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TIDAL TAILS OF GLOBULAR CLUSTERS

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ABSTRACT

We present N-body simulations of globular clusters including gravitational field of the Galaxy, in order to study effects of tidal field systematically on the shape of outer parts of globular clusters using NBODY6. The Galaxy is assumed to be composed of central bulge and outer halo. We investigate the cluster of multi-mass models with a power-law initial mass function (IMF) starting with different initial masses, initial number of particles, different slopes of the IMF and different orbits of the cluster. We have examined the general evolution of the clusters, the shape of outer parts of the clusters, density profiles and the direction of tidal tails. The density profiles appear to become somewhat shallower just outside the tidal boundary consistent with some observed data. The position angle of the tidal tail depends on the location in the Galaxy as well as the direction of the motion of clusters. We found that the clusters become more elongated at the apogalacticon than at the perigalacticon. The tidal tails may be used to trace the orbital paths of globular clusters.

Key words: celestial mechanics, stellar dynamics — globular clusters: general — methods: numerical

I. INTRODUCTION

Globular clusters are one of the most suitable systems for studying of dynamical evolution in our Galaxy. Globular clusters are believed to have undergone substantial dynamical evolution because of long ages compared to the relaxation time scale. The dynamical evolution of globular clusters are affected by several different processes. Tidal interaction with the Galaxy and two-body relaxation are responsible for the evaporation of stars. Two-body relaxation engages in internal evolution and the Galactic tidal force affects both the structure and evolution of the outer regions. For these reasons, the course of dynamical evolution is determined by the interplay between the internal processes and the Galactic tidal interaction.

As a globular cluster moves through space, it will be subject to gravitational force of the Galaxy, in addition to the self-gravity generated by the stars in the cluster. The Galactic tidal field causes the clusters to be limited by finite radii called tidal radii (e.g., Lee 1990). Two-body relaxation drives stars beyond the tidal boundary, resulting continuous mass loss from globular clusters. By balancing the tidal force and the self gravity, the tidal radii can be expressed as a function of cluster mass and Galactic mass within R_G, where R_G is the distance of the cluster from the Galactic Center.

The cluster which moves along a circular orbit about Galactic Center experiences time-independent tidal force if the Galactic potential is spherical. On the other hand, cluster which has non-circular orbit is affected by time-dependent tidal force. There have been a substantial number of investigations of the effects of the steady tidal field on the cluster evolution (e.g., Hènon 1961, Lee & Ostriker 1986, Lee & Goodman 1995, Takahashi & Portgies Zwart 1998, Takahashi & Lee 2000).

When the duration of the perturbing tidal force is usually much shorter than the period of a star in the cluster, the star is affected by variation of abrupt tidal force at almost motionless state. Such a process is called tidal shock or gravitational shock.

Rapid variation of tidal force occurs when the cluster passes through the Galactic disk (disk shock) or bulge (bulge shock). The disk shock compresses the cluster while the bulge shock stretches. The tidal shock provides energy to the individual stars in the cluster. The general evolution and the mass evaporation are both accelerated by the shocks (e.g., Gnedin, Lee & Ostriker 1999). Kim & Oh (1999) and Vesperini & Heggie (1997) investigated the evolution of multi-mass models with a power-law initial mass function of globular clusters driven by relaxation, stellar evolution and disk shocking, and effects of the tidal field of the Galaxy using direct N-body simulation. Gnedin, Lee & Ostriker(1999) investigated the effects of tidal shocks on the evolution of globular clusters using the Fokker-Planck method. However, most of the above mentioned works are restricted to simple model of spherical geometry or approximate description of the stellar escape process.

The stars escape continuously due to the tidal force in the cluster. In addition, because equipotential surface is not spherical, the cluster is expected to have tidal tails. The long axis of the equipotential surface is aligned along the direction to the Galactic center. However, the elongation direction is not expected to coincide with the direction to the Galactic center be-

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cause of the orbital motion of the cluster.

There are a few researches about such a complex aspect. By carefully analyzing the stellar distribution around globular clusters, Grillmair et al. (1995) reported the presence of tidal tails. Leon, Meylan & Combes (2000) carried out a systematic study of tidal tails of 20 Galactic globular clusters using POSS plates. However, the tails examined in their study are not fully understood theoretically; the directions appear to have no clear correlation with the Galactic plane or center direction. This could mean that the direction of the tail is related to both the location of the cluster relative to the Galactic Center and the orbital phase.

The distribution of the Galactic gravitational field is found from the mass distribution of several components. Therefore, it is possible that the information of the cluster orbit can be obtained from the tidal tail direction, if we know how the direction of tidal tail is determined. In principle the radial velocity and proper motion are needed at the same time to determine the cluster orbit, but finding the proper motion is very difficult. It will be very useful if we can constrain the three-dimensional velocity vector using the tidal tail.

The cluster orbit is important in several aspects. First of all, the globular cluster is utilized in order to measure mass of the Galaxy, but the result varies greatly according to distribution of the orbit. Current estimate of Galactic mass is very uncertain because we do not have good information on the cluster orbits. The orbits of the globular cluster are also essential in understanding formation process of the Galaxy. If the clusters are formed in the early stage of the Galaxy formation, most of globular clusters will have radial orbits. If they are formed in the nearly equilibrium stage of the Galaxy, they will have predominantly tangential orbits. Furthermore, the orbit of the cluster is closely connected with the fate of the cluster. Clusters on radial orbits are very likely to pass through the central part of the Galaxy. Through such process, the clusters can be easily destroyed by bulge shocking. Therefore the distribution of the cluster orbit provides important information from the Galaxy formation up to now.

The purpose of this paper is to study how the tidal tails are related with the cluster orbits. We carry out extensive N-body simulations for stellar systems embedded in external potential. We examine 1) the shape of tidal tails of clusters moving on circular and elongated orbits, 2) the correlation between the direction of the tidal tail and the orbital phase, and 3) the density profiles beyond the tidal boundary in order to confirm recent findings of shallower density distribution outside the tidal boundary.

