.24$, 0.48, and $0.90 \times 10^{44}\ h^{-2}\mathrm{erg\ s}^{-1}$.

and depth, and applied galactic latitude or declination dependence corrections. Even with these variations they have consistently obtained CFs with amplitude higher than that reported by Dalton et al. (1994a) or by Croft et al. (1997), and could not find the sources for discrepancy. Dalton (1998) has pointed out that our redshift sample contains the clusters with redshifts drawn from the Las Campanas Redshift survey or from the Edinburgh-Durham-Milano cluster redshift survey which cover only parts of the APM Galaxy Survey region, and that these clusters could produce an inhomogeneity in the distribution of clusters on the sky. Our statistical samples, however, contain very few of those clusters, and we have found no effect on the CF by them. For example, our magnitude-limited sample with $\mathcal{R} \geq 70$ does not contain any of the clusters in question.

(b) X-ray Clusters

Figure 3c shows the CFs of the XBACs samples with $L_\mathrm{x} \geq 0.24$ (triangle), 0.48 (circle), and $1.00 \times 10^{44}\ h^{-2}$ erg s^{-1} (square). The dashed line represents Abadi et al. (1998)'s fit $\xi(r) = (r/24.6\ h^{-1}\mathrm{Mpc})^{-1.80}$ for the XBACs clusters with $L_\mathrm{x} \geq 0.54 \times 10^{44}\ h^{-2}\mathrm{erg\ s}^{-1}$. The correlation lengths reported by Abadi et al. (1998) agree well with our results, except for the highest L_x subsample with $L_\mathrm{x} \geq 1.00 \times 10^{44}\ h^{-2}\mathrm{erg\ s}^{-1}$. It should be considered, however, that the uncertainties in the fitting parameters are very large both for their and our highest L_x subsamples. The CFs of the BCS clusters are shown in Figure 3d.

In Table (c) we list the parameters of CFs of the XBACs and the BCS clusters. Our X-ray cluster subsamples have very low number density and consist of the rarest clusters which might be very massive. As expected, the correlation length is very large and tends to increase as the luminosity limit increases. Borgani, Plionis, & Kolokotronis (1999) calculated the CFs from the XBACs and their resulting r_0 from the sample with the same flux-limit as ours, $f_x \geq 5.0 \times 10^{-12}$ erg cm^{-2} s^{-1}, is 26.0 h^{-1}Mpc, which agrees excellently with our result of the lowest L_x subsample; $r_0 = 25.7\ h^{-1}$Mpc.

IV. RICHNESS DEPENDENCE

We compare the amplitudes of CFs of the Abell and the APM clusters having the same level of richness. Richness level of a cluster sample is often represented by the mean separation $d_c = n_c^{-1/3}$ rather than richness itself (Szalay & Schramm 1985; Bahcall 1988; Bahcall & West 1992; Croft et al. 1997) because the definition on richness is different for different catalogs.

Our results on the richness dependence of the cluster CF are shown in Figure 4. Filled circles are the relations measured from the Abell sample, and filled squares are from the APM sample. We find that the clustering strength of the APM clusters is consistent with that of the Abell clusters, contrary to what Croft et al. (1997) have claimed. On the other hand, our results agree well with the previous studies on the Abell clusters (see Table 1 of Nichol et al. 1994 and other references presented in Fig. 4), but disagree with those of the APM team and Nichol et al. (1992). The solid line in Figure 4 is a linear fit to the r_0 versus d_c distribution found from our optical cluster samples. The relation we found is $r_0 = 0.40 d_c + 3.2$. The long and short dashed lines are the relations of Bahcall & West (1992) and Croft et al. (1997), respectively. Our relation is very similar to Bahcall & West (1992)'s law except for a constant offset. However, our result strongly disagrees with the slope found by Croft et al. (1997).