This paper is organized as follows. § II describes the method used in this study, together with model and parameters of Galaxy, and initial conditions of globular cluster. In § III, we present the results of simulations and provides detailed discussions. In § IV, we discuss relationship between the direction of the tidal tail and the orbital phase. Finally, the conclusions are summarized in § V.

II. METHOD AND MODEL

Dynamical evolution of many body systems can be studied by various methods. Direct integration of N-body equation of motion is conceptually very simple, but computationally very difficult. Even with the fastest computers available nowadays, the limit is thought to be $\lesssim 10^5$ stars.

There are several alternatives to direct N-body integration method. Fokker-Planck equation describes the diffusion of stars in energy and angular momentum space by two-body relaxation. Cohn (1979) has pioneered in applying this equation to stellar systems. Since then Fokker-Planck equation has been widely used to study various aspects of globular cluster evolution. Lee & Ostriker (1987) introduced the tidal boundary to the Fokker-Planck equation and confirmed earlier finding by Hènon (1961) that the life time of the single-mass cluster is $\sim 20t_{rh}$, where t_{rh} is the half-mass relaxation time. The effect of tidal field on the multi-mass cluster is further examined by Lee & Goodman (1995). Gnedin, Lee & Ostriker (1999) extended the Fokker-Planck equation to take into account tidal shocks by disks.

Other approximate methods to study dynamical evolution of globular clusters include gas dynamical and Monte Carlo method. More detailed description of these methods can be found in a monograph by Spitzer (1987).

The advantage of Fokker-Planck method is that we can study the evolution without much computing power. However, this method has many serious restrictions. First, the shape of the cluster is restricted to sphere (or oblate spheroid for rotating models; see Einsel & Spurzem 1999). Second, the external gravitational field cannot be taken into account. The previous works including tidal field assumed tidal boundary rather than external gravity. Finally, the gravitational shock that occurs on dynamical time scale cannot be properly accounted. Therefore we use the N-body method for our study.

(a) N-body Method

N-body calculation is most desirable method to study the cluster evolution within tidal field because of the complex geometry of the problem. However, we have to be careful in applying this method and interpreting the results because we cannot use realistic number of stars in the simulations. All the N-body simulations for our investigation have been carried out using NBODY6(Aarseth 1999) code. The code uses a direct summation method in computing the gravitational potential and adopts a Hermite integration scheme (Makino 1991) and Ahmad-Cohen neighbor scheme (Ahmad & Cohen 1973, Aarseth 1985). It also includes the formation of binaries via three-body processes and their interactions with other stars. Since we follow the evolution of the clusters until near complete disruption, binary heating is necessary in order to let the cluster evolve beyond the core collapse.

Neighbor scheme is based on the idea of separating the total force on a particle into irregular and regular components,

$$\mathbf{F} = \mathbf{F}_{irr} + \mathbf{F}_{reg}. \tag{1}$$

Each part is represented by the high-order force polynomial on its own time-step. The regular distant force due to a particle which exists outer region of given radius changes slowly, but the irregular neighbor force due to a particle of inside the radius change rapidly.

We calculate the motions of particles in inertial frame. So we do not to include centrifugal and coriolis force for the force calculation. We modify the external force parts of the code NBODY6 and the external force is included by regular force,

$$\mathbf{F}_{reg} = -\nabla\Phi_{Gal} \tag{2}$$

where Φ_{Gal} is the potential due to the Galaxy. The Galactic model is described below.

(b) Model of the Galaxy

The gravitational field of Galaxy is produced by various components. The main components are central bulge, disk, halo, and bar. Among these four components we have ignored the contribution from disk and the bar, since they affect only clusters very close to them. Our adopted model for the Galaxy has only halo and bulge components. We have used simple models for the potentials of these component following Lee et al. (1999).

The halo component, which gives rise to the flat rotation curve at large radii, is assumed to have a logarithmic potential,

$$\Phi(r)_{halo} = \frac{1}{2}{v_0}^2 \ln({R_c}^2 + r^2) + const, \tag{3}$$

where R_c is the core radius of halo and v_0 is the constant rotation velocity at large r.

The bulges of real galaxies can be usually represented by de Vaucouleurs' $R^{\frac{1}{4}}$ law, but this model is difficult to convert into gravitational potential. Therefore, for simplicity, we have assumed the Plummer model to allow a steep outward velocity increase from the Galactic center,

$$\Phi(r)_{bulge} = -\frac{GM_{bulge}}{\sqrt{r^2 + {r_c}^2}}, \tag{4}$$

where r_c is a parameter that controls the size of bulge.

The model parameters which are necessary for these analytical model are v_0, R_c, M_{bulge}, r_c. The unit of length in our calculation is chosen to be $R_{sc} = 10$ kpc. The total mass of the Galaxy within 10 kpc is assumed to be $M_{10} = 1.24\times10^{11}$ $M_\odot$ (Caldwell & Ostriker 1981). We have used the following units for velocity:

$$V_{sc} = (GM_{10}/R_{sc})^{1/2} \approx 230.7 \text{ km s}^{-1}.$$

In this unit, the rotation speed v_0 at the Galactocentric distance of 10 kpc is 0.92 (Caldwell & Ostriker 1981). Other adopted parameters are listed in Table 1.

Table 1. Model Parameters

M_{bulge}	R_c	r_c
$9.4 \times 10^9 M_\odot$	7 kpc	240 pc

(c) Initial Conditions of Globular Cluster

The greatest concern of our study is evolution of the cluster's outer region. The evolution of the cluster's outer region is known to be insensitive to the initial model through the study using Fokker-Planck method (e.g., Kim, Morris & Lee 1999). We have chosen the King model with $W_0 = 4$ as initial model, where W_0 is the scaled central potential, which determines the degree of central concentration (King 1966). The density profiles of all mass components are assumed to be the same.

We choose the initial mass spectrum as a simple power law

$$dN(m) \propto m^{-\alpha}\, dm,$$

where $dN(m)$ is the initial number of stars with masses between 1 and 150 $M_\odot$, and $\alpha = 2.35$ gives a Salpeter initial mass function. The mass range is much larger than the actual one in globular clusters, but we have chosen such a parameter set to get sufficient relaxation effect with a small number of stars because we do not have enough computing power to compute clusters with N comparable to the number of stars in real globular clusters at this moment. The unit of mass in our models is also unrealistically high: more realistic range of mass should be between 0.1 $\sim$ 1 $M_\odot$. We adopted such a high mass in order to have large total mass with small N. We omit the effects of stellar evolution for simplicity. We carried out simulation with various initial parameters in oder to check whether results depend on the initial parameters.

We have two types of models for the cluster orbits: circular and non-circular orbits. For models with circular orbits, we have varied the slope of IMF, total mass and Galactocentric distance. The parameters of these models are listed in Table 2. For models with non-circular orbits, we have examined various ellipticities (ε). The ellipticity is defined as

$$\varepsilon = \frac{R_a - R_p}{R_a + R_p} \tag{5}$$

Table 2. Parameters for Models with Circular Orbits

Model	α	M ($M_\odot$)	N	R_g kpc
C4	2.35	1×10^4	3135	4
C5	2.35	2×10^4	6270	5
C6	2	2×10^4	3968	6
C15	1.5	1×10^5	8163	15

R_g : Distance from the Galactic Center

Table 3. Parameters for Models with Non-circular Orbits

Model	α	M ($M_\odot$)	N	R_p kpc	R_a kpc	ε
E3a	2.35	2×10^4	6270	1.40	12.00	0.79
E3b	2	3×10^4	5952	1.40	12.00	0.79
E4	2	2×10^4	3968	2.80	10.00	0.56
E5a	2	2×10^4	3968	2.60	7.00	0.46
E5b	1.5	1×10^5	8163	2.60	7.00	0.46
E7	2	2×10^4	3968	1.70	3.50	0.35

R_p : Perigalacticon
R_a : Apogalacticon

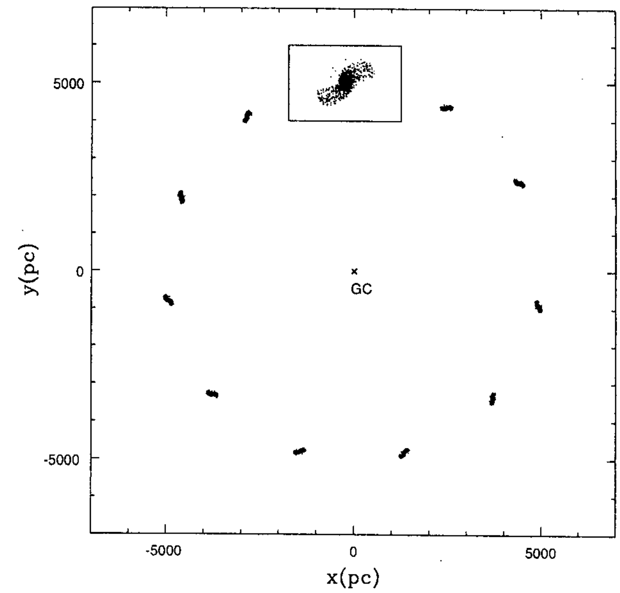

Fig. 1.— The shape of clusters for the model C5. The box shows the enlarged shape of the cluster at its location.

where R_a and R_p are apogalacticon and perigalacticon, respectively. The parameters for the models with non-circular orbits are listed in Table 3.

III. RESULTS FOR CIRCULAR ORBITS

The cluster moving along a circular orbit about Galactic Center experiences time-independent tidal force if there is no disk component. On the other hand, the cluster with extremely elliptical orbit is affected by time-dependent tidal force. We studied the shape and direction of the tails in the time-independent and time-dependent tidal field.

Each simulation was continued until the number of stars becomes as small as N $\sim$ 25. We have carried out a large number of simulation with various orbital parameters to cover wide range of different orbits of the cluster, as summarized in the previous chapter.

Tidal force restricts cluster's size and the size (tidal radius) can be expressed as follows,

$$R_t = \left(\frac{M}{2M_g}\right)^{1/3} R_g. \qquad (6)$$

where R_g is the Galactocentric radius and M_g is the enclosed Galactic mass within R_g. For a circular orbit, R_g remains to be a constant and the tidal radius does not change with time. In this section we present the effect of the time-independent tidal field.

The models C4, C5, C6 and C15 are the clusters which have circular orbits with an external potential field of host Galaxy. Fig. 1 shows the cluster shape along the orbit of model C5. The small box presents an enlarged image of the cluster at its location. The outer part clearly shows the spiral shaped tidal tail. It is evident that the tail is not aligned with the GC direction.

In oder to investigate the directions of tidal tails we have defined the position angle θ as the angle between long axis of the tail and the GC direction, as shown in Fig. 2. We attempted to find the position angle by fitting the shape of the tails to ellipses. However, since the tidal tails deviate significantly from simple ellipse, the fitting does not give reliable results for the elongation direction. Therefore we determined the long axis by eyes. The position angle of the circular orbit is found to be about 60° in all cases. That is, the models C4, C5, C6 and C15 have the same position angle ($\sim$ 60°) regardless of their mass, particle number, and galactocentric distance.

We use the contour of cluster to examine the shape of tidal tails. To obtain the contour, we smoothed each star with Gaussian window function weighted proportional to the mass of each star and calculated the densities on 100 $\times$ 100 cells (each cell has a size, 5 $\times$ 5 pc^2). The examples of resulting contours are shown in Fig. 3 for model C5. Also shown in this figure are the direction to the Galactic center (as solid arrows) and the direction of cluster's motion (as dotted arrows). In all cases, the position angle remains to be around 60°. So we can easily see that the angles between solid arrow and long axis of the tail are almost same.