The r_0 versus d_c relations of the XBACs and the BCS clusters are consistent with each other, but are much lower than the relation for optical clusters. The slope of the relation is uncertain, but tends to increase as $d_c > 80\ h^{-1}$Mpc. The correlation lengths calculated by Borgani et al. (1999) from the XBACs and by Collins et al. (2000) from the ROSAT-ESO Flux-Limited X-ray (REFLEX) galaxy cluster survey (Böhringer et al. 2001) are consistent with our results and their distributions show rather weak dependence with L_x like ours, or look like even randomly distributed. As discussed in the following section, we think the X-ray luminosity of rich clusters is not a very good measure of richness or mass. It should be also noted that the uncertainties in r_0 are rather large.

V. DISCUSSION

We have measured the cluster-cluster correlation function by using four catalogs of optical and X-ray clusters. We have applied the same analysis method and the same method of characterizing the resulting CFs to all observational samples. Our CF for the Abell cluster sample is consistent with the previous works by Batuski & Burns (1985), Ling, Frenk, & Barrow (1986), Retzlaff et al. (1998), and Miller et al. (1999). The correlation lengths reported by Postman et al. (1992) and Bahcall & Soneira (1983) are slightly larger than ours. Those estimated by Huchra et al. (1990) and Peacock & West (1990) are very close to ours for the Abell clusters with $RC \geq 1$, but are somewhat larger than ours in the case of $RC \geq 0$. On the other hand, our CF for the APM cluster sample has the correlation length significantly larger than those reported by Dalton et al. (1992, 1994a) and Croft et al. (1997), in particular for richer subsamples. We have not been able to find the reason for this discrepancy (Park & Lee 1998).

Sutherland (1988) and Efstathiou et al. (1992) have suggested that projection effects of the Abell cluster catalog caused an artificial radial correlation, which could lead to an overestimation of CF relative to the APM sample. They have attempted to correct the Abell sample for the projection effects, and reported that the correlation length was reduced to $r_0 \simeq 14\ h^{-1}$Mpc. However, several other studies have pointed out that the uncertainties inherent in this correction are very large, and that the correction may have been overdone (Nichol et al. 1992). Miller et al. (1999) found that the excess radial correlation is not artificial but due to two elongated superclusters, Ursa Majoris and Corona Borealis. Postman et al. (1992) have estimated the magnitude of the projection contamination and have found that only 17 clusters (5%) in their 351 Abell clusters may be contaminated. They also have measured the effects of these contaminated clusters on the CF and have found that the amplitude and slope of the CF do not change significantly. Our study indicates that the APM team's CFs of the APM clusters are underestimated for richer subsamples, and the CF of the Abell sample is consistent with that of the APM sample at similar richness.

We have detected a rather strong dependence of the clustering amplitude on the richness of clusters. The correlation length r_0 versus mean separation d_c relation we have found from the Abell and the APM samples, $r_0 = 0.40 d_c + 3.2$, has a slope equal to that found by Bahcall & West (1992), but has a finite correlation length at zero separation. We think this is more reasonable because the correlation length should be finite at the continuum limit, namely for the matter field.

By using the X-ray selected clusters one can greatly overcome the projection effects which might have affected the optical cluster samples selected from the distribution of galaxies projected on the sky. Although the XBACs clusters are not genuine X-ray selected clusters and have incompleteness due to the missing poor ACO clusters, they are indeed three-dimensionally bound systems and the sampling is nearly free from false detection or overestimation of richness. The BCS clusters are selected purely from their X-ray properties, and also have negligible projection effects. However, since