There have been several measurements of the surface density profiles of globular clusters beyond the tidal boundary (Grillmair et al. 1995, Leon, Meylan

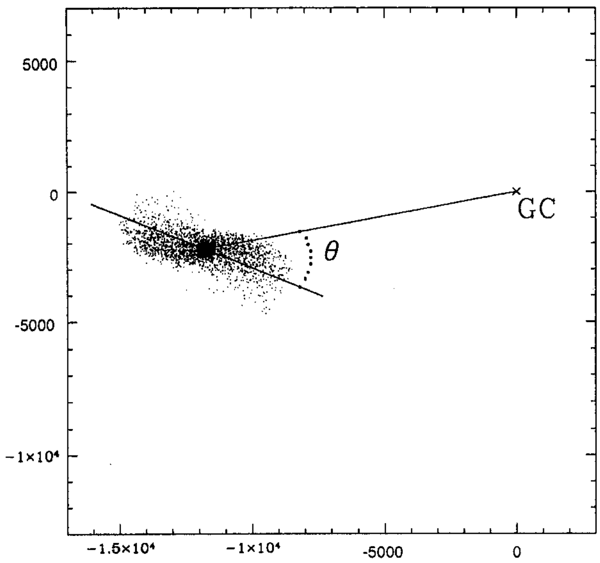

Fig. 2.— Definition of the position angle. This θ is the angle between the long axis of the tail and the GC direction. The long axis is determined by eyes.

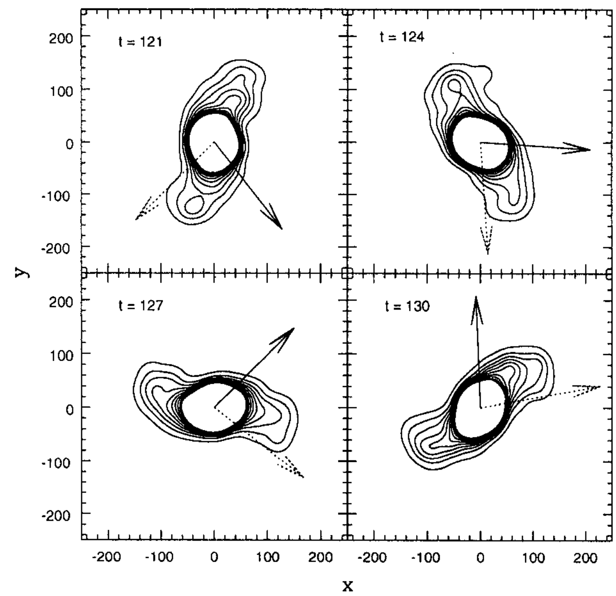

Fig. 3.— Contours of the cluster in four different epochs for the model C5. The arrows represent the directions of the Galactic Center (solid arrow) and the cluster orbital direction (dotted arrow).

& Combes 1999). They concluded that the slope of the surface density of some clusters appears to become shallower at $R > R_t$. We also attempted to determine surface density profiles of simulated clusters. The local density is calculated, along tail direction, as the average of four adjacent cells of contour data shown in Fig. 3 The result is displayed in Fig. 4. Although the data are very noisy, the surface density becomes flatter outside the tidal radius, consistent with observations.

IV. RESULTS FOR NON-CIRCULAR ORBITS

Because the potential of our Galaxy is not spherically symmetric and the clusters are not likely to follow along a circular orbit, the tidal field usually varies with time. If the duration of the perturbing tidal force is much less than the period of a star in the cluster, the time-dependent tidal field play a role similar to shock in hydrodynamics. The orbital periods of stars differ by large amount depending on the location of stars. The "shocking" is most effective for stars with long orbital periods. Those are stars located mostly at the outer parts. On the other hand, stars near the center have orbital periods shorter than the duration of tidal force variation. These stars are not much affected by the external field variation. Precise understanding of the effect of the tidal shock requires complex analysis of interplay between the field variation and internal orbital motion. Spitzer (1987) presented a thorough analysis assuming the harmonic motion for stars. Weinberg (1994) revised the theory of Spitzer for stars in the in-

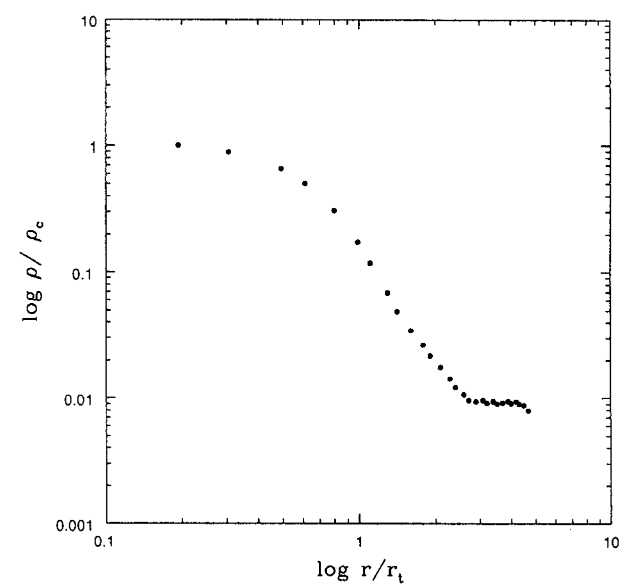

Fig. 4.— Surface density profile from N-body simulations at t = 130 (1.2 Gyr) in Fig. 3. The profile is obtained along the long axis.

ysis assuming the harmonic motion for stars. Weinberg (1994) revised the theory of Spitzer for stars in the inner parts.

Our N-body simulation provides more precise account of tidal shock in principle. In this work, however, we are not concerned about the energy input to stars in the cluster by tidal shock. Instead, our interests are rather concentrated to the general shape of the tidal tail. We will not discuss the effects of tidal shocks on the internal dynamics of stars any further.

In this section we present the effect of the time-dependent tidal field through the evolution of cluster whose orbit is non-circular.

(a) Shapes of the Tidal Tails

Fig. 5 shows the evolution of the cluster (model E3a) having rather large ellipticity (ε = 0.79). The solid arrow and triangle on the small box represent starting location. The cluster shape which is spherical at the beginning changes into the elongated shape because of the formation of the tail. Since then, the cluster shape changes almost periodically. The models E3a and E3b move on the same orbits but have different total mass and slope of IMF. There is virtually no difference in the shapes of clusters. We conclude that the cluster's shape depends mostly on cluster's orbit. More detailed shapes of the tidal tails are shown in Fig. 6 for one orbital period. In this figure we found that the cluster has a longer tail near apogalacticon and a shorter tail near bulge. Moreover the tidal tails of the cluster align more vertically about the orbital path of the cluster when the cluster moves from perigalacticon to apogalacticon and the tails align more horizontally about that when the cluster moves from apogalacticon to perigalacticon in all orbital period.

Therefore we observe anti-correlation between the strength of the tidal force and the length of the tails. Similar to the case of circular orbits, we use the contour of the cluster to examine the change of tail's shape and direction when the cluster undergoes the bulge shocking. Such a tendency is more clearly shown in Fig. 7.

The contours (Fig. 7) of the model E3a, which has very eccentric orbit, show the changes of the tidal tail direction and shape when the cluster passes close to the Galactic Central region. The number in each box represents the epoch number shown in Fig. 5. The different arrows represent the directions of the Galactic Center (solid arrow) and the cluster orbital path (dotted arrow). The cluster is located close to the perigalacticon at epochs 2 and 10, so the cluster experiences the bulge shocking. The tails of cluster undergoing the bulge shocking are relatively short. After the tails of cluster have undergone bulge shocking they become relatively long. The cluster of second row in Fig. 7 shows that the tidal tail around globular cluster is extended when the cluster is more distant from Galactic Center. The cluster of first and third row in Fig. 7 shows that the tidal tail becomes shorter when the cluster is close

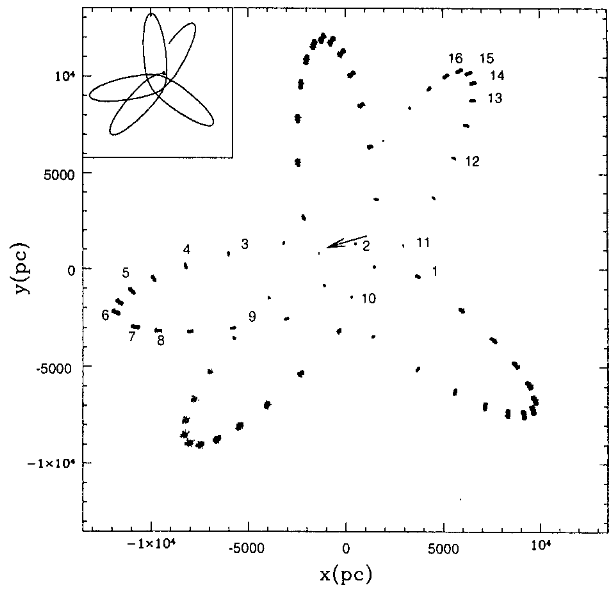

Fig. 5.— The shape of the cluster at various locations in orbit for the model E3a which has the largest ellipticity of ε = 0.79. The solid arrow and triangle on the small box indicate the initial position of the simulation. The orbital epochs are identified by the numbers shown close to the location of the cluster.

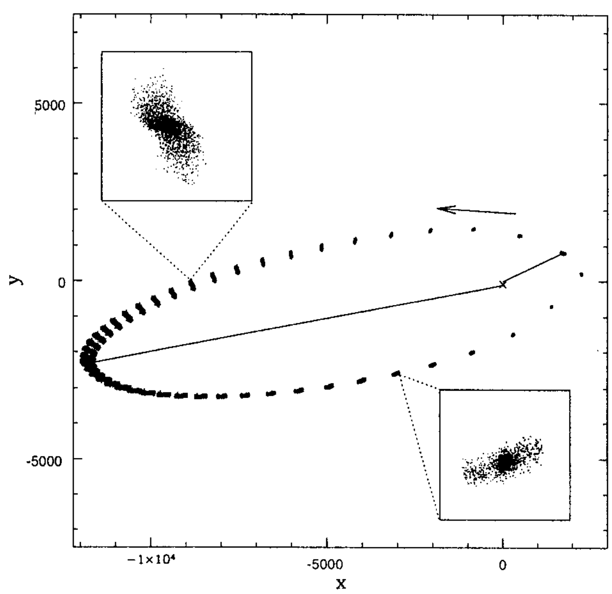

Fig. 6.— The variation of the cluster shape during one azimuthal period. Note that the tidal tail has large position angle when the cluster moves from perigalacticon to apogalacticon and small position angle when it moves the opposite direction. The boxes are convexes of the cluster those locations.