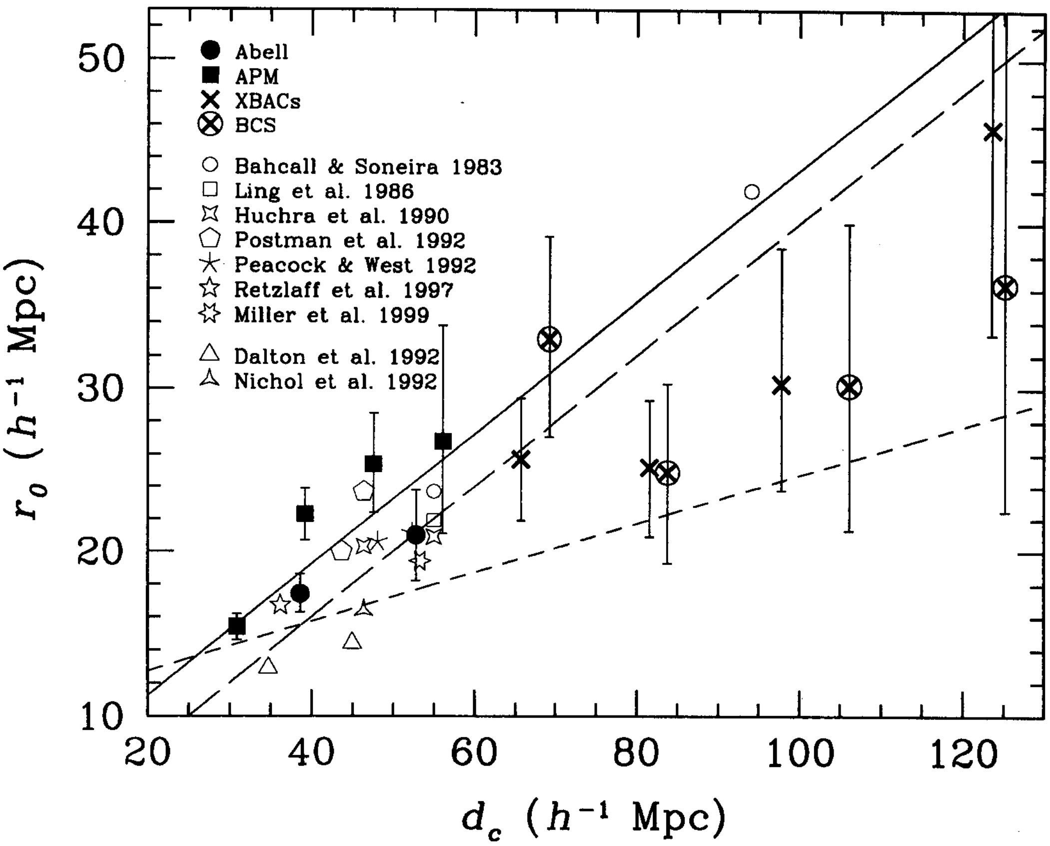

Fig. 4.— Dependence of the correlation length r_0 on the richness or the mean intercluster separation d_c. The solid line is $r_0 = 0.40d_c + 3.2$, which is a best fit to our results for the Abell and the APM samples. The long dashed line is the relation reported by Bahcall & West (1992), and the short dashed line is by Croft et al. (1997).

the number of X-ray clusters is not yet large enough to make a sufficiently large volume-limited sample, the CFs measured from the XBACs and the BCS clusters have large uncertainties compared to those of optical clusters. The clustering strength of these X-ray clusters is in general lower than what is expected from the r_0 versus d_c relation of optical clusters. This is probably due to the fact that the X-ray luminosity of X-ray clusters is not an excellent measure of the cluster richness or mass. The X-ray luminosity of a cluster depends on the environment and activity as well as its dynamical mass. In fact, Figure 4 of David, Forman & Jones (1999) shows that the clusters very bright in X-ray belong to various richness classes. Gilbank, Bower, & Castander (2001) also found that X-ray selection misses some significant rich clusters while optical selection can detect all X-ray clusters. However, there is a trend of the clustering strength of the X-ray clusters increasing as the mean separation increases. Despite the large uncertainties, the correlation length of the X-ray clusters is higher in amplitude compared to Croft et al. (1997)'s expectation.

We conclude that the Abell and the APM clusters do have CFs statistically consistent with each other and show a similar richness dependence relation of the clustering strength. It should be again emphasized that our results have been obtained by applying the same method of analyzing the observational data and calculating the CF.

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