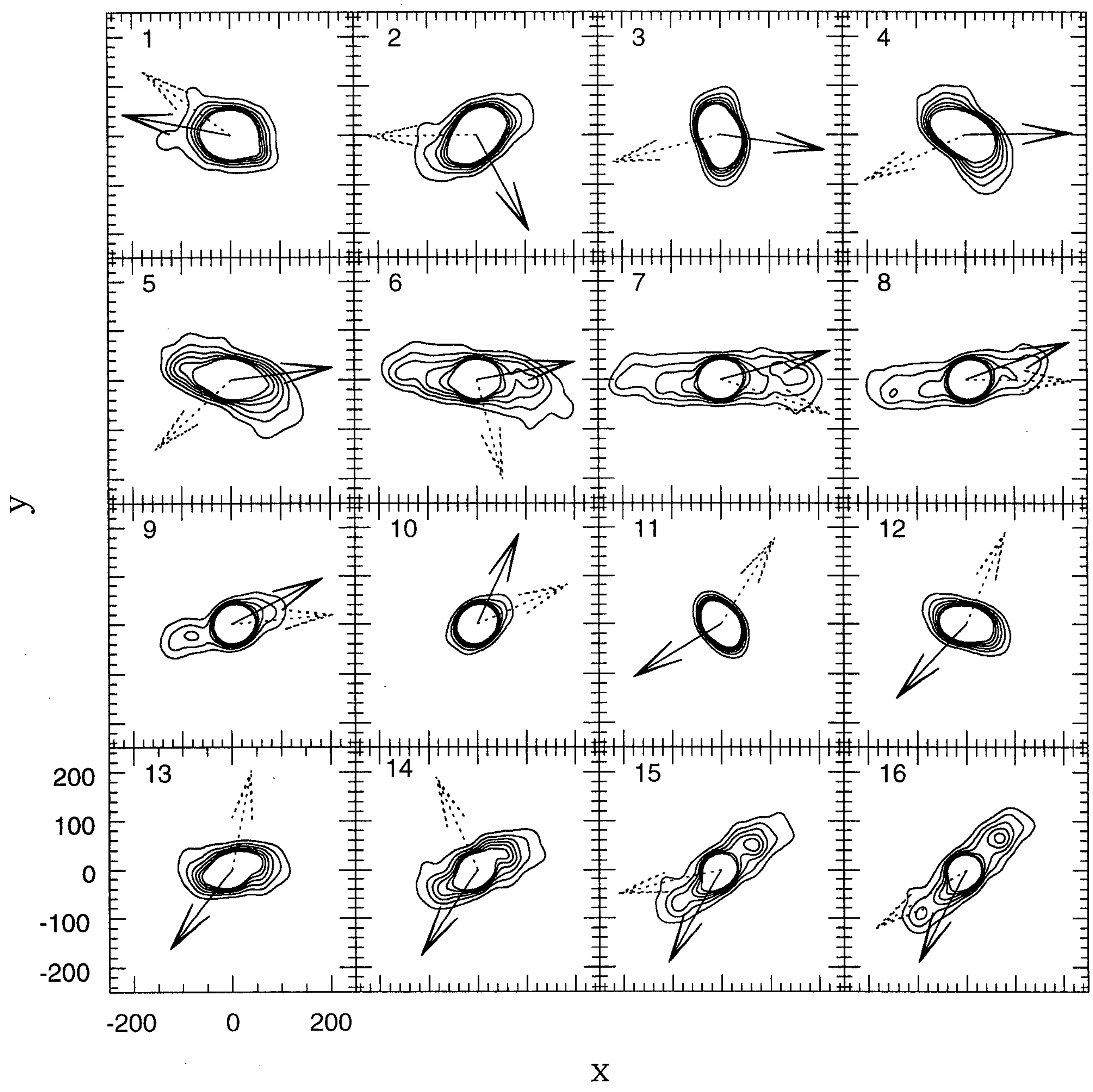

Fig. 7.— Contours of the cluster in sixteen different epochs for the model E3a. The arrows represent the directions of the Galactic Center (solid arrow) and the cluster orbital direction (dotted arrow).

to the bulge. The tidal tail direction changes quickly when the cluster passes the bulge, but it changes slowly when the cluster is located far away from the bulge. The bulge shocking causes changes of the tail shape and its direction.

The fact that the length of the tidal tail is shorter when the cluster is near perigalacticon where the tidal force is strongest needs some explanation. The stars usually do not respond instantaneously to the external force. Since the stars near the tidal boundary have orbital periods similar to the cluster's orbital time around the Galaxy, the effect of the strong tidal force appears when the tidal force is weakest.

(b) Angular Momentum

The clusters have no angular momentum at the beginning, but they could acquire it through tidal interactions with the Galaxy. We have shown the specific angular momentum (angular momentum per unit mass) of cluster as a function of time for E3a model in Fig. 8. The cluster acquires angular momentum gradually, although angular momentum is lost just before the perigalactic passages which are shown as arrows. The gain and loss of the angular momentum is related to the fact that there is phase lag between the tidal tail and the GC direction. The tidal torque acting on the cluster causes the angular momentum to the cluster. The direction of tidal torque becomes opposite while the cluster approaches to the perigalacticon, but the specific angular momentum increases in the long run. The actual value of the angular momentum is very small. We expect that the cluster rotates in the same direction as the orbital motion. In the final phase of the evolution, the specific angular momentum tends to increase rather sharply. Oh & Lin (1992), however, found that the the cluser rotates in retrograde direction due to the fact that the stars on prograde orbit has higher escape rate. More careful study on the angular momentum will be necessary.

Some of the observed globular clusters are known to be rotating. Even clusters with no kinematic data to support the rotation, some amount of rotation is implied by the flattened shape. It would be interesting to examine the rotation axis relative to the orbital plane. If the clusters rotation is induced by tidal torque, the rotation axis must be perpendicular to the orbital plane. If there is no correlation between the rotation axis and orbital plane, the rotation we observe today should be remnant of initial rotation. The rotation is an important factor in determining the time scale of dynamical evolution of clusters (e.g., Kim et al. 2002).

(c) Surface Density Profile

Leon, Meylan & Combes (2000) showed that the gradient of surface density becomes shallower outside the tidal boundary from observed cluster NGC 288 and NGC 6254 which have recently undergone a gravitational shock. In oder to verify our simulation data, we plotted the surface density profile of the cluster which has undergone a gravitational shock.

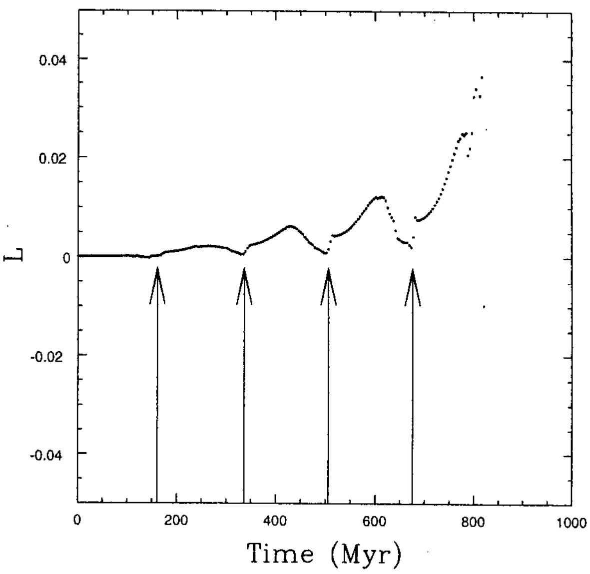

Fig. 8.— Specific angular momentum of model E3a as a function of the time. The arrows represent the perigalacticon.

Unlike the tidal tail of clusters on circular orbit, the shape of tidal tails varies depending on the orbital phase for clusters on non-circular orbits. Fig. 9a shows density profile of the cluster at epoch 2 of Fig. 7. The cluster is located near the perigalacticon and the cluster has relatively short tail. Therefore the figure of density profile has no shallow gradient outside the tidal boundary. Fig. 9b shows density profile of the cluster (epoch 6 in Fig. 7) near apogalacticon. Contrary to the density profile of the Fig. 9a, the surface density profile of the Fig. 9b shows a shallow gradient just outside the tidal boundary like that of the observed clusters such as NGC 288 and NGC 6254. Lee et al. (2002) recently analyzed the density profile of M92 using deep CCD image obtained by CFHT 3.5 m telescope. This cluster does not appear to show shallower density profile outside the tidal radius. Our simulation clearly shows that the density profile depends on the orbital phase. M92 probably lies near the perigalacticon, judging from the density profile.

(d) Position Angle (θ)

As defined in § III(a), the position angle θ of the tidal tail is the angle between the direction of the tail and the direction toward GC. As seen in § IV(a), θ appears to be sensitively dependent on the orbital phase. We now examine the behavior of θ more carefully.

The variation of the tail directions shows a regular

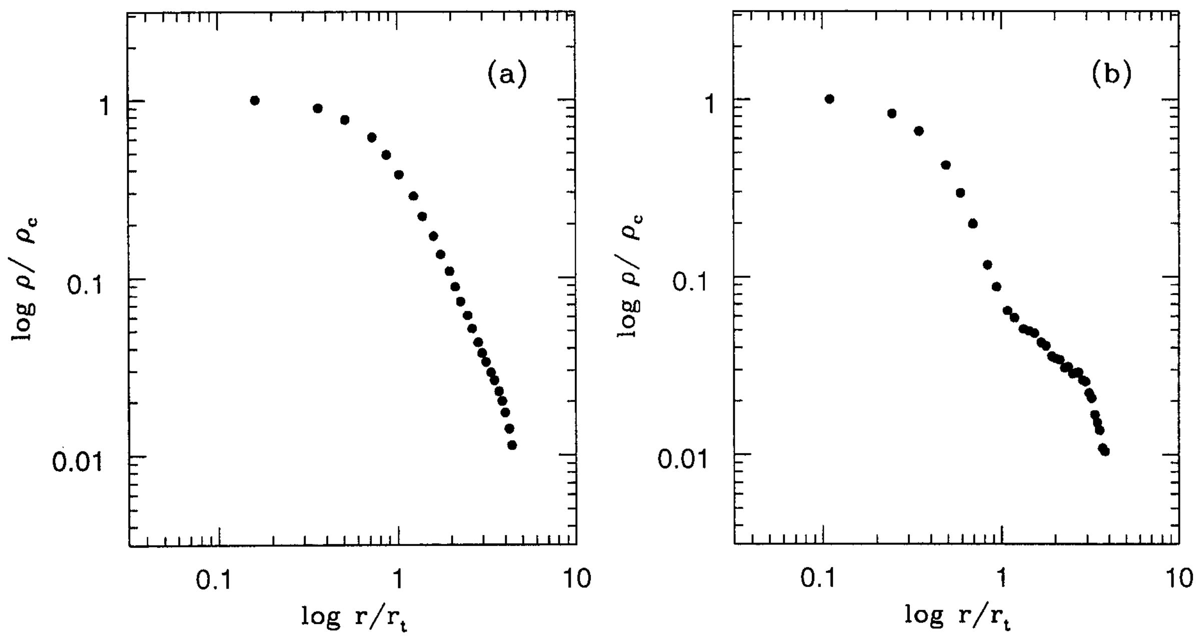

Fig. 9.— Surface density profiles (a) ID number 2 cluster and (b) ID number 6 cluster in Fig. 7 from N-body simulation data. The profile is obtained along the long axis.

pattern in all non-circular models during orbital period of each cluster. We have plotted the position angles along the galactocentric distance in a period of orbit in Fig. 10 for models E3a and E5a. Model E3a has the largest ellipticity and model E7 has the smallest ellipticity among them. Fig. 10a and Fig. 10b show the position angles of the models E3a (Fig. 6) and E5a at each galactocentric distance. The arrows represent orbital direction. The crosses in Fig. 10 represent the position angles of the cluster during the motion from perigalacticon to apogalacticon, and the circles represent the position angles of the cluster moving apposite direction. Generally speaking, the position angle is large during the phase of perigalacticon to apogalacticon and small during the other half-period.

In Fig. 11 we show the position angles as a function of R_G, normalized by each distance from pericenter to apocenter of model E3a, E5a and E7, because this makes it easier to see the difference among the position angles of these orbits. Here zero indicates the perigalacticon and 1 indicates the apogalacticon. The position angle of E3a ($\varepsilon = 0.79$) varies from 15° to 90° while that of E7 ($\varepsilon = 0.35$) varies from 40° to 70°. Therefore we could estimate the orbital phase of the cluster from the position angle. For example, if a cluster has small position angle (about 10° $\sim$ 20°), we may conclude that the cluster's orbit is very eccentric and the cluster is heading for perigalacticon. The presence of tidal tails give us a potentially useful tool for determining cluster orbital phase.

V. SUMMARY AND CONCLUSION

The tidal tails are produced by the tidal field of the Galaxy and the exact shape of tail is determined by the gravitational field of the Galaxy and cluster orbits. In this work we have carried out morphological investigation of the tidal tails using the results obtained by N-body calculation. For simplicity, we assumed spherically symmetric gravitational field of the Galaxy. We have assumed that the Galactic potential is generated by bulge and halo only.

We studied the tidal tails of clusters whose orbit is circular or non-circular. We have found that the tidal tails of the clusters have a definite direction and regular shape regardless of galactocentric distance, mass of the cluster, and initial particle number for clusters moving on circular orbits.

On the other hand, the shapes and tail directions of the clusters whose orbit is non-circular vary significantly with orbital phase. The tidal tails become shorter at near perigalacticon than near the apogalacticon, because the effect of strong tidal field appears after the typical orbital periods of stars at the outer parts of the cluster. This phenomenon repeat during all orbital period.

We have examined the surface density of the cluster which has undergone a gravitational shock. We found that the surface density profile outside the tidal radius becomes shallower when the cluster is located near the apogalacticon. Leon, Meylan & Combes (2000)

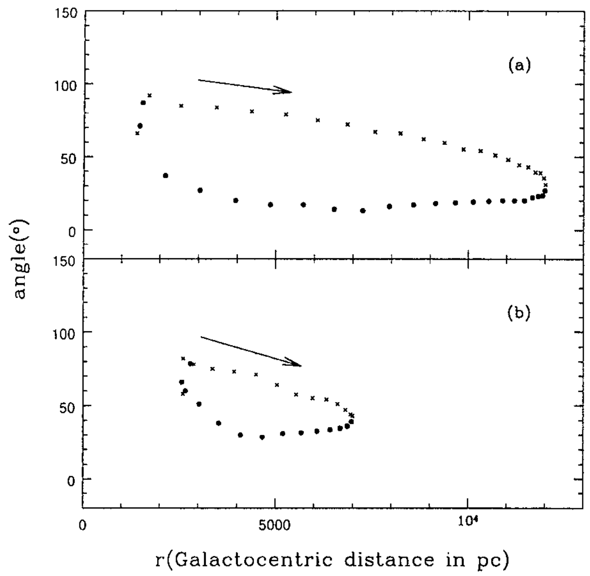

Fig. 10.— Position angles as a function of galactocentric distance for the models (a) E3a and (b) E5a. The crosses represent the position angles of the cluster which is heading for apogalacticon from perigalacticon and the circles represent the angles of the cluster which is heading for perigalacticon from apogalacticon. The arrows represent the cluster orbital direction.

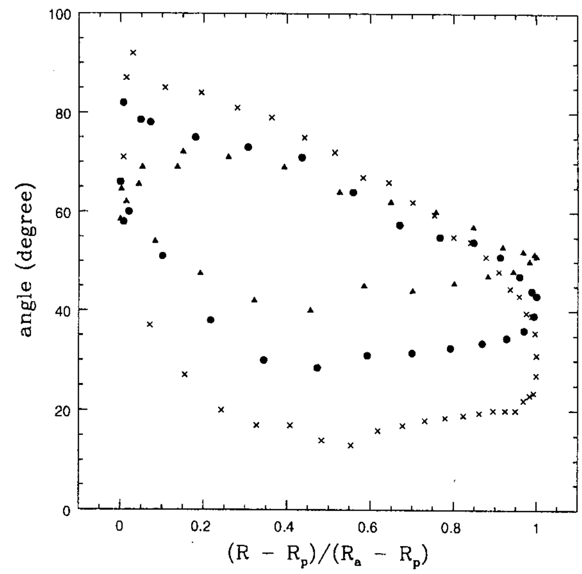

Fig. 11.— Position angles as a function of R_G, normalized by each distance from pericenter to apocenter of model E3a, E5a and E7. The crosses represent the angle of model E3a, the circles represent the angle of model E5a, and the triangles represent the angle of model E7. Here zero indicates the perigalacticon and 1 indicates the apogalacticon. The R_p and R_a mean the perigalacticon and apogalacticon distances, respectively

showed that the observed cluster NGC 288 and NGC 6254 which have recently undergone a gravitational shock have shallow density profiles outside the tidal radius. Probably these clusters are now located near the apogalacticon. On the other hand, clusters showing no significant flattening if the surface density profile may be located near the perigalacticon. M92 may be such an example.

The cluster acquires angular momentum gradually while the cluster passes the perigalacticon, and the cluster loses it during the apogalactic passage. Therefore the direction of tidal torque changes around apogalacticon. The angular momentum which increases and decreases repeatedly augments in the end. We anticipate that the cluster rotate in the same direction as the orbital motion if the intrinsic angular momentum is not large enough.

We examine the correlation between orbital phases and position angles. The tidal tails of clusters with circular orbits always maintain definite position angle ($\sim 60°$) in all cases. But the position angles of clusters with non-circular orbits vary according to orbital path and phase. The small position angle occurs that the cluster heads for perigalacticon and moves along non-circular orbit with large ellipticity. On the other hand, the large position angle occurs that the cluster heads for apogalacticon and moves along elliptical orbit with small ellipticity. The smallest value of the position angles appears in the elliptical orbit with the largest ellipticity.

We have omitted the Galactic disk component although it also provides strong shock to the cluster. The direction of the tidal tail would become different from our simple models. We will study the effects of disk shock in the forthcoming papers.

